JOURNAL OF APPLIED SCIENCES RESEARCH

JOURNAL home page: http://www.aensiweb.com/jasr.html 2013 December; 9(10): pages 6414-6418.

Published Online: 15 January 2014 **Research Article**

A Gradation ditopological texture space

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Received: 12 November 2013; **Revised:** 14 December, 2013; **Accepted:** 20 December 2013.

ABSTRACT

The concept of a gradation ditopology on a texture space is introduced as a generalization of a ditopological space on a texture space, a descending family of ditopological spaces on the same texture space are constructed. Many properties for a gradation ditopological space are studied, also the concepts of dineighbourhood system, the subspace of the gradation ditopological space, and some separation axioms for this spaces are introduced.

Key words: Gradation of openness, texture space, ditopological space, separation axioms.

INTRODUCTION

The theory of texture spaces was introduced by L.M. Brown [9] under the name "fuzzy structure". The basic concept of texture spaces and many results are appear in several articles [2,3,5,6]. In Chattapadhyay *et al.,* [11] and Chattapadhyay *et al.,* [12] introduced concept of a gradation of openness which is a generalization of a fuzzy topology on a non - empty set. In this paper, the concepts of a gradation ditopological structure is introduced and many of its properties are studied

2 Preliminaries:

Let S be a non-empty set. A texture Φ on S is a point separated, complete completely distributive sub-lattice of P(S) with respect to inclusion, which contains S, φ and for which arbitrary meets coincides with intersections and finite joins coincides with unions.

If Φ is a texturing of S, the pair (S, Φ) is called a texture space. In general a texturing of S need not be closed under set complementation, but it is called a complemented texture with complementation λ if there exists a mapping $\lambda: \Phi \to \Phi$
Satisfies $\lambda(\lambda(A)) = A \ \forall A \in \Phi$ and if $A \subseteq B$, $\lambda(B) \subseteq \lambda(A)$

In any texture space (S, Φ), any for every s ∈ S, two sets are defined and have an Important role in the study ditopological spaces P_s and Q_s. The set P_s is defined by
$P_s = \cap \{ A \in \Phi: s \in A \}$ and is therefore the smallest elements of Φ containing s and $Q_s = V\{ A \in \Phi: s \notin A\} = V\{ Pu \in \Phi: u \in S, s \notin Pu\}$ Also, for any subset of A of S, the core of A is defined by $A^b = \cap \{ U\{ A_{j:}\ j \in J\}: A = U\{A_j: j \in J\}\}$. The relation between these concepts in an: arbitrary texture space are given in the following theorem.

Theorem 2-1:

[3,5,6,8]: Let (S, Φ) be a texture space, then:
(1) $s \notin A \Longrightarrow A \subseteq Q_s \Longrightarrow s \in A^b \ \forall s \in S, A \in \Phi$
(2) $A^b = \{ s: A \not\subset Q_s, s \in S \}, \forall A \in \Phi$
(3) For any collection $\{ A_i \in \Phi: i \in J\}$, $(V_i A_i)^b = U_i A_i^{\ b}$
(4) A is the smallest element of Φ containing A^b, $\forall A \in \Phi$
(5) For A, B ∈ Φ, if $A \not\subset B$, then $\exists\ s \in S$ s. t $A \not\subset Q_s$ and $P_s \not\subset B$
(6) $A = \cap \{ Q_s: P_s \not\subset A \} = U\{ P_s: A \not\subset Q_s \}, \forall A \in \Phi$

Definition 2-2:

[3,6,8]: Let (S, Φ) be a texture space. The pair (τ, σ) is called a ditopology on (S, Φ) if:
I) τ satisfies the following axioms:
i) φ, S ∈ τ
ii) $G_1, G_2 \in \tau \Longrightarrow G_1 \cap G_2 \in \tau$
iii) $G_\alpha \in \tau \ \forall \alpha \in \Gamma \Longrightarrow U_\alpha G_\alpha \in \tau$

II) σ satisfies the following axioms:
i) φ, S ∈ σ
ii) $F_1, F_2 \in \sigma \Longrightarrow F_1 U F_2 \in \sigma$
iii) $F_\alpha \in \sigma \ \forall \alpha \in \Gamma \Longrightarrow \cap_\alpha F_\alpha \in \sigma$

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Definition 2-3:

[11,12]: Let $X \neq \varphi$, a mapping $\tau: I^X \longrightarrow I$ is called a gradation of openness on X if it satisfies the conditions:
(o1) $\tau(o) = 1 = \tau(1)$
(o2) $\tau(\mu_1) \wedge \tau(\mu_2) \leq \tau(\mu_1 \wedge \mu_2)$
(o3) $\tau(V_i\ \mu_i) \geq \wedge_i\ \tau(\mu_i)$, $\forall\ i \in \Gamma$

Definition 2-4:

[11,12]: Let $X \neq \varphi$, a mapping $\sigma: I^X \longrightarrow I$ is called a gradation of closeness on X if it satisfies the conditions:
(o1) $\sigma(o) = 1 = \sigma(1)$
(o2) $\sigma(\mu_1) \wedge \sigma(\mu_2) \leq \sigma(\mu_1 \vee \mu_2)$
(o3) $\sigma(\wedge_i\ \mu_i) \geq \wedge_i\ \sigma(\mu_i)$, $\forall\ i \in \Gamma$
Where I = [0, 1] and I^X is the family of all fuzzy sets on X.

Definition 2-5:

[11,12]: A family $\{G_i: i \in \Gamma\}$ is called a descending family if it satisfies the condition $i \leq j \Longrightarrow G_j \subseteq G_i$

3 Gradation of ditopological space:

In the following, the concept of a gradation ditopology, many of its properties and some relations between the ditopological spaces are studies.

Definition 3-1:

Let (S, Φ) be a texture space, the pair of mappings (τ, σ) where, $\tau, \sigma: \Phi \longrightarrow I$ is called a gradation ditopology on the space (S, Φ) if it satisfies the conditions:
(GD1) $\tau(\varphi) = 1 = \tau(S)$
(GD2) $\tau(A_1) \wedge \tau(A_2) \leq \tau(A_1 \wedge A_2)$
(GD3) $\tau(V_i\ A_i) \geq \wedge_i\ \tau(A_i)$, $\forall\ i \in \Gamma$
(GD4) $\sigma(\varphi) = 1 = \sigma(S)$
(GD5) $\sigma(A_1) \wedge \sigma(A_2) \leq \sigma(A_1 V A_2)$
(GD6) $\sigma(\wedge_i\ A_i) \geq \wedge_i\ \sigma(A_i)$, $\forall\ i \in \Gamma$
The pair (S, Φ, τ, σ) is called a gradation ditopological texture space.

Proposition 3-2:

Let (τ, σ) be a gradation ditopology on the texture space (S, Φ)

Then, the pair (τ_r, σ_t) is ditopology on the space (S, Φ), $\forall$ r, t $\in$ (0, 1], where
$\tau_r = \{A \in \Phi: \tau(A) \geq r\}$, $\sigma_t = \{A \in \Phi: \sigma(A) \geq t\}$

Proof:
(1) Since $\tau(\varphi) = 1 = \sigma(\varphi) \Longrightarrow \varphi \in \tau_r, \varphi \in \sigma_t$
Also, $\tau(S) = 1 = \sigma(S) \Longrightarrow S \in \tau_r, S \in \sigma_t$
(2) Let A, B $\in \tau_r \Longrightarrow \tau(A), \tau(B) \geq r \Longrightarrow$
$\tau(A \wedge B) \geq \tau(A) \wedge \tau(B) \geq r \Longrightarrow A \wedge B \in \tau_r$
Similarly, Let A, B $\in \sigma_t \Longrightarrow \sigma(A), \sigma(B) \geq t \Longrightarrow$
$\sigma(A \vee B) \geq \sigma(A) \wedge \sigma(B) \geq t \Longrightarrow A \vee B \in \sigma_t$
(3) Let $A_i \in \tau_r$, $\forall\ i \in \Gamma$, then $\tau(A_i) \geq r \Longrightarrow \wedge_i\ \tau(A_i) \geq r$
$\Longrightarrow \tau(V_i\ A_i) \geq \wedge_i\ \tau(A_i) \geq r \Longrightarrow V_i\ A_i \in \tau_r$
Similarly, if $F_i \in \sigma_t$, $\forall\ i \in \Gamma$, then $\sigma(F_i) \geq t$
$\Longrightarrow \sigma(\wedge_i\ F_i) \geq \wedge_i\ \sigma(F_i) \geq t$
$\Longrightarrow \wedge_i\ F_i \in \sigma_t$
Then, the pair (τ_r, σ_t) is a ditopology on the space(S, Φ), $\forall$ r, t $\in$ (0, 1] and it is called the (r, t) - level of the space (S, Φ, τ, σ).

Remark 3-3:

In general case the gradation of openness τ and the gradation of closeness σ on the space (S, Φ) are independent. If the following relation between τ and σ, $\tau(A) = \sigma(A^c)\ \forall\ A \in \Phi$ holds then, the space (S, Φ, τ, σ) is called a complemented gradation ditopological space.

Proposition 3-4:

Let (τ, σ) be a gradation ditopology on the texture space (S, Φ)
If $(r_1, t_1) \leq (r_2, t_2)$ then $\tau_{r2} \subseteq \tau_{r1}$ and $\sigma_{t2} \subseteq \sigma_{t1}$

Proof:

It is Clearly.
The above proposition show that, the collection $\{(\tau_r, \sigma_t): r, t \in (0, 1]\}$ is a descending family for any space (S, Φ, τ, σ).

Proposition 3-5:

If $\{(\tau_i, \sigma_j): i, j \in (0, 1]\}$ is a descending family of ditopologies on the space (S, Φ). which satisfies the conditions $\tau_k = \cap_{I<k} \tau_i$ and $\sigma_k = \cap_{j<k} \sigma_j$, then the pair (τ, σ) is a gradation of ditopology on (S, Φ), where $\tau(A) = V\{i: A \in \tau_i\}$ and $\sigma(A) = V\{j: A \in \sigma_j\}$

Proof:
(1) Since $\varphi, S \in \tau_i\ \forall$ i Then, $\tau(S) = 1 = \tau(\varphi)$
(2) Let $A_1, A_2 \in \Phi$ and suppose that $\tau(A_1 \wedge A_2) < \tau(A_1) \wedge \tau(A_2)$, then there exists r $\in$ (0, 1] such that $\tau(A_1 \wedge A_2) < r < \tau(A_1) \wedge \tau(A_2)$ implies that $A_1, A_2 \in \tau_r$, consequently $A_1 \wedge A_2 \in \tau_r$, but $\tau(A_1 \wedge A_2) < r$ which implies $A_1 \wedge A_2 \notin \tau_r$ which contradiction.
(3) Let $\{A_i: i \in (0, 1]\}$ be a family of sets of the space (S, Φ) and suppose that $\tau(V_i\ A_i) < \wedge_i\ \tau(A_i) \leq \tau(A_i)\ \forall\ i \in (0, 1]$ then, there exist r $\in$ (0, 1] such that $\tau(V_i\ A_i) < r < \tau(A_i)$ which implies that $A_i \in \tau_r\ \forall$ i, so $V_i\ A_i \in \tau_r$. Consequently $r < \tau(V_i\ A_i)$ which

contradicts that $\tau(V_i A_i) < r$, then $\wedge_i \tau(A_i) \leq \tau(V_i A_i)$

Hence, τ is a gradation of openness on the space (S, Φ). Similarly we can show that σ is a gradation of closeness. Then, the pair (τ, σ) is a gradation ditopology on (S, Φ) generated by the family $\{ (\tau_i, \sigma_j): i, j \in (0, 1] \}$.

4 Subspaces of a gradation ditopological space:

Since the restriction of a texture structure on a non -empty subset is also a texture structure, then we have

Lemma 4 -1:

Let (S, Φ) be a texture space and let A be a non –empty subset of S then, the family $\Phi_A = \{ H \cap A: H \in \Phi \}$ is a texture on the set A and the pair (A, Φ_A) is called a texture subspace of (S, Φ).

Definition 4-2:

Let (τ, ρ) be a gradation di-topological structure on a texture space (S, Φ) and, let A be a non-empty subset of S. Define the mappings $\tau_A: \Phi_A \longrightarrow I$, $\sigma_A: \Phi_A \longrightarrow I$ as follows:

$\tau_A(G) = V \{ \tau(H): H \cap A = G, H \in \Phi \}$

$\sigma_A(F) = V \{ \sigma(H): H \cap A = F, H \in \Phi \}$

Proposition 4-3:

Let (τ, σ) be a gradation ditopological structure on a texture space (S, Φ), and let $A \neq \varphi$, $A \subseteq S$. Then, the pair (τ_A, σ_A) is a gradation ditopological structure on the texture subspace (A, Φ_A).

Proof:

(o 1) Since $\tau(\varphi) = 1$ implies $\tau_A(\varphi) = 1$. Also, $\tau(S) = 1$ implies $\tau_A(A) = 1$

(o 2) $\tau_A(G_1 \cap G_2) = V \{ \tau(H): H \cap A = G_1 \cap G_2, H \in \Phi \}$

Suppose that, $\tau_A(G_1 \cap G_2) < \tau_A(G_1) \wedge \tau_A(G_2) \Longrightarrow \exists\, r \in (0, 1)$ such that

$\tau_A(G_1 \cap G_2) < r < \tau_A(G_1) \wedge \tau_A(G_2)$

$\Longrightarrow \tau_A(G_1) > r$ and $\tau_A(G_2) > r \Longrightarrow \exists\, H_1, H_2 \in \Phi$ s. t $H_1 \cap A = G_1$,

$H_2 \cap A = G_2$, $\tau(H_1) > r$ and $\tau(H_2) > r$

$\Longrightarrow \tau(H_1 \cap H_2) > r$, $(H_1 \cap H_2) \cap A = G_1 \cap G_2$

$\Longrightarrow \tau_A(G_1 \cap G_2) > r$ which is contradiction. So,

$\tau_A(G_1 \cap G_2) \geq \tau_A(G_1) \wedge \tau_A(G_2)$

(0 3) By the definition of the gradation of openness

$\tau_A(V_i G_i) = V \{\tau(H): H \cap A = V_i G_i, H \in \Phi \}$

Suppose that, $\tau_A(V_i G_i) < \wedge_i (\tau_A(G_i)) \Longrightarrow \exists\, r \in (0, 1)$ s.t

$\tau_A(V_i G_i) < r < \wedge_i (\tau_A(G_i)) \Longrightarrow \tau_A(G_i) > r\ \forall i$

$\Longrightarrow \exists\, H_i \in \Phi$ s. t $H_i \cap A = G_i$, $\tau(H_i) > r\ \forall i$

$\Longrightarrow \tau(V_i H_i) \geq \wedge_i \tau(H_i) \geq r$, $(V_i H_i) \cap A = V_i G_i$

$\Longrightarrow \exists\, H_0 = V_i H_i \in \Phi$ s. t $\tau(H_0) \geq r$, $H_0 \cap A = V_i G_i$

$\Longrightarrow \tau_A(V_i G_i) \geq r$ which is contradiction. So $\tau_A(V_i G_i) \geq \wedge_i (\tau_A(G_i))$. Thus τ_A is a gradation of openness on the space (A, Φ_A). The rest of the proof is similarly. The space $(A, \Phi_A, \tau_A, \sigma_A)$ is called a subspace of the gradation ditopological texture space (S, Φ, τ, σ).

Theorem 4-4:

If (τ, σ) is a complemented gradation ditopological structure on a texture space (S, Φ), then for every $r \in (0, 1]$,the pair (τ_r, σ_r) is a complemented ditopological structure on (S, Φ).

Proof:

Let (S, Φ, τ, σ) be a complemented gradation ditopological space $A \in \tau_r \Longleftrightarrow \tau(A) \geq r \Longleftrightarrow \sigma(A^c) \geq r \Longleftrightarrow A^c \in \sigma_r$, $\forall\, r \in (0, 1]$ So, $(S, \Phi, \tau_r, \sigma_r)$ is a complemented ditopological texture space.

Theorem 4 -5:

If the collection $\{ (\tau_r, \sigma_r): r \in (0, 1] \}$ is a descending of a Complemented ditopologies on the texture space (S, Φ). Then, the pair (τ, σ) is acomplemented gradation ditopological structure on a texture space (S, Φ), where $\tau(A) = V \{ r: A \in \tau_r \}$ and $\sigma(A) = V \{ r: A \in \sigma_r \}$

Proof:

(1) For every $A \in \Phi$, $\tau(A) = V \{ r: A \in \tau_r \}$ then, we have $\tau(A) = V \{ r: A \in \tau_r \} = V \{ r: A^c \in \sigma_r \}$ i. e $\tau(A) = \sigma(A^c)$. Hence, the pair (τ, σ) is complemented gradation ditopological structure on the space (S, Φ).

Examples:

[1] A gradation ditopology (τ, σ) on the space (S, Φ) is called a discrete structure If $\tau(A) = 1\ \forall\, A \in \Phi$ and it is called a codiscrete structure

If $\sigma(F) = 1\ \forall\, F \in \Phi$

[2] A gradation ditopology (τ, σ) on the space (S, Φ) is called indiscrete structure

$$\text{if } \tau(A) = \begin{cases} 1 & if\ A = \emptyset\ or\ S \\ 0 & otherwise \end{cases}$$

and it is called co-indiscrete structure if

$$\sigma(F) = \begin{cases} 1 & if\ F = \emptyset\ or\ S \\ 0 & otherwise \end{cases}$$

[3] Let $S = \{a, b, c\}$, $\Phi = \{\varphi, S, \{b\}, \{a, b\}, \{b, c\}, \{c\}\}$ define a maps

$\tau, \sigma: \Phi \longrightarrow I$ by $\tau(A) = \begin{cases} 1 \text{ if } A = \emptyset \text{ or } S \\ \frac{1}{2} \text{ if } A = \{b\} \\ \frac{1}{3} \text{ if } A = \{a, b\} \\ 0 \text{ otherwise} \end{cases}$ And $\sigma(A) = \begin{cases} 1 & \text{if } F = \emptyset \text{ or } S \\ \frac{1}{2} & \text{otherwise} \end{cases}$

Then, (τ, σ) is a gradation ditopology on the texture space (S, Φ) if r = 1/3, then the pair (τ_r, σ_r) is a ditopology on (S, Φ) where τ_r = { φ, S, {b}, {a, b} }, $\sigma_r = \Phi$

Definition 4-6:

On the texture space (S, Φ). The pair (τ, σ) is called:

[1] di-discrete if τ is discrete and σ is co-discrete.

[2] di-indiscrete if τ is indiscrete and σ is co-indiscrete.

Proposition 4-7:

A subspace of a gradation di-discrete (resp. di-indiscrete) di topological space is a gradation di-discrete (resp. di-indiscrete) space.

Proof:

It is obvious. In the following, we study some properties for the gradation ditopological texture space.

Definition 4-8:

Let (τ, σ) be a gradation di-topology on the texture space (S, Φ).

For every r ∈ (0, 1], the space (S, Φ, τ, σ) is called:

(i) r -compact if whenever S = $\bigvee_{i \in J} G_i$, $\tau(G_i) \geq r$, r ∈ (0, 1]

⟹ ∃ finite subset K of J s. t S = $\bigvee_{i \in K} G_i$.

(ii) r - co-compact if whenever $\bigcap_{i \in J} F_i = \varphi$, $\sigma(F_i) \geq r$

⟹ ∃ finite subset K of J s. t $\bigcap_{i \in K} F_i = \varphi$

(iii) s r - stable if ∀ F ∈ Φ, σ (F) ≥ r, F ≠ S is s -compact

i. e whenever $F \subseteq \bigcup_{i \in J} A_i$, $\tau(A_i) \geq s$

⟹ ∃ finite subset K of J s. t $F \subseteq \bigcup_{i \in K} A$

(iv) s r-co-stable if ∀ F ∈ Φ, τ (F) ≥ r, F ≠ S is s -co-compact

i. e whenever $\bigcap_{i \in J} F_i \subseteq F$, $\sigma(F_i) \geq s$

⟹ ∃ finite subset K of J s. t $\bigcap_{i \in K} F_i \subseteq F$

Proposition 4 -9: Let (τ, σ, γ) be a complemented gradation ditopology on the complemented texture space (S, Φ, γ).Then

(i) (S, Φ, τ, σ, γ) is r-compact iff it is r-co-compact

(ii) (S, Φ, τ, σ, γ) is sr - stable iff it is sr- co-stable

Proof:

(i) (⟹)Let (S, Φ, τ, σ, γ) is r-compact and let $\bigcap_{i \in J} F_i = \varphi$, $\sigma(F_i) \geq r$

⟹ S = $\bigcup_{i \in J} (F^c_i)$, $\tau(F^c_i) \geq r$

⟹ ∃ finite subset K of J s.t S = $\bigcup_{i \in K} (F^c_i)$

⟹ ∃ finite subset K of J s.t $\bigcap_{i \in K} F_i = \varphi$

⟹ it is r –co-compact

(⟸) By a similar way

(ii) Let (S, Φ, τ, σ, γ) is s r - stable and A ∈ Φ, τ (A) ≥ r, and suppose $\bigcap_{i \in J} F_i \subseteq A$, $\sigma(F_i) \geq s$ ⟹ $A^c \subseteq (\bigcap_{i \in J} F_i)^c$

$= \bigcup_{i \in J} (F_i)^c$, $\tau(F_i)^c = \sigma(F_i) \geq s$

⟹ ∃ finite subset K of J s. t $A^c = \bigcup_{i \in K} (F_i)^c$

⟹ ∃ finite subset K of J s.t $\bigcap_{i \in K} F_i \subseteq A$, $\sigma(F_i) \geq s$

⟹ it is s- co-stable. The converse is similar

5 A Gradation dineighbourhood systems:

Definition 5-1:

Let (τ, σ) be a gradation ditopology on the texture space (S, Φ)

(I) If s ∈ S^b, N∈ Φ is called r-neighbourhood of s if ∃G ∈ Φ s. t τ (G) ≥ r, $p_s \subseteq G \subseteq N \not\subseteq Q_s$

(ii) If s ∈ S^b, M ∈ Φ is called t-co-neighbourhood of s I f ∃ F ∈ Φ s.t σ (F) ≥ t, $p_s \not\subseteq M \subseteq F \subseteq Q_s$

We denoted the set η_r (s) = { N ∈ Φ: N is r-neighbourhood of s } and the set ξ_t (s) = { F ∈ Φ: F is t- co-neighbourhood of s } and We refer to the pair (η_r (s), ξ_t (s))as the (r, t) di-neighbourhood system of a point s ∈ S^b with respect to the space (S, Φ, τ, σ).

Theorem 5-2:

For a gradation ditopology (τ, σ)on the texture space (S, Φ),

Then We have:

For each s ∈ S^b the family η_r (s) satisfies the following conditions: [1]

(i) η_r (s) ≠ φ

(ii) N∈ η_r (s) ⟹ $N \not\subseteq Q_s$

(iii) N∈ η_r (s), N ⊆ M ⟹ M∈ η_r (s)

(iv) N_1, N_2∈ η_r (s), $N_1 \cap N_2 \not\subseteq Q_s$ ⟹ $N_1 \cap N_2$∈ η_r (s)

(v) N ∈ η_r (s) ⟹ ∃ N^* ∈ S, $p_s \subseteq N^* \subseteq N$ s. t $N^* \not\subseteq Q_t$ and N^* ∈ η_r (t) ∀ t ∈ S^b

[2] For each s ∈ S the family ξ_t (s) satisfies the following conditions:

(i) ξ_t (s) ≠ φ

(ii) M ∈ ξ_t (s) ⟹ $p_s \not\subseteq M$

(iii) M ∈ ξ_t (s), N ⊆ M ⟹ N ∈ ξ_t (s)

(iv) M_1, M_2 ∈ ξ_t (s) ⟹ $M_1 \cup M_2$ ∈ ξ_t (s)

(v) M ∈ ξ_t (s) ⟹ ∃ M^* ∈ S, $M \subseteq M^* \subseteq Q_s$, s.t $P_t \not\subseteq M^*$, M^* ∈ ξ_t (s) ∀ t ∈ S

Proof:

It is obvious

Definition 5-3:

Let (τ, σ)a gradation di-topology on the texture space (S, Φ) And A ∈ S We define two operators int_r, cl_t: $\Phi \longrightarrow \Phi$ by int_r (A) = V { G ∈ Φ: τ (G) ≥ r, G ⊆ A} and cl_t (A) = ∧ { F ∈ Φ: σ (F) ≥ t, A ⊆ F}

Proposition 5-4:

Let (τ, σ)a gradation ditopology on the texture space (S, Φ), then the two operators int_r, cl_t satisfies the following conditions:

[I](i) int_r (φ) = φ, int_r (S) = S

(ii) int_r (A) ⊆ A

(iii) A ⊆ B ⟹ int_r (A) ⊆ int_r (B)

(iv) int_r int_r (A) = int_r (A)

[II] (i) cl_t (φ) = φ, cl_t (S) = S

(ii) cl_t (A) ⊇ A

(iii) A ⊆ B ⟹ cl_t (A) ⊆ cl_t (B)

(iv) cl_t cl_t (A) = cl_t (A)

The set int_r (A) is called the interior of A with r-degree of openness, and the set cl_t (A) is the closure of A with t – degree of closness

Proof:

It is clear

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