Journal of Applied Sciences Research, 8(4): 2109-2112, 2012
ISSN 1819-544X
This is a refereed journal and all articles are professionally screened and reviewed

ORIGINAL ARTICLES

Determination of the optimal axisymmetric fluid flow moving from two equidistant sources

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ABSTRACT

This paper intends to study the axisymmetric fluid flow moving from two sources separated by a variable distance. Through this study, we determine the optimal distance between these sources such that the production of fluid in these sources passing through the boundary of a fluid domain can reach an optimal value and then we estimate this optimal flow.

Key words: Axisymmetric fluid flow, axis of symmetry, equidistant fluid sources, optimal fluid flow, boundary of a fluid domain.

Introduction

Fluid mechanics forms a theoretical basis of hydraulics; nevertheless perfect fluid flows are sometimes resistant to the theoretical analysis. It means that in hydraulics nowadays we need fundamentally, empirical experiments to describe phenomena. It follows that it is important to get the initial hypothesis in any theoretical formulation. In fluid mechanics, generally, we distinguish the flow regimes of the fluids e.g. laminar and non-laminar. One of the characteristics of the flow is the Reynolds number (Prandtl, L., 1952) and (Streeter, V.L., 1961):
-for the Reynolds number $\boldsymbol{Re}$ smaller than the critical Reynolds number $\boldsymbol{Re}_{cr} = 2320$, the flow is laminar;
-for the Reynolds number $\boldsymbol{Re}$ bigger than the critical Reynolds number, the flow is turbulent.

In general, because of the viscosity, perfect flows are rotational. Nevertheless, in some cases, when the viscosity is negligible, the rotational flow becomes irrotational. It is especially the case of flow through porous media.

In the reservoir mechanics, the disposition of the geothermal wells is similar to that of wells in petroleum engineering; hence, the usual disposition of the geothermal wells is such that the rate of flow of the production wells is identical, the capacity of re-injection wells is similar and the distance between two arbitrary wells is constant (Boldizsár, T.V., 1968).

In this study, we consider the variable distance between two fluid sources lying on the z- axis. The objective of the investigation is to find the values of the variable distance such that the axisymmetric fluid flow moving from these sources through the boundary of a fluid domain can reach an optimal value.

In the section 2, we analyze the problem and we propose an approach for its solving; in section 3, we provide a conclusion.

State of the problem and its solving:

Consider the fluid flow in a three dimensional space, endowed with the cartesian coordinates system (x, y, z) and the origin O of coordinates. We assume that the flow is permanent and axisymmetric with the z-axis as axis of symmetry. For this kind of flow, the corresponding potential of the velocity field is defined by (Eglit, M.E., 1996), (Selivanov, V.V., Zarubine, V.S. and V.N., Ionov, 1994) and (Asszony, G., 1974):

$$\phi(x, y, z) = q\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \qquad \forall (x, y, z) \in D, \tag{1}$$

Where q is the intensity, a the parameter, D the bounded fluid domain,

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$R_{1,2} = \sqrt{x^2 + y^2 + (z \pm a^2\ )^2},\ \forall (x,y,z) \in \mathrm{D}$

the distances from the point (x,y,z) to the two sources lying in the points $(0,0,-a)$, $(0,0,a)$. Thus we can determine the three components of the flow velocity field v_x, v_y, v_z according to the x-axis, y-axis, z-axis respectively. Let us remark that a two dimensional fluid domain can represent the model of a three dimensional one if the intersection of a three dimensional domain with a plane defined by the equation $= c$, $(c = const)$ does not change with $\boldsymbol{c}$. Then we analyze the case when $z = 0,\ v_z = 0$ and the potential of velocity reduces to

$$\phi(x,y) = \frac{2q}{\sqrt{x^2+y^2+a^2}},\ \forall (x,y) \in \Omega, \qquad (2)$$

Where
the fluid domain $\Omega = \{(x,y): -b \le x \le b,\ 0 < y < \varphi(x)\}$ with b positive constant and φ a continuous function on [-b, b].

Consider Γ the boundary of a fluid domain Ω and keep in mind that the flow is irrotational . Assume that the components v_x, v_y of the velocity field v are continuously differentiable in the domain $\Omega \cup \Gamma$ and the unit normal vector to Ω is outside .

Suppose a $\in [a_1; a_2]$ strictly positive parameter. The flow of the fluid passing through the boundary Γ is defined by (Fichtengolts, G.M., 1968), (Liachko, I.I., Boyarchuk, A.K., Gayi, Y.G. and G.P., Golovach, 1977)and (Smirnov, V., 1979) :

$$Q(a) = \oint_{\Gamma} -v_y(x,y)dx + v_x(x,y)dy$$

According to the Green-Riemann formula, we get:

$$Q(a) = \iint_{\Omega} \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) dxdy,$$

Equivalent to

$$Q(a) = \int_{-b}^{b} h(x,a)dx, \qquad (3)$$

where

$$h(x,a) = \frac{2q\varphi(x)(x^2-2a^2)}{(x^2+a)^2\sqrt{x^2+a^2+\varphi^2(x)}} + \frac{2qa^2\varphi^3(x)}{(x^2+a)^2\sqrt{\left(x^2+a^2+\varphi^2(x)\right)^3}}.$$

If Q reaches an optimal value for $\overline{a} \in [a_1; a_2]$, then

$$\frac{dQ}{da}(\overline{a}) = 0 \quad \text{and} \quad \frac{d^2Q}{da^2}(\overline{a}) < 0 \quad \text{or} \quad \frac{d^2Q}{da^2}(\overline{a})>0.$$

Computing the derivative of the function h with respect to a, we get :

$$\frac{dh(x,a)}{da} = \frac{2q\varphi(x)a}{(x^2+a^2)^3\left(a^2+x^2\varphi^2(x)\right)^{5/2}}\left[6a^6 + \left(3x^2 + 5\varphi^2(x)\right)a^4\right.$$

$$\left. +(2\varphi^4(x) - 10x^2\varphi^2(x) - 12x^4)a^2 - \left(9x^6 + 15x^4\varphi^2(x) + 6x^2\varphi^4(x)\right)\right]$$

Thus

$$\frac{dQ}{da}(a) = 0 \Leftrightarrow$$
$$\int_{-b}^{b} 2q\varphi(x)a\left[6a^6 + \left(3x^2 + 5\varphi^2(x)\right)a^4\right.$$

$$+(2\varphi^4(x) - 10x^2\varphi^2(x) - 12x^4)a^2 - \left(9x^6 + 15x^4\varphi^2(x) + 6x^2\varphi^4(x)\right)\Big]$$

$$\times \frac{dx}{(x^2+a^2)^3\left(x^2+a^2+\varphi^2(x)\right)^{5/2}} = 0 \qquad (4)$$

Computing the second derivative of h and Q with respect to a, we obtain

$$\frac{d^2h(x,a)}{da^2} = \frac{2q\varphi(x)}{(x^2+a^2)^3\left(x^2+a^2+\varphi^2(x)\right)^{5/2}} \times \left[6a^6 + \left(3x^2+5\varphi^2(x)\right)a^4\right.$$
$$+(2\varphi^4(x) - 10x^2\varphi^2(x) - 12x^4)a^2 - \left(9x^6 + 15x^4\varphi^2(x) + 6x^2\varphi^4(x)\right)\Big]$$

$$+a\left[36a^5 + 4\left(3x^2+5\varphi^2(x)\right)a^3 + 2(2\varphi^4(x) - 10x^2\varphi^2(x) - 12x^4)a\right]\Big\}$$

$$-\frac{2q\varphi(x)a^2}{(x^2+a^2)^4\left(x^2+a^2+\varphi^2(x)\right)^{7/2}}\left[6a^6 + \left(3x^2+5\varphi^2(x)\right)a^4\right.$$

$$+(2\varphi^4(x) - 10x^2\varphi^2(x) - 12x^4)a^2 - \left(9x^6 + 15x^4\varphi^2(x) + 6x^2\varphi^4(x)\right)\Big]$$

$$\times\left(11x^2 + 11a^2 + 6\varphi^2(x)\right)$$

And

$$\frac{d^2Q(x,a)}{da^2} = \int_{-b}^{b} \frac{2q\varphi(x)}{(x^2+a^2)^3\left(x^2+a^2+\varphi^2(x)\right)^{5/2}} \times \left[6a^6 + \left(3x^2+5\varphi^2(x)\right)a^4\right.$$
$$+(2\varphi^4(x) - 10x^2\varphi^2(x) - 12x^4)a^2 - \left(9x^6 + 15x^4\varphi^2(x) + 6x^2\varphi^4(x)\right)\Big]dx$$
$$-60\int_{-b}^{b} \frac{q\varphi(x)a^2}{(x^2+a^2)^4\left(x^2+a^2+\varphi^2(x)\right)^{7/2}}\left\{a^8 + \frac{1}{30}\left(15x^2+35\varphi^2(x)\right)a^6\right.$$
$$+\frac{1}{30}(28\varphi^4(x) - 105x^2\varphi^2(x) - 135x^4)a^4$$
$$+\frac{1}{30}(8\varphi^6(x) - 112x^2\varphi^4(x) - 315x^4\varphi^2 - 195x^6)a^2$$
$$\left.-\frac{1}{30}(140x^4\varphi^4(x) + 175x^6\varphi^2(x) + 40x^2\varphi^6(x) + 175x^8)\right\}dx$$

Taking into account the equation (4), we obtain :

$$\frac{d^2\ Q(a)}{da^2} = -60qa^2d_0(a^8 + d_1a^6 + d_2a^4 + d_3a^2 + d_4)$$

Where the quantities

$$d_1 = \frac{f_1}{d_0}\ ; d_2 = \frac{f_2}{d_0}\ ; d_3 = \frac{f_3}{d_0}\ ; d_4 = \frac{f_4}{d_0},$$

depending on the parameter **a** are defined by

$$d_0 = \int_{-b}^{b} \frac{\varphi(x)dx}{(x^2+a^2)^4\left(x^2+a^2+\varphi^2(x)\right)^{\frac{7}{2}}}\ ,$$

$$f_1 = \frac{1}{30}\int_{-b}^{b} \frac{\varphi(x)(15x^2+35\varphi^2(x))}{(x^2+a^2)^4\left(x^2+a^2+\varphi^2(x)\right)^{\frac{7}{2}}}dx\ ,$$

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$$f_2 = \frac{1}{30}\int_{-b}^{b} \frac{28\varphi^4(x) - 105x^2\varphi^2(x) - 135x^4}{(x^2 + a^2)^4\left(x^2 + a^2 + \varphi^2(x)\right)^{\frac{7}{2}}}\varphi(x)\,dx\,,$$

$$f_3 = \frac{1}{30}\int_{-b}^{b} \frac{8\varphi^6(x) - 112x^2\varphi^4(x) - 315x^4\,\varphi^2(x) - 195x^6}{(x^2 + a^2)^4\left(x^2 + a^2 + \varphi^2(x)\right)^{\frac{7}{2}}}\varphi(x)\,dx$$

$$f_4 = -\frac{1}{30}\int_{-b}^{b} \frac{140x^4\varphi^4(x) + 175x^6\varphi^2(x) + 40x^2\varphi^6(x) + 75x^8}{(x^2 + a^2)^4\left(x^2 + a^2 + \varphi^2(x)\right)^{\frac{7}{2}}}\varphi(x)\,dx$$

We arrive at the following result

Lemma. The flow Q reaches its maximum if $\bar{a}$ is a solution of the equation (4) and

$\bar{a}^8 + \bar{d}_1\,\bar{a}^6 + \bar{d}_2\bar{a}^4 + \bar{d}_3\bar{a}^2 + \bar{d}_4 > 0$,

its minimum if $\bar{a}$ is a solution of the equation (4) and

$\bar{a}^8 + \bar{d}_1\,\bar{a}^6 + \bar{d}_2\bar{a}^4 + \bar{d}_3\bar{a}^2 + \bar{d}_4 < 0$,

With $\bar{d}_1, \bar{d}_2, \bar{d}_3, \bar{d}_4$ the values of d_1, d_2, d_3, d_4 for $a = \bar{a}$.

Therefore, the optimal flow Q can be defined by

$$Q(\overline{a}) = \int_{-b}^{b}\left[\frac{2q\varphi(x)(x^2 - 2a^2)}{(x^2 + a^2)^2\sqrt{x^2 + a^2 + \varphi^2(x)}} + \frac{2qa^2\varphi^3(x)}{(x^2 + a^2)^2\sqrt{\left(x^2 + a^2 + \varphi^2(x)\right)^3}}\right]dx$$

for $a = \bar{a}$.

Conclusion:

In this paper, we have investigated the optimal distance and flow. Under the defined conditions, the minimal and maximal values of the production of fluid through the boundary of a fluid domain are reached.These results allow to use the optimal disposition of the geothermal wells, wells in petroleum engineering,etc...

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