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ORIGINAL ARTICLES

Application of Simplest Equation Method to Nonlinear Lienard Equation

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ABSTRACT

In this paper, the simplest equation method has been used for finding the general and particular exact solutions of the nonlinear Linard equation. The solutions obtained here are expressed in exponential functions. The method appears to be easier and faster by means of a symbolic computation system.

Key words**:** Lienard equation, Simplest equation method, Riccati equation.

Introduction

The research area of nonlinear equations has been very active for the past few decades. There are many kinds of nonlinear equations that appear in various areas of physical and mathematical sciences. Much effort has been made on the construction of exact solutions of such nonlinear equations. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optimal fiber, biology, solid state physics, chemical physics and geometry.

In recent years, many powerful and efficient methods to find analytic solutions of nonlinear equations have been presented by a diverse group of scientists. These methods include the tanh-function method, the extended tanh-function method (Fan, E., 2000; Wazwaz, A.M., 2007), the sine-cosine method (Wazwaz, A.M., 2004; Wazwaz, A.M., 2004), the $(\frac{G'}{G})$-expansion method (Abazari, R., 2010; Wang, M., 2008).

The simplest equation method that has been widely used to obtain exact solutions of nonlinear equations was developed by Kudryashov (1991; 2005; 2008) on the basis of a procedure analogous to the first step of the test for the Painleve property (Hone, A., 2009). In this paper, we utilize the simplest equation method to obtain exact solutions of Lienard equation (Zhang, F., 2002):

$$\begin{cases} u''(x) - u(x) + 4u^3(x) - 3u^5(x) = 0. \\ u(0) = \dfrac{\sqrt{2}}{2}, \\ u'(0) = \dfrac{\sqrt{2}}{4}. \end{cases}$$

2 The basic idea of simplest equation method:

In this section we recall the basic idea of the simplest equation method. Let we have a partial differential equation and by means of an appropriate transformation this equation is reduced to a nonlinear ordinary differential equation as follow:

$$P(u, u', u'', u''', ...) = 0. \qquad (1)$$

Exact solution of this equation can be constructed as finite series

$$u(x) = \sum_{i=0}^{n} a_i (G(x))^i \qquad (2)$$

Where $G(x)$ is a solution of some ordinary differential equation referred to as the simplest equation, and $A_0, A_1, A_2, ..., A_M$ are parameters to be determined.

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In this paper we use the equation of Riccati, as the simplest equation

$$G'(x) = cG(x) + dG(x)^2 \quad (3)$$

This equation is well-known nonlinear ordinary differential equation which process exact solution constructed by elementary function. In this paper we work with the following solutions of Riccati equation

$$G(x) = \frac{c\exp[c(x+x_0)]}{1-d\exp[c(x+x_0)]} \quad (4)$$

for case $d < 0$, $c > 0$, here x_0 is a constant of integration. and

$$G(x) = -\frac{c\exp[c(x+x_0)]}{1+d\exp[c(x+x_0)]} \quad (5)$$

for case $d > 0$, $c < 0$, similar above x_0 is a constant of integration.
the simplest equation has two properties: (i)The order of simplest equation is lesser than equation (1), (ii) we know the general solution of the simplest equation or we know at least exact analytical particular solution(s) of the simplest Eq(3).
Now $u(x)$ can be determined explicitly by using the following three steps:

• Step (1). By considering the homogeneous balance between the highest nonlinear terms and the highest order derivatives of $u(x)$ in Eq.(1), the positive integer n in (2) is determined.

• Step (2). By substituting (2) with Eq.(4) or (5) into (1) and collecting all terms with the same powers of G together, the left hand side of Eq.(1) is converted into a polynomial. After setting each coefficient of this polynomial to zero, we obtain a set of algebraic equations in terms of $A_i(i = 0,1,2,...,n), c, d$.

• Step (3). Solving the system of algebraic equations and then substituting the results and the general solutions of (4) or (5) into (2) gives solutions of (1).

3 Exact solutions of the Lienard equation:

In this section, we will utilize the simplest equation method to obtain exact solutions of Lienard equation. Let us consider the Lienard equation:

$$u''(x) - u(x) + 4u^3(x) - 3u^5(x) = 0. \quad (6)$$

By the balancing procedure we get $n = \frac{1}{2}$, By the transformation $u(x) = v^{\frac{1}{2}}(x)$, Eq.(6) becomes:

$$\frac{1}{2}v''v - \frac{1}{4}v'^2 - v^2 + 4v^3 - 3v^4 = 0 \quad (7)$$

With the balancing procedure we get $n = 1$, therefore the solution of (7) can be expressed by a polynomial as follows:

$$v(x) = A_0 + A_1 G, \quad A_1 \neq 0, \quad (8)$$

where G is the solution of (3). Substituting (8) into (7) and making use of (4) and equating each coefficient of this polynomial to zero, we obtain a set of nonlinear algebraic equations for A_0, A_1, c . Solving this system using *Mathematica*, we obtain

$$\begin{cases} Case1: \quad A_0 = 1, \quad A_1 = -\frac{d}{2}, \quad c = -2, \quad d \neq 0 \\ Case2: \quad A_0 = 0, \quad A_1 = -\frac{d}{2}, \quad c = 2, \quad d \neq 0 \end{cases} \quad (9)$$

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3.1 Exact solutions of the Lienard equation where $d < 0$ *and* $c > 0$*:*

Using these values (Case1) of A_0 and A_1, into (8) we obtain

$$v(x) = 1 - \frac{d}{2}G \tag{10}$$

Substituting the general solution of (4) into (10) we obtain following solution

$$v(x) = 1 + \frac{d\exp[-2(x+x_0)]}{1 - d\exp[-2(x+x_0)]}; \tag{11}$$

where x_0 is constant of integration. Using the transformation $u(x) = v^{\frac{1}{2}}(x)$, we have

$$u(x) = \left\{1 + \frac{d\exp[-2(x+x_0)]}{1 - d\exp[-2(x+x_0)]}\right\}^{\frac{1}{2}}; \tag{12}$$

If integration constant is assumed zero with consider initial conditions, we obtain:

$$u(x) = \left\{1 - \frac{\exp(-2x)}{1+\exp(-2x)}\right\}^{\frac{1}{2}}; \tag{13}$$

Now using values of (9) (Case2) of A_0 and A_1, into (8) we obtain

$$v(x) = -\frac{d}{2}G \tag{14}$$

Substituting the general solutions of (4) into (14) we obtain following solution

$$v(x) = -\frac{d\exp[2(x+x_0)]}{1 - d\exp[2(x+x_0)]}; \tag{15}$$

where x_0 is constant of integration. Using the transformation $u(x) = v^{\frac{1}{2}}(x)$, we have

$$u(x) = \left\{-\frac{d\exp[2(x+x_0)]}{1 - d\exp[2(x+x_0)]}\right\}^{\frac{1}{2}}; \tag{16}$$

If integration constants is assumed zero with consider initial conditions, we obtain:

$$u(x) = \left\{\frac{\exp(2x)}{1+\exp(2x)}\right\}^{\frac{1}{2}}; \tag{17}$$

3.2 Exact solutions of the Lienard equation where $d > 0$ *and* $c < 0$*:*

Using values of (9) (Case1) of A_0 and A_1, into (8) we obtain

$$v(x) = 1 - \frac{d}{2}G \tag{18}$$

Substituting the general solutions of (5) into (18) we obtain following solution

$$v(x) = 1 - \frac{d\exp[-2(x+x_0)]}{1 + d\exp[-2(x+x_0)]}; \tag{19}$$

where x_0 is constant of integration. Using the transformation $u(x) = v^{\frac{1}{2}}(x)$, we have

$$u(x) = \left\{1 - \frac{d\exp[-2(x+x_0)]}{1+d\exp[-2(x+x_0)]}\right\}^{\frac{1}{2}}; \tag{20}$$

If integration constant is assumed zero with consider initial conditions, we obtain:

$$u(x) = \left\{1 - \frac{\exp(-2x)}{1+\exp(-2x)}\right\}^{\frac{1}{2}}; \tag{21}$$

Now using values of (9) (Case2) of A_0 and A_1, into (8) we obtain

$$v(x) = -\frac{d}{2}G \tag{22}$$

Substituting the general solutions of (5) into (22) we obtain following solution

$$v(x) = \frac{d\exp[2(x+x_0)]}{1+d\exp[2(x+x_0)]}; \tag{23}$$

where x_0 is constant of integration. Using the transformation $u(x) = v^{\frac{1}{2}}(x)$, we have

$$u(x) = \left\{\frac{d\exp[2(x+x_0)]}{1+d\exp[2(x+x_0)]}\right\}^{\frac{1}{2}}; \tag{24}$$

If integration constants is assumed zero with consider initial conditions, we obtain:

$$u(x) = \left\{\frac{\exp(2x)}{1+\exp(2x)}\right\}^{\frac{1}{2}}; \tag{25}$$

4 Conclusion:

In this paper we obtained exact solutions for nonlinear Lienard equation. The method used to obtain these solutions was simplest equation method. These exact solutions included the exponential function solutions. It was also concluded that the simplest equation method is direct, concise, very effective and powerful method for solving nonlinear equations of mathematical physics. We have assured the correctness of the obtained solutions by putting them back into the original equation.

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