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Decomposition Method for Solving Integer Linear Programming Problems with Trapezoidal Fuzzy Numbers

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ABSTRACT

In this paper, integer linear programming with trapezoidal fuzzy numbers is studied. The trapezoidal fuzzy numbers are assumed as the coefficient of the problem and it is also assumed that decision variables are trapezoidal fuzzy variables. We prove that four crisp linear systems are needed to be solved in order to solve the trapezoidal fuzzy linear system. We also prove that solving a trapezoidal fuzzy integer linear programming problem requires solving four crisp integer linear programming problems. Based on the theorems, a decomposition method is proposed to solve the problem. A numerical example is presented to illustrate the method.

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INTRODUCTION

Linear programming (LP) [1] is a mathematical approach to specify a method to achieve the best result in a given mathematical model for several constraints represented as linear relationships. in real life, information is not crisp, certain, and precise but it is uncertain. In order to overcome this difficulty, the fuzzy concept is proposed by Zadeh [2]. Fuzzy linear programming (FLP) is applied while the experts and decision makers are unable to determine the exact values of parameters or there exist some vagueness in decision environment. FLP has been received significant attention of researchers because of its application in decision making. The FLP on a general level of the concept was first studied by Tanaka et al. [3]. Thereafter, several approaches are proposed to solve different FLP problems. Most of these approaches are based on the idea of comparison of fuzzy numbers which is applied fuzzy ranking functions. The variety of such methods provides a lot of different models of formal linear programming problems from which fuzzy solutions to the former problem can be obtained [4,5,6]. Shams et al. [7] have conducted a comprehensive survey on existing methods and approaches for FLP problems.

An integer programming problem [8] is a mathematical optimization in which some or all of the variables are integers. In many researches, the term refers to integer linear programming (ILP), in which the objective function and the constraints are linear. Like FLP problems, sometimes experts and decision makers are not able to determine precisely for ILP problem. In order to handle this, fuzzy integer linear programming (FILP) is applied. Herrera and Verdegay [9] present methods to solve FILP problems with either fuzzy constraints, or fuzzy numbers in the objective function or fuzzy numbers defining the set of constraints. Their methods are based on the representation theorem and on fuzzy number ranking methods. Nasseri [10] has proposed a new method to solve the FILP problems in which the fuzzy ranking method for converting the fuzzy objective function into crisp objective function is applied.

Recently, Pandian and Jayalakshmi [11] have proposed a decomposition method for solving ILP problems with triangular fuzzy variables by using classical ILP. They have provided a new method for solving ILP problems with fuzzy variables without using any ranking functions.

This paper is organized as follows. The notations and principles are presented in Section 2. in Section 3, we prove that four crisp linear systems are needed to be solved in order to solve the trapezoidal fuzzy linear system and also prove that solving a trapezoidal FILP problem requires solving four crisp ILP problems. Based on the theorems, a decomposition method is also proposed to solve the problem in Section 3. A numerical example is presented to illustrate the method in Section 4 and finally, the conclusions are given in Section 5.

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2. Preliminaries:

In this paper, basic arithmetic operators on trapezoidal fuzzy numbers should be defined. Dubios and Prade [12] and Kauffman and Gupta [13] have proposed basic arithmetic operators on trapezoidal fuzzy numbers. The following definitions and notations are introduced based on the principles which they have proposed.

Notation 1:

A trapezoidal fuzzy number is presented by: $\tilde{A}$ (left support, left core, right core, right support). In this way, a fuzzy number which is shown in the Figure 1 is denoted by $\tilde{A}(1,4,6,8)$.

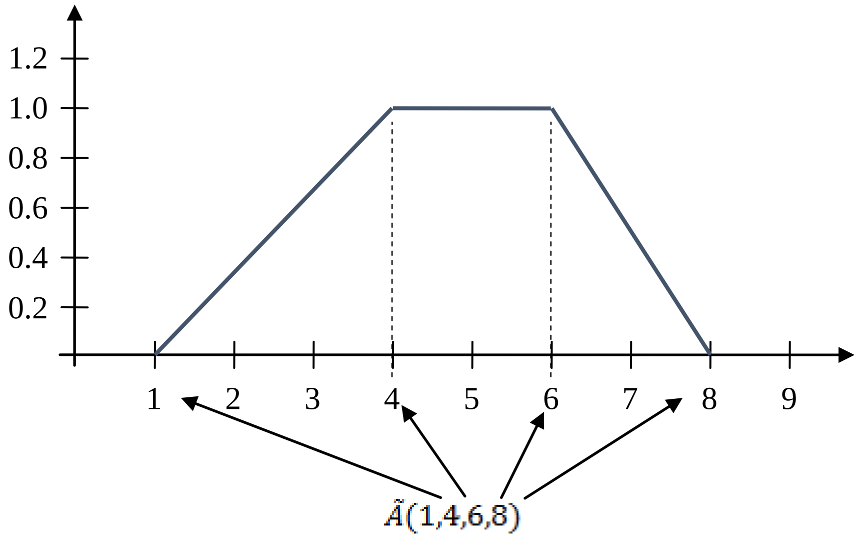

Fig. 1: Notation for a trapezoidal fuzzy set.

Definition 1:

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if $a_1 \leq a_2 \leq a_3 \leq a_4$ and its membership function is given as follows.

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 < x \leq a_2 \\ 1, & a_2 < x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 < x \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

where $a_1, a_2, a_3, a_4 \in \mathbb{R}$

Definition 2:

In order to define arithmetic operations on trapezoidal fuzzy numbers let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two real trapezoidal fuzzy numbers; then the one has the following.

(i) $(a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $-\tilde{A} = (-a_4, -a_3, -a_2, -a_1)$

(iii) $(a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(iv) $(a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

(v) $k(a_1, a_2, a_3, a_4) = (ka_1, ka_2, ka_3, ka_4)$, for $k \geq 0$ and $k \in \mathbb{R}$

(vi) $k(a_1, a_2, a_3, a_4) = (ka_4, ka_3, ka_2, ka_1)$, for $k < 0$ and $k \in \mathbb{R}$

Definition 3:

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number in $F(\mathbb{R})$ and $F(\mathbb{R})$ is the set of all real trapezoidal fuzzy numbers then the following statements are defined.

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(i) $\tilde{A}$ is said to be positive if $a_i \geq 0$, for all i and i = 1, 2, 3, 4
(ii) $\tilde{A}$ is said to be negative if $a_i < 0$, for all i and i = 1, 2, 3, 4
(iii) $\tilde{A}$ is said to be symmetric if $a_4 - a_3 = a_2 - a_1$

Definition 4:

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are in $F(\mathbb{R})$ Then
(i) $\tilde{A} = \tilde{B} \Leftrightarrow a_i = b_i$, for all i and i = 1, 2, 3, 4
(ii) $\tilde{A} \leq \tilde{B} \Leftrightarrow a_i \leq b_i$, for all i and i = 1, 2, 3, 4

Definition 5:

A real fuzzy vector $\tilde{b} = (\tilde{b}_i)_{m\times 1}$ is called non-negative and denoted by $\tilde{b} \geq 0$, if each element of $\tilde{b}$ is a non-negative real fuzzy number, that is $\tilde{b}_i \geq 0$, for all i and i = 1, 2, 3, 4, …, m.

Consider the following $m \times n$ fuzzy linear system with non-negative real trapezoidal fuzzy numbers, formulation (1).

$$A\tilde{x} \leq \tilde{b} \tag{1}$$

where $A = (a_{ij})_{m\times n}$ is a non-negative crisp matrix and $\tilde{x} = (\tilde{x}_j)$, $\tilde{b} = (\tilde{b}_i)$ are non-negative fuzzy vector and $\tilde{x}_j, \tilde{b}_i \in F(\mathbb{R})$, for all $1 \leq i \leq m$, $1 \leq j \leq n$

Definition 6:

A non-negative fuzzy vector $\tilde{x}$ is said to be a feasible solution for the fuzzy linear system (1) if $\tilde{x}$ satisfies in the formulation (1).

Theorem 1:

Assume that $A\tilde{x} \leq \tilde{b}$ is an $m \times n$ fuzzy linear system where $A = (a_{ij})_{m\times n}$ is a non-negative crisp matrix, $\tilde{x} = (\tilde{x}_j)$, $\tilde{b} = (\tilde{b}_i)$ are non-negative real trapezoidal fuzzy vector and $\tilde{x}_j = (x_1{}^j, x_2{}^j, x_3{}^j, x_4{}^j)$ and $\tilde{b}_i = (b_1{}^i, b_2{}^i, b_3{}^i, b_4{}^i)$ in $F(\mathbb{R})$ for all $1 \leq i \leq m$, $1 \leq j \leq n$.

Consider the following real linear systems with solutions in Table 1.

Table 1: Real linear systems with solutions.

Solution	Linear System	
$x_2^* = (x_2{}^{*j})_{n\times 1}$	$Ax_2 \leq b_2$ $x_2 \geq 0$	$x_2 = (x_2{}^j)_{n\times 1}$ $b_2 = (b_2{}^i)_{m\times 1}$
$x_1^* = (x_1{}^{*j})_{n\times 1}$	$Ax_1 \leq b_1$ $x_1 \geq 0$ $x_1 \leq x_2^*$	$x_2 = (x_2{}^j)_{n\times 1}$ $b_2 = (b_2{}^i)_{m\times 1}$
$x_3^* = (x_3{}^{*j})_{n\times 1}$	$Ax_3 \leq b_3$ $x_3 \geq x_2^*$	$x_3 = (x_3{}^j)_{n\times 1}$ $b_3 = (b_3{}^i)_{m\times 1}$
$x_4^* = (x_4{}^{*j})_{n\times 1}$	$Ax_4 \leq b_1$ $x_4 \geq 0$ $x_4 \geq x_3^*$	$x_4 = (x_4{}^j)_{n\times 1}$ $b_4 = (b_4{}^i)_{m\times 1}$

Then $\tilde{x}^* = (\tilde{x}_i^*)$ is a solution of system $A\tilde{x} \leq \tilde{b}$ where $\tilde{x}_i^* = (x_1^*, x_2^*, x_3^*, x_4^*)$.

Note that if the $\tilde{x} = (\tilde{x}_j)$ and $\tilde{b} = (\tilde{b}_i)$ are symmetric trapezoidal fuzzy vector, there is no need to solve the last linear system. The solution is obtained by the following formulation (2).

$$x_4^* = x_3^* + x_2^* - x_1^* \tag{2}$$

3. Fuzzy integer linear programming:

Consider the following fuzzy integer linear programming **(P)**, with trapezoidal fuzzy variables.

(P)	$\max \tilde{z} = c\tilde{x}$		(3)
subject to	$A\tilde{x} \leq \tilde{b}$		(4)
	$\tilde{x} \geq 0$	and are integers	(5)

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where $A = (a_{ij})_{m\times n}$ is a non-negative crisp matrix, $c = (c_1, \dots, c_n)$ is non-negative crisp vector, $\tilde{x} = (\tilde{x}_j)$, $\tilde{b} = (\tilde{b}_i)$ are non-negative real trapezoidal fuzzy vector and $\tilde{x}_j = (x_1{}^j, x_2{}^j, x_3{}^j, x_4{}^j)$ and $\tilde{b}_i = (b_1{}^i, b_2{}^i, b_3{}^i, b_4{}^i)$ in $F(\mathbb{R})$ for all $1 \le i \le m$, $1 \le j \le n$.

Difinition 7:

A fuzzy vector $\tilde{x}^*$ is said to be a feasible solution of the problem **(P)** if $\tilde{x}$ satisfies in (4) and (5).

Definition 8:

A feasible solution $\tilde{x}^*$ is said to be the optimal solution of the problem **(P)** if for any feasible solution of the problem **(P)**, say $\tilde{t} = (\tilde{t}_j)_{m\times 1}$, the following inequality (6) satisfies.

$$c\tilde{t} \ge c\tilde{x} \tag{6}$$

The following result is obtained by applying Theorem 1 and the arithmetic operations fuzzy numbers.

Theorem 2:

A fuzzy vector $\tilde{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ is an optimal solution of the problem **(P)** if x_3^*, x_2^*, x_1^*, and x_4^* is optimal solution of the following crisp integer linear programming problems **(P_1)**, **(P_2)**, **(P_3)**, and **(P_4)** respectively.

(P_2) max $z_2 = cx_2$
subject to
$Ax_2 \le b_2$
$x_2 \ge 0$ and are integers

(P_1) max $z_1 = cx_1$
subject to
$Ax_1 \le b_1$
$x_1 \le x_2^*$
$x_1 \ge 0$ and are integers

(P_3) max $z_3 = cx_3$
subject to
$Ax_3 \le b_3$
$x_3 \ge x_2^*$
$x_3 \ge 0$ and are integers

(P_4) max $z_4 = cx_4$
subject to
$Ax_4 \le b_4$
$x_4 \ge x_3^*$
$x_4 \ge 0$ and are integers

Proof:

Assume $\tilde{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ is an optimal solution of the problem **(P)**. Let $\tilde{x} = (x_1, x_2, x_3, x_4)$ which is a feasible solution of the problem **(P)**.

Since the parameter c is a crisp number and $\tilde{x}$ is a trapezoidal fuzzy number, the following inequalities (7) are obtained based on trapezoidal fuzzy number definitions.

$$cx_1 \le cx_1^* \qquad cx_2 \le cx_2^* \qquad cx_3 \le cx_3^* \qquad cx_4 \le cx_4^* \tag{7}$$

On the other hand, we assume that $\tilde{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ is an optimal solution of the problem **(P)**, therefore $\tilde{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ is basically a feasible solution. The following inequalities (8) are obtained from the problem **(P)**.

$$\begin{matrix} Ax_1^* \le b_1 & Ax_2^* \le b_2 & Ax_3^* \le b_3 & Ax_4^* \le b_4 \\ x_1^* \ge 0 & x_2^* \ge 0 & x_3^* \ge 0 & x_4^* \ge 0 \end{matrix} \tag{8}$$

In addition, let $\tilde{z}^* = (z_1{}^*, z_2{}^*, z_3{}^*, z_4{}^*)$ is the objective function of the problem **(P)** for the optimal solution $\tilde{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)$.

Now, from (8) the following (9) are obtained.

$$\max z_1{}^* = cx_1^* \qquad \max z_2{}^* = cx_2^* \qquad \max z_3{}^* = cx_3^* \qquad \max z_4{}^* = cx_4^* \tag{9}$$

Now from (8) and (9), it is observed that x_1^*, x_2^*, x_3^*, and x_4^* are optimal solutions of the crisp integer linear programming problems **(P_1)**, **(P_2)**, **(P_3)**, and **(P_4)** respectively.

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Assume that x_1^*, x_2^*, x_3^*, and x_4^* are optimal solutions of the crisp integer linear programming problems **(P_1)**, **(P_2)**, **(P_3)**, and **(P_4)** with the objective function values $z_1{}^*$, $z_2{}^*$, $z_3{}^*$, and $z_4{}^*$ respectively. Therefore, $\tilde{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ is an optimal solution of the problem **(P)** with the objective function value $\tilde{z}^* = (z_1{}^*, z_2{}^*, z_3{}^*, z_4{}^*)$.

4. Numerical example:

In order to illustrate the proposed method, one numerical example is presented. In the example 1, the approach using decomposition method is applied in order to solve the problem with trapezoidal fuzzy numbers as their coeficients and trapezoidal fuzzy variables as the decision variables. In other words, the problem decomposed into four problems and the optimal solutions of these four problems are fed to each other as it has been mentioned before.

Example 1:

Consider the following integer linear programming problem with trapezoidal fuzzy variables.

(P)
subject to

$$\begin{aligned} &\max\ \tilde{z} = 8\tilde{x}_1 + 16\tilde{x}_2 \\ &4\tilde{x}_1 + 6\tilde{x}_2 \le (40, 46, 50, 60) \\ &\tilde{x}_1 + 2\tilde{x}_2 \le (4, 13, 16, 20) \\ &\tilde{x}_1, \tilde{x}_2 \ge 0 \qquad \text{and are integers} \end{aligned}$$

Let $\tilde{z} = (z_1, z_2, z_3, z_4)$, $\tilde{x}_1 = (x_1^1, x_2^1, x_3^1, x_4^1)$, $\tilde{x}_2 = (x_1^2, x_2^2, x_3^2, x_4^2)$. Consider the following crisp integer linear programming problems (P_1), (P_2), (P_3), and (P_4). In order to solve the problem **(P)**, we need to solve the crisp integer linear programming problems. Based on the proposed method,the first problem which is needed to be solved is (P_2), the optimal solution for the problem (P_2) is fed to the problems (P_1) and (P_3), and the optimal solution for the problem (P_3) is fed to the problem (P_4). The crisp integer linear programming problems for the example and the solutions are as follows.

As it was mentioned, the first problem which is needed to be solved is (P_2), the optimal solution for the problem is obtained by using an algorithm ILP problem. The problem is as follows and the optimal solution is shown in Table 2.

(P_2)
subject to

$$\begin{aligned} &\max\ z_2 = 8x_2^1 + 16x_2^2 \\ &4x_2^1 + 6x_2^2 \le 46 \\ &x_2^1 + 2x_2^2 \le 13 \\ &x_2^1, x_2^2 \ge 0 \qquad \text{and are integers} \end{aligned}$$

Table 2: The optimal solution for the problem (P_2).

The Problem	The Optimal Solution		
	x_2^{1*}	x_2^{2*}	$z_2{}^*$
(P_2)	1	6	104

As it was mentioned, the optimal solution for the problem (P_2) is fed to the problems (P_1) and (P_3) which are needed to be solved. The optimal solutions for the problems are obtained by using an algorithm ILP problem. The problems are as follows and the optimal solution for the problem (P_1) is shown in Table 3 and the optimal solution for the problem (P_3) is shown in Table 4.

(P_1)
subject to

$$\begin{aligned} &\max\ z_1 = 8x_1^1 + 16x_1^2 \\ &4x_1^1 + 6x_1^2 \le 40 \\ &x_1^1 + 2x_1^2 \le 4 \\ &x_1^1 \le 1 \\ &x_1^2 \le 6 \\ &x_1^1, x_1^2 \ge 0 \qquad \text{and are integers} \end{aligned}$$

Table 3: The optimal solution for the problem (P_1).

The Problem	The Optimal Solution		
	x_1^{1*}	x_1^{2*}	$z_1{}^*$
(P_1)	0	2	32

(P$_3$) $\max \ z_3 = 8x_3^1 + 16x_3^2$
subject to
$4x_3^1 + 6x_3^2 \le 50$
$x_3^1 + 2x_3^2 \le 16$
$x_3^1 \ge 1$
$x_3^2 \ge 6$
$x_3^1, x_3^2 \ge 0$ and are integers

Table 4: The optimal solution for the problem (P$_3$).

The Problem	The Optimal Solution		
	x_3^{1*}	x_3^{2*}	z_3^*
(P$_3$)	2	7	128

As it was mentioned, the optimal solution for the problem (P$_3$) is fed to the problem (P$_4$) which is needed to be solved. The optimal solution for the problem is obtained by using an algorithm ILP problem. The problem is as follows and the optimal solution for the problem (P$_4$) is shown in Table 5.

(P$_4$) $\max \ z_4 = 8x_4^1 + 16x_4^2$
subject to
$4x_4^1 + 6x_4^2 \le 60$
$x_4^1 + 2x_4^2 \le 20$
$x_4^1 \ge 2$
$x_4^2 \ge 7$
$x_4^1, x_4^2 \ge 0$ and are integers

Table 5: The optimal solution for the problem (P$_1$).

The Problem	The Optimal Solution		
	x_4^{1*}	x_4^{2*}	z_4^*
(P$_4$)	3	8	152

Therefore, the solution for the given fuzzy integer programming problem **(P)** is as follows.

$$\tilde{z}^* = (z_1{}^*, z_2{}^*, z_3{}^*, z_4{}^*) = (32, 104, 128, 152)$$

$$\tilde{x}_1{}^* = (x_1^{1*}, x_2^{1*}, x_3^{1*}, x_4^{1*}) = (0, 1, 2, 3), \qquad \tilde{x}_2{}^* = (x_1^{2*}, x_2^{2*}, x_3^{2*}, x_4^{2*}) = (2, 6, 7, 8)$$

5. Conclusion:

In this paper, the integer linear programming with trapezoidal fuzzy coeficient and variables is studied. First, definitions and notations from literatures are recalled. Then, in order to propose a decomposition method to solve the problem, two theorems are proved. The decomposition method is proposed and finally a numerical example is presented to illustrate the method. This approach is able to serve to managers by providing a solution simply and effectively in compare with different approaches in literature.

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