Advances in Environmental Biology, 8(9) Special 2014, Pages: 212-222

AENSI Journals

Advances in Environmental Biology

ISSN-1995-0756 EISSN-1998-1066

Journal home page: http://www.aensiweb.com/aeb.html

Providing Criteria for Determining the Validity of the Results of the Stability of Slopes using Lower Bound Limit Analysis Theory.

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ARTICLE INFO

Article history:
Article History
Received 25 March 2014
Received in revised form 20 April 2014
Accepted 15 May 2014
Available online 5 June 2014

Key words:
three-dimensional slope, slope stability, bound analysis, lower bound

ABSTRACT

This study is based on linear theory, the lower bound limit analysis and finite element method and its main subject is to calculate safety factor of slope stability under the influence of bed depth. Although the traditional method of two-and three-dimensional limit equilibrium is used to calculate the stability factor of slopes, however, according to the hypothesis that this pathway considers the stability analysis, the response status remains questionable. Furthermore, the responses obtained from the lower bound limit analysis of the status and accuracy are in specified accuracy and location, and therefore, the answers of this method can be used with high boundary responses as criteria for determining the validity of other methods.

To Cite This Article: Seyed Mostafa Haghighatjoo, Arash Totonchi, Providing criteria for determining the validity of the results of the stability of slopes using lower bound limit analysis theory. ***Adv. Environ. Biol., 8(9),*** *212-222, 2014*

INTRODUCTION

The problem of stability of straigth slopes is one of the most important and yet most complicated topics studied soil mechanics that it is impossible to be investigated without knowing and understanding the concepts of soil mechanics. The complete solution of stability problems in soil mechanics considering both the transition from elastic to plastic behavior (mainly using numerical methods) are available, however, due to the relatively high complexity of these solutions with regard to in many problems of soil mechanics, investigating the final bound states is considered, ways in which you can analyze them without having to solve the whole problem, have been considered by engineers and researchers. Such methods are known as bound methods, and in response to the most fundamental issues of stability in soil mechanics and slope stability have been highly developed. Due to the problems manually is acceptable to create a stress field; the majority of previous studies were focused on using upper bound method of limit analysis, and although the upper limit can be a very good guess is to answer the questions but the bottom line is in interest because they are used to ensure performance problems. With the present study, we try to extend the solution, promotion of the shortcomings and provide a criterion for determining the validity of the of other researchers by an optimal presentation and a mesh roof, three-dimensional flattened gables. The hypothesis that is used generally in current calculations of sustainability of gables is plane strain behavior, or in other words, a two-dimensional problem solution, there is a variety of three-dimensional soil behavior, and it is assumed that the two-dimensional problem does not lead to good solutions. For example, failure to take into account the three-dimensional behavior effects on the soil shear strength parameters in the broken gable roofs, these parameters are obtained higher than the actual values.

Research Literature:

Three-dimensional analysis of Azzouz & Baligh was among the first works has been proposed based on the limit equilibrium method [2, 3]. The two researchers in 1975, 1978.1981 and 1983 have been investigated the lateral surfaces slippery pile in cohesive soils. The analysis results of the research are presented in diagrams of figure 1. In 1985, Ugai has used the combination of the calculating methods of changes and bound balance which were presented by Garbr and Baker (1977) for the two-dimensional stability analysis of the gables and was extended to three-dimensional case [4]. In this method, as shown in Figure 2, the slide mechanism includes a cylinder with a limited length and attached to curved surfaces side, and the minimum of safety factor satisfying moments balance equation is obtained.

Three-dimensional stability analysis of soil friction is more difficult and requires more assumptions. After analysis of Kryzek and Geiger and the works of Steiner in 1977 and in 1975 Azuz and Baligh were among the

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first efforts in this regard are based on the limit equilibrium method [5]. The sliding surface and the shear stresses are considered in this analysis as in Figure 1, and suppose that each point of the surface of the slide:

A) Vertical effective stress is a principal stress.

B) Horizontal stress in cross-section, depending on the sign of the slope tangent to the sliding surface, is the largest and smallest principal stresses.

C) The third principal stress, K is equal to the vertical stress.

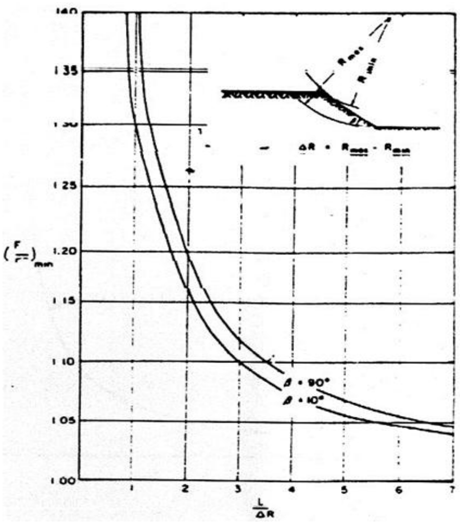


Fig. 1: The ratio of the three-dimensional to two-dimensional safety factors of Azuz and Baligh mode analysis [5].

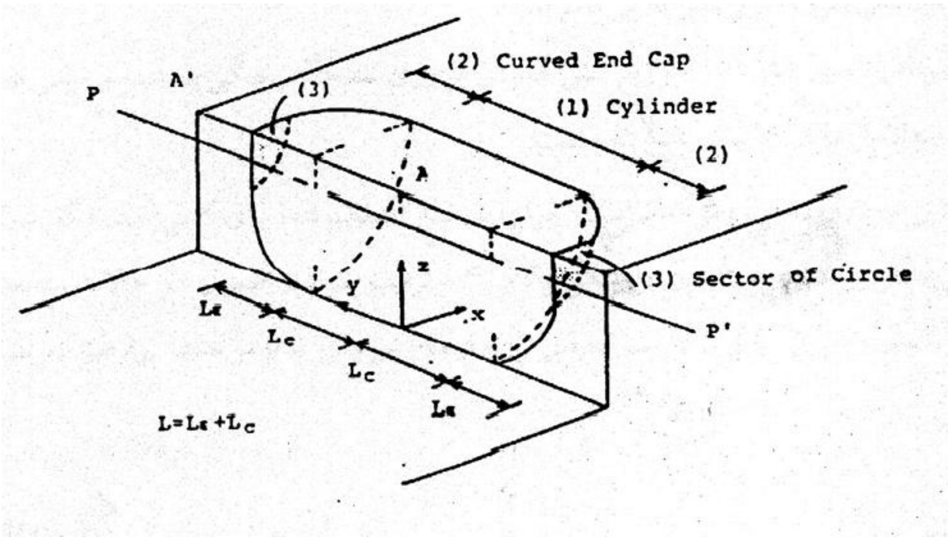


Fig. 2: Mechanism slip in Ugai technique [6]

In 1977, Hovland provided a method for three-dimensional analysis of soil cohesion and internal friction [7]. In this method, an extension of the method is from two-dimensional to three-dimensional pieces.

In 1928, Chen & Chameau designed a way that its time limitations was lower than the methods presented in [8]. In this method, assuming a rotational mechanism, the sliding mass is divided into vertical columns. This mechanism includes a cylindrical symmetry and connects the two sides of the ellipsoid that is shown in figure 3. As mentioned, the use of the calculus of variations based on the limit equilibrium is another trend that some researchers have used it in the stability analysis. In this context we can note the analyze of Kopacsy in 1957. The sliding surface shape, the distribution of normal stress and shear forces on the sliding surfaces of unknowns equations have been used Kopacsy analysis [9].

After him, Baker & Garber in 1977 and 1978, Castilla & Revilla in 1977, Ramamurthy in 1977 and Leshchinsky et al. in 1985, 1986, 1987, 1988 and 1991 were used this method in slope stability analysis [10].

Duncan in an article published in 1996, while performing a comprehensive overview of the history of the undertaken works by the slope stability questioned the formulation of some recent methods. For example, the observed responses of these methods are actually less than 30% of the responses. However, the combination of the limit equilibrium method and the calculus of variations produces an appropriate three-dimensional stability analysis of the gable [9].

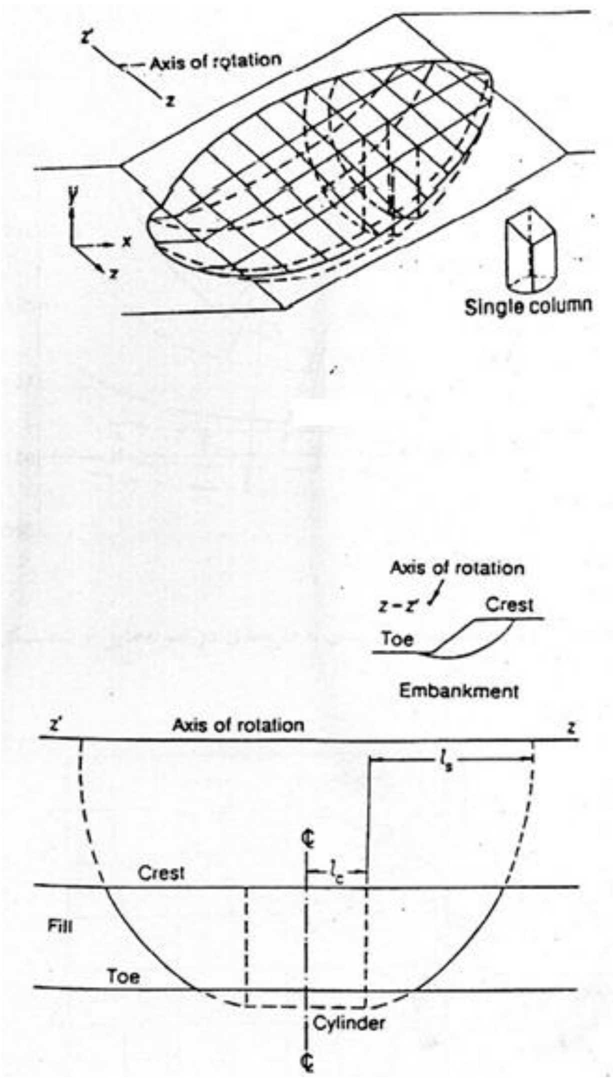


Fig. 3: Mechanism analysis of landslide Chen [8]
(1) Overview of the sliding mass
(2) Front view of a sliding mechanism

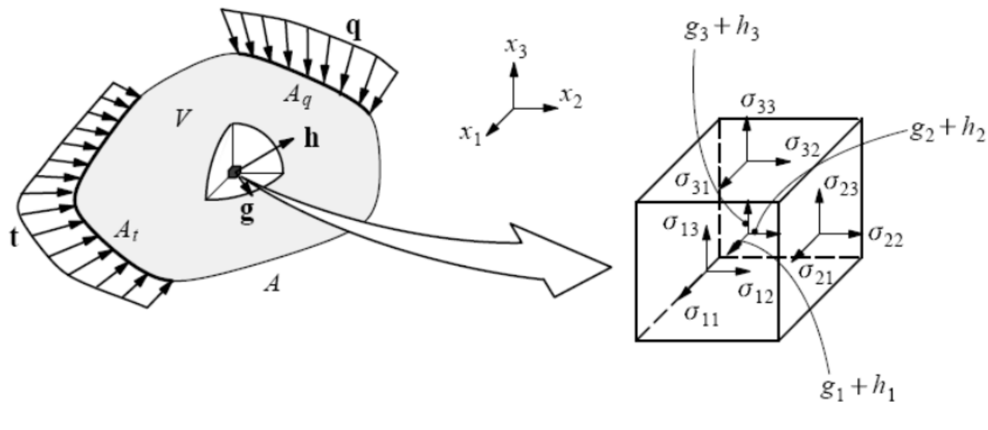


Fig. 4: The object that is under volume and surface forces.

Analysis Formulation:

A body with volume V and area A is considered, as seen in Figure 5, and let t and q, respectively, are transactions on At and Aq and g and h are fixed and unknown dynamics that the volume V is applied. By these hypotheses, the main aim of these calculations that are constructed based on lower bound theory is to find the tension distribution which provides the equilibrium equations in V volume, bound circumstances in Aq and At and surrender index in total environment and maximizes the followed integral.

$$Q = \int_{A_q} \mathbf{q}\, dA + \int_V \mathbf{h}\, dV \tag{3}$$

This problem can be solved for very simple modes, but using the Sloan relationships, acceptable tension fields for complex problems regarding materials geometry, loading and heterogeneity and the best answer can be evaluated that is the nearest to the accurate response. The best method to do this is the finite element method. By renouncing the kind of the element that is used to calculate the tension level, each formulation results to an optimization problem based on lower bound theory.

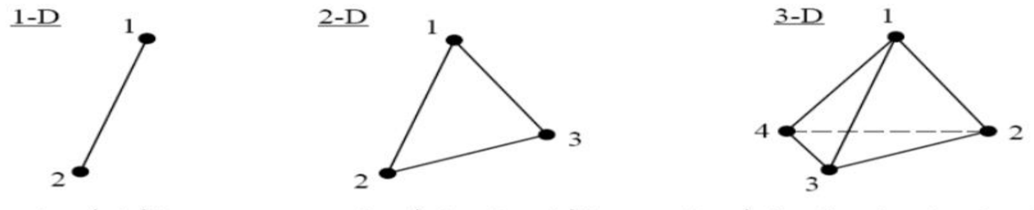
$$\begin{aligned} &\text{maximise } Q(\mathbf{x}) \\ &a_i(\mathbf{x}) = 0 \quad ; \quad i \in I = \{1,\ldots,m\} \\ &f_j(\mathbf{x}) \le 0 \quad ; \quad j \in J = \{1,\ldots,r\} \\ &\qquad \mathbf{x} \in \mathrm{R}^n \end{aligned}$$

The main efficiency of the linear finite element method is:

- The final optimization equations are linear in this method. This method causes to gain the best and the nearest answer in each repetition.
- Since the changes are linear between the points, if the surrender equation is satisfied, it is satisfied for all the points between them certainly and because of it, tension index is provided for all the points related to the model. Therefore, the number of the equations that are used for satisfying the surrender index in the entire model is reduced and increases the speed of the analysis.
- In this method, the separations of the tension between the neighboring elements easily are identified.
- There are bounds in the most of the geotechnical problems that should be modeled accurately in the situations that loading is done on a curve surface, which can be provided by a better approximate of the equations relating to the bound situations.

Linear finite element:

In this numerical formulation that is written according to the lower bound theory, the simplest elements are in the form of the Figure 6 that is used in modeling.



$\boldsymbol{\sigma}^l = \{\sigma^l_{11}\}^{\mathrm{T}}$ $\quad \boldsymbol{\sigma}^l = \{\sigma^l_{11}\ \sigma^l_{22}\ \sigma^l_{12}\}^{\mathrm{T}}$ $\quad \boldsymbol{\sigma}^l = \{\sigma^l_{11}\ \sigma^l_{22}\ \sigma^l_{33}\ \sigma^l_{12}\ \sigma^l_{23}\ \sigma^l_{13}\}^{\mathrm{T}}$

Fig. 5: Simplest kind of elements for different dimensions.

Separation Equations:

To describe the separation between the neighboring elements, it should be provided a set of conditions in the form of adverb on variables of each element. Figure 7 shows a tension separation in a two-dimensional mode.

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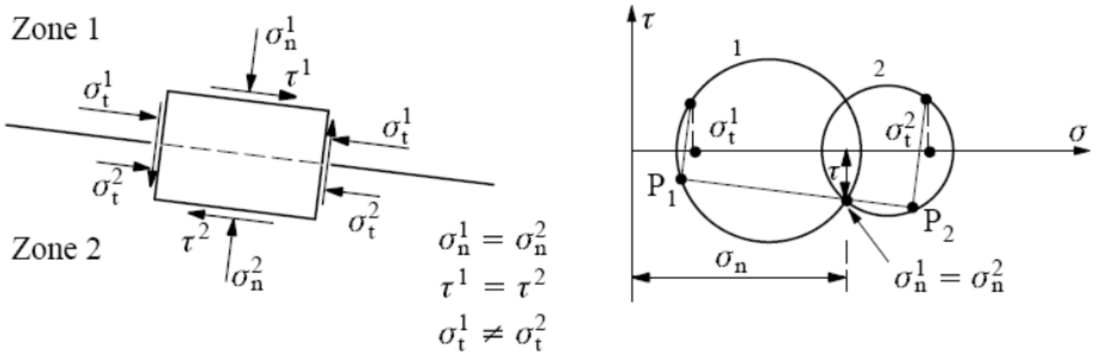

Fig 6: Tension separation & its mouher circles in all parts.

Since the tensions are changed between the elements, if normal and cutting tensions are equal in all neighboring elements, the equation is maintained, the problem is shown in Figure 8.

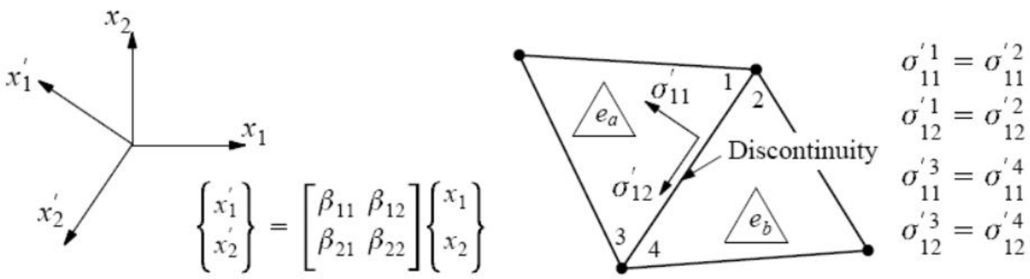

Fig. 7: Tension separation in neighboring elements.

Bound Situations:

It is assumed that a finite surface force t_p, (p is a set of finite constraints of N_p) is applied on a part of bound surface of A_t (Figure 9). In such situation that the forces are acted like a total coordinates and on a linear bound surface A^d_t, the bound tension can be defined for each L point as follows.

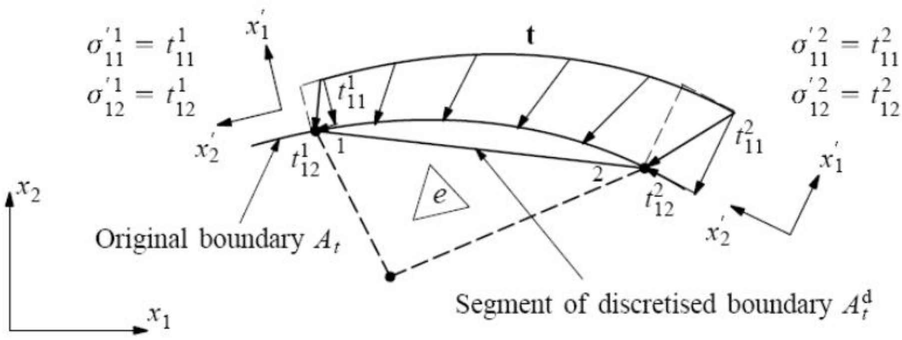

Fig. 8: Bound situation of tension in a two-dimensional mode.

$$\sigma^l_{pi}\,\beta^l_{ik} = t^l_p \;;\; p \in P \tag{4}$$

Surrendering Condition:

The solving method that is provided in this research is not dependent to a special soil behavioral model, but the models that define the surrendering condition should be convex and having a straight surface.

Convexity is necessary to ensure the best absolute optimized responses exist and also the complete plastic problems hypotheses should be provided. Also, having a straight surface is important in order to calculate the first and second function derivatives regarding the unknown tensions. For surrendering functions that have the Takin points like Tereska and Mouher Colomb, it is necessary to select a straight approximate.

Developing Tension Field in Middle of Infinite:

As mentioned above, there is the surrendering condition, bound situations and lower bound theory analysis based on satisfying the tension separations of equation problems for all the existing points. The conditions are provided for all points of the models having a middle infinite space. Therefore, this method can't be used. But this method also can approximate a suitable response near to the accurate response, but we can't be sure about the responses of lower bound that the conditions are provided for all existing points in the model. Moreover, we should find a solution for the middle infinite region to support the parts using the lower bound condition. This subject can often follow with some big problems, because some vast bounds are in unknown shapes.

Two different methods can be used to solve this problem. In the first method, the complete plastic region is approximated, and is supported by some elements. When the target function is calculated based on this mesh, another element layer is added to the first layer. Therefore, a new account is obtained for the target function (Figure 10).

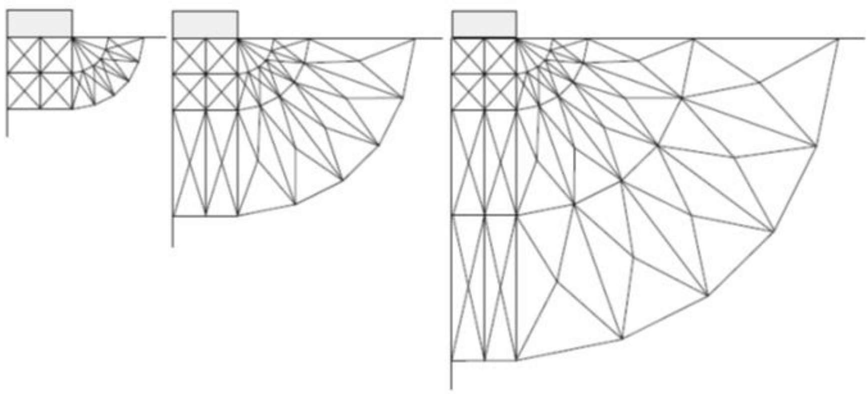

Fig. 9: Developing tension field using the mesh increasing.

This process is continued until the response of target function remains unchanged. Surely, this method does not provide a guarantee to have an answer for lower bound, but actually this method have the suitable responses. It can be used for all the kind of surrendering indexes, and also it is not difficult for the programs that have automated mesh.

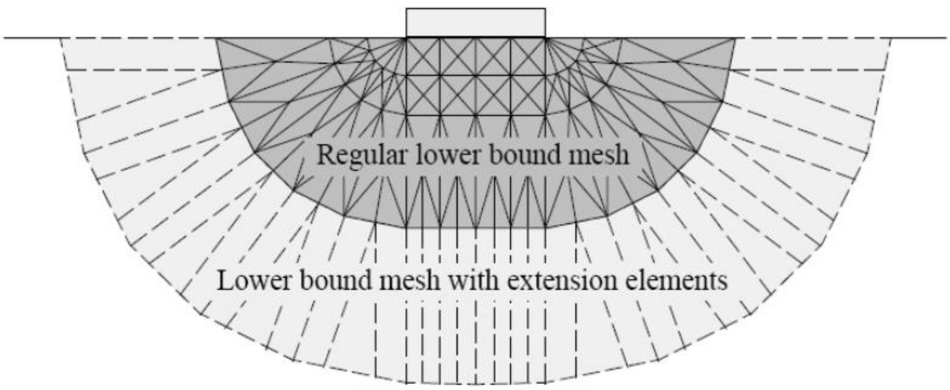

Fig. 10: Developing tension field in the middle infinite region by develop elements.

This mesh is provided to develop the tension field in the middle infinite region. To do modeling in a two-dimensional environment, PASTOR (1978) show that the maximum of two different elements is needed. It is shown in this research that for a D dimensional space, D number of development elements is necessary. Using these elements removes exam and trial process of the previous method. The development elements should provide, in addition to some special adverbs, all bound conditions of analysis include equilibrium equations, tension separation, bound condition and surrendering index. Assuming the two-dimensional condition in Figure 11, the development elements including 3 points are shown. If the equilibrium equations and bound conditions are provided for these elements, all points between them will be supported by these conditions.

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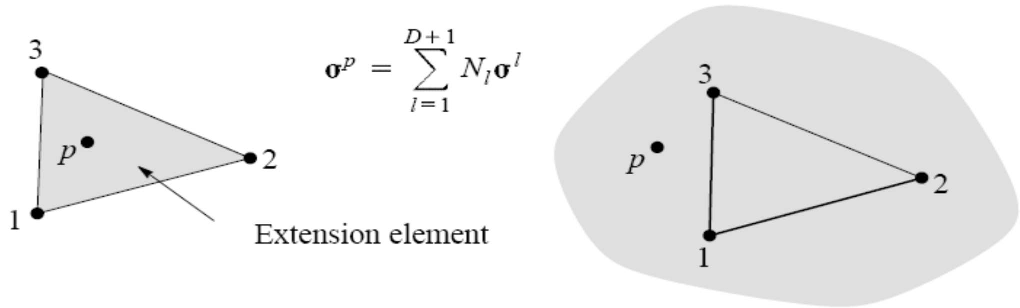


Fig. 11: approximation of internal & external tension field of development elements.

It is clear in surrendering situations that another condition should be added to these elements. As mentioned above, the condition of $f(\sigma^L) \leq 0$ for L=1,2,3 points should be satisfied that surrendering condition is considered for a point that is not outside of this space. Thus, more care should be applied to add a condition in the form of an adverb support a point outside pf this environment. For make it simpler, surrendering indexes are more considered to have not curved main tensions (including Tereska, Van Mices, Dargraper and Mouher Colomb).

Adverbs Total Equation:

In some parts of this research, the necessary stages for formulation of the theory to lower bound analysis are investigated in a finite element environment. This formulation is ready to be given to an optimization program. Just one remaining of this stage, is providing an adverb matrix and coefficient of target function for total meshing the model.

Using the equilibrium equations, tension separation and surrendering index, different equal adverbs are shown like following equal equation.

$$\mathbf{A} = \sum_{e=1}^{E} \mathbf{A}^{e}_{equil} + \sum_{d=1}^{Ds} \mathbf{A}^{d}_{equil} + \sum_{b=1}^{Bn} \mathbf{A}^{b}_{bound}.$$

Where E is the number of the all elements, Ds is the number of all of the tension separations and Bn is the number of all of the bound points that are bounded by the surface forces. All the coefficients are placed suitably in related rows and columns according to summing rules of the elements. Similarly b vector is given as follows.

$$\mathbf{b} = \sum_{e=1}^{E} \mathbf{b}^{e}_{equil} + \sum_{d=1}^{Ds} \mathbf{b}^{d}_{equil} + \sum_{b=1}^{Bn} \mathbf{b}^{b}_{bound}.$$

When the tension field is modeled using the finite element method, target function and adverbs equations act linearly and surrendering condition acts between the variables. Then, the final target function and adverbs equations are summarized as follows.

$$\text{maximise } \mathbf{c}^{\mathrm{T}}\mathbf{x}$$

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$f_j(\mathbf{x}) \leq 0 \quad ; \; j \in J$$

$$\mathbf{x} \in \mathrm{R}^n$$

Where c vector of target function's coefficients by n length, A of M×N matrix should be calculated by the coefficient of adverbs of the equation, (X), fj (x) is surrendering function and X is the variables vector.

In this part, formulation of the linear finite elements method is investigated for the next 1, 2, 3 modes, according to this formulation, the obtained results are all based on the lower bound that has the ability of applying the force, volume forces modeling, solving the geometric complex problems and heterogeneous materials. To ensure that the providing method for all the points, gives the lower bound conditions, the theory and relations of the development elements for half infinite regions.

Target function:

The aim of the lower bound analysis is to find an acceptable tension field which optimizes (maximizes) the target function. Target function is defined based on the combination of the surface and internal forces (Figure 12). Safety coefficient is defined as target function for strength of the gables. In another words, the general shape of the surrendering condition has the plastic act completely in soil structures that is defined as follows:

$$f(\sigma_{ij}) \leq 0$$

For the indexes that are in regions with individual points, for example, Mouher Colomb index, the straighten approximation should be used in surrendering surface. A profile of the optimized index of Maouher Colomb is shown in Figure 13.

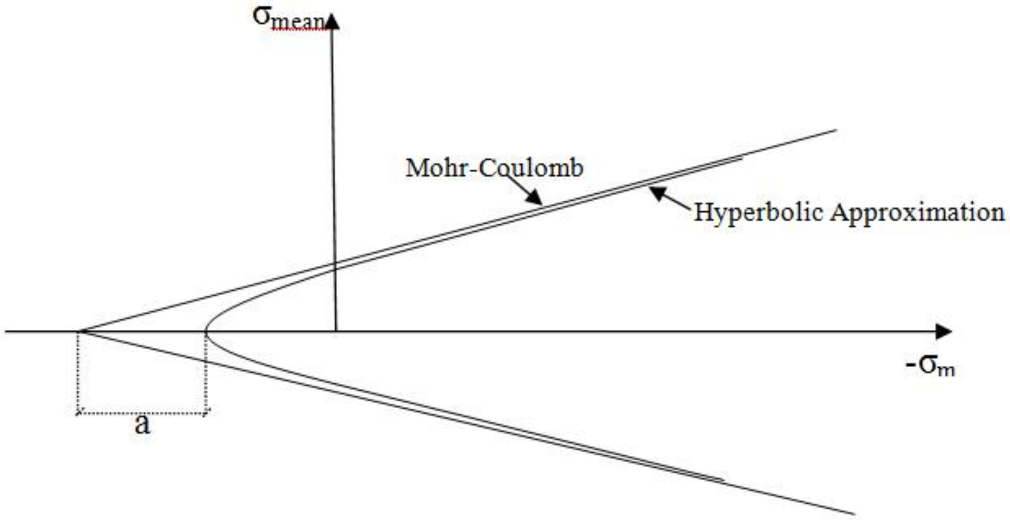


Fig. 12: optimized index of Mouher Colomb.

The trial of the method that could be placed for other analysis methods was impossible without providing suitable software because of the complexity of the three-dimensional in optimization methods. Thus, programming software with the desire abilities becomes necessary. The software is provided by the name of FELAB and is used in order to the research be continued. (Finite Element Method + Limit Analysis + Lower Bound).

It should be noted that the FELAB software is used for modeling the gables with the system of finite elements and developed elements, then equal and unequal adverbs resulting from lower bound conditions (equilibrium equations, tension separations equations and bound conditions equations) and surrendering condition are defined, then considering the target function of maximum account, the safety coefficient of gables is obtained in lower bound conditions.

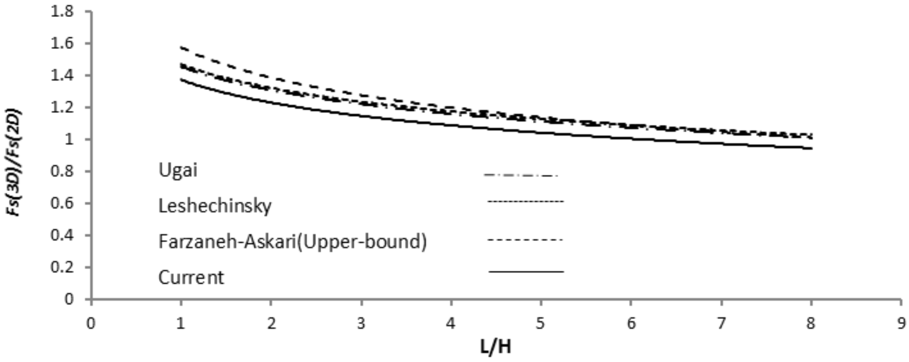
After finishing the analysis, the results are shown in a vector including an acceptable tension field. By placing these tensions in surrendering index, it is obvious that some points are in plastic and some in elastic mode.

Using the present method shows that research results give an interval with upper bound results (Frazaneh & Askari [11]) that surely provide the accurate response.

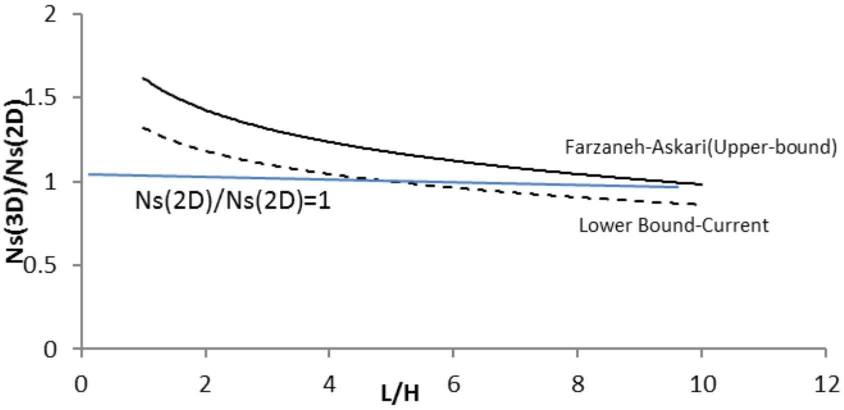
By considering the new meshing method that is used in this research, the research has the capability of gables modeling which have the variations in the plan, and up to now none of the previous methods don't have the ability of this kind of modeling based on the lower bound theory.

Comparing the results of three-dimensional analyses:

In Figure 14a, the results of three-dimensional analyses of Ugai [8], Farzaneh & Askari [17] are provided in 1985, 1986 and 2003 respectively, are compared with the results of the present study. As it is obvious, the results of FELAB program are very satisfying. These results are obtained from analysis of the models with 72 elements. By comparing the results of the Bishop and present method, it is seen that in lower L/Hs, in addition to weak results of the present method, more results obtained comparing the Bishop method. The comparison is shown in Figure 14b.



(A)



(B)

Fig. 13(a): The comparison of the relation of three-dimensional to two-dimensional coefficient in Ugai, Leshinski, Frazaneh-Askari (upper bound) and the present study, (b) the comparison of the results of Bishop and the present study (Tootoonchi 2012 [60]).

Applied Comparison:

In order to do a comparison between the identified and two-dimensional results, the bound equilibrium method (simplified Bishop method), the comparison of an applied example relating to the straight gables is provided.

The characteristics of the gable:

Slope angle (β): 60 degree, (H) height of the gable: 4 m, (L) width of the gable: 16m, (ϒ) specific weigh of soil: 17 kN/m^3 , (ϕ) internal friction angle of oil: 17 degree, and (C) adhesion of soil: 10 kN/m^3 .

Solving the above problem in summarized multiple stages

1. Considering $\lambda_{\phi c}$ parameter with obscure dimension and gable slope, relating to present gable analysis diagram is found.

$$\lambda_{\phi c} = \frac{(17 \times 4)}{10} \tan 17 = 2$$

One of the present diagram weaknesses is that these diagrams are provided just for $\lambda = 2$. In order to comprehensiveness of these diagrams, the more performance of these diagrams is needed.

2. For $\beta = 60^\circ$, the 18-5 diagram is chosen.

3. In this diagram, a straight line resulting from $L/H = 4$ is designed. This line cuts the lower and upper bound curves.

4. The two-dimensional lasting number is derived using the table (1-5) that the number of it is $Ns(2D) = 8.4$ equals to Fs=1/23. Using the cutting points, the values of $\frac{Ns_{3D}}{Ns_{2D}}$ is obtained. By these accounts, the values of safety coefficient of lower bound, Fs=1/29, and safety coefficient of upper bound, Fs=1/35, are obtained.

The average of the upper bound and lower bound, and safety coefficient is calculated for $L/H = 4$ that equals to 1/23. By considering the other values for L/H, $L/H = 1$, $L/H = 2$, $L/H = 3$ will equal to 1/45, 1/42 and 1/39. As mentioned, three-dimensional safety coefficient in 1-1/16 equals to two-dimensional safety coefficient. It could be concluded that two-dimensional analysis of the gables acts for designing to ensure and return analysis in counter assurance. In other side, the difference between the safety coefficient of upper and lower bound is calculated approximate to 18% that this value is reduced slowly by increasing the L/H.

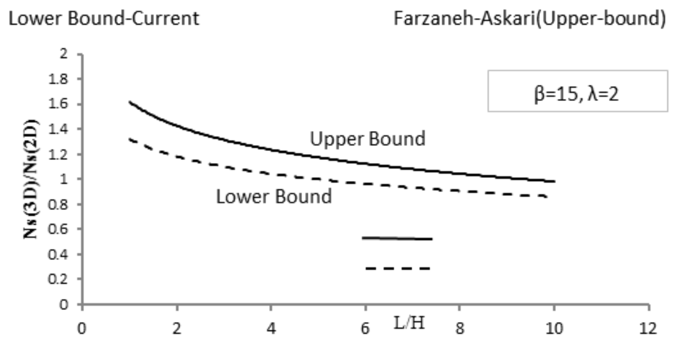


Fig. 14: The results of lower bound of lasting the gable in β=15, λ=2

Mode:

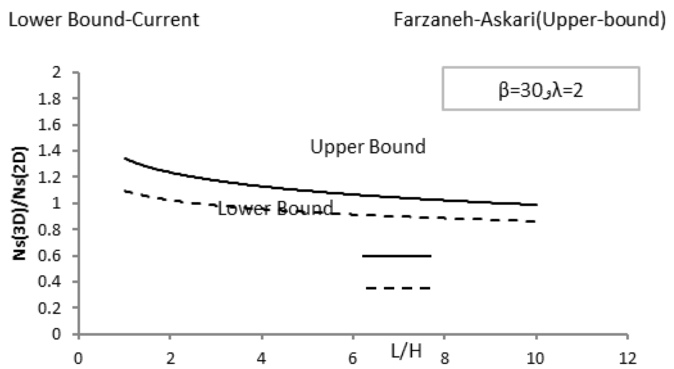


Fig. 15: The results of lower bound of lasting of gable in β=30, λ

Mode:

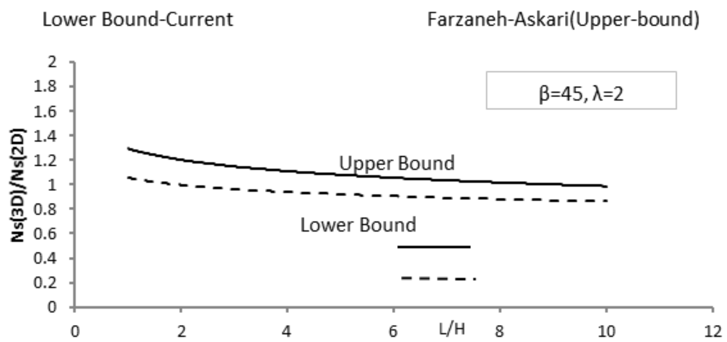


Fig. 16: The results of lower bound of lasting of gable in β=45, λ=2.

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Conclusion:

1. Providing the vague dimensional diagrams, the results of the program were compared by other researchers' results, and their difference will be shown as diagrams.
2. The obtained results from the present method limit these results and the responses of the upper bound in an interval of ±9% or lower. This can help to have an accurate response and evaluate the obtained ones. As it can be investigated from diagrams, the value of $\frac{F_{3D}}{F_{2D}}$ increases by reducing the β and L/H.
3. By considering the applied example, the results of three-dimensional gable for 4 modes of $L/H = 1, 2, 3, 4$ in intervals of 1-1/16 equals the results of two-dimensional analysis.
4. The strength number Ns reduces by increasing β and L/H, and if L/H moves towards infinite, the Ns value minimizes for an identified number of β, and Ns value reaches to its minimum number. This shows that the reduction of safety coefficient causes the increase of L/H.
5. Considering the applied diagrams, it is clear that if the soils has more adhesion, the relation of Fs(3D)/Fs (2D) increases.

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