Medical Application for Ordered Topological Approximation Spaces

M.E. El-Shafei, A.M. Kozae and M. Abo-Elhamayel

Mathematics Department, Faculty of Science, Mansoura University, Egypt
Mathematics Department, Faculty of Science Tanta University, Egypt

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ABSTRACT

Although, rough set theory is a powerful mathematical tool for dealing with inexact, uncertain or vague information. The purpose of this paper is to use new approximations based on ordered topological spaces and rough sets. This operator explores that ordered topological spaces can be used to analyze many real life problems.

Key words: mathematical tool, ordered topological, rough sets

INTRODUCTION


Approximation operators draw close links between rough set theory and topology. The purpose of using rough sets is to find the core, that is the set of all indispensable features. The purpose of the present work is to put a starting point for the applications of abstract topological theory into rough set analysis.

2. Preliminaries:

In this section, we give an account for the basic definitions to be used in the paper.

Definition 2.1:

[3]. A triple \( (U, \tau, \rho) \), where \( U \) be any non empty set, \( \tau \) is a topology on \( U \) and \( \rho \) is a partially order relation on \( U \), is said to be a topological ordered space.

Definition 2.3:

[7]. An information system is a pair \( (U, A) \), where \( U \) is a non-empty finite set of objects and \( A \) is a finite set of attributes.

Definition 2.4:

[4]. Let \( (U, A) \) be an information system. A reduct is a subset of attributes that are sufficient to describe the decision attributes.

Definition 2.5:

[4]. Let \( (U, A) \) be an information system. The core is the intersection of all possible reducts.

Definition 2.6:

[2]. A general approximation space is a pair \( (U, R) \), where \( U \) is a non-empty finite set of objects and \( R \) is a general relation on \( U \).
[11]. Let \((U, R)\) be a general approximation space. For \(A \subseteq U\). The lower and upper approximations of \(A\) in \((U, R)\) are respectively defined as:

\[ \overline{R}A = \{x \in U : R_S(x) \subseteq A\}, \]

\[ \overline{R}X = \{x \in U : R_S(x) \cap A \neq \emptyset\}, \]

Where \(R_S(x) = \{y \in U : (x, y) \in R\}\).

According to these definitions, we can see that lower approximation set of \(A\) is a subset of \(A\) and \(A\) is a subset of the upper approximation set of \(A\).

Definition 2.8:

[2]. Let \((U, R)\) be a general approximation space and \(A \subseteq U\). The positive and negative regions of \(A\) are respectively defined as:

\[ \text{Pos}(A) = \overline{R}(A), \text{Neg}(A) = U - \overline{R}(A). \]

3. New approximations via ordered topological spaces:

Definition 3.1:

A non empty set \(U\) equipped with a general relation which generate a topology \(\tau_R\) on \(U\) and a partially order relation \(\rho\) is said to be a generalized ordered topological approximation space (GOTAS, for short) and has the form \((U, \tau_R, \rho)\).

Definition 3.2:

Let \((U, \tau_R, \rho)\) be a GOTAS and \(A \subseteq U\). We introduce the following concepts:

1. \(\overline{R}^{-\text{Inc}}(A) = A^{-\text{Inc}}, A^{-\text{Inc}}\) is the greatest increasing open subset of \(A\) is called \(\overline{R}\) lower increasing of \(A\).

2. \(\overline{R}^{-\text{Dec}}(A) = A^{-\text{Dec}}, A^{-\text{Dec}}\) is the greatest decreasing open subset of \(A\) is called \(\overline{R}\) lower decreasing of \(A\).

Example 3.3:

Here we consider the problem of Flu, a disease transmitted to humans by tiny wet drops produced when a person coughs, sneezes or talks. A person can get the flu by breathing in these wet drops or by touching items and surfaces covered with these drops and then touching their mouth, nose or eyes. It may happen that the decision attribute depends not on the whole set of condition attributes but on a subset of it taking in mind the ordering of attributes by a partially order relation and hence we are interested to find this subset which is given by the core.

Consider the following information table which giving data about 8 patients. \(P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\} \) and \(P_8\) and the attributes are Caugh (C), Headache (H), Nausea (N) and decision flu. The columns of the table represent the attributes (the symptoms for flu) and the rows represent the objects (the patients). The entries in the table are the attribute values. The patient \(P_5\) is characterized by the value set (Cough, No), (Headache, Yes), (Nausea, Yes), (Temperature, High) and (Flu, No), which gives information about the patient \(P_5\). In the table, the patients \(P_1, P_2, P_3, P_4, P_5, P_6, P_7\) and \(P_8\) are indiscernible with respect to the attribute 'cough'.

Let \(R = \{(P, q) : P, q \text{ has the same value attribute Caugh}\}\). Then the attribute 'Cough' generates two equivalence classes, namely, \(\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}\) and \(\{P_4, P_5\}\).

<table>
<thead>
<tr>
<th>Patients</th>
<th>Caugh(C)</th>
<th>Headache(H)</th>
<th>Nausea(N)</th>
<th>Temperature(T)</th>
<th>Flu</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_1</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>High</td>
<td>Yes</td>
</tr>
<tr>
<td>P_2</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>High</td>
<td>No</td>
</tr>
<tr>
<td>P_3</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>High</td>
<td>Yes</td>
</tr>
<tr>
<td>P_4</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Very high</td>
<td>No</td>
</tr>
<tr>
<td>P_5</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Normal</td>
<td>No</td>
</tr>
<tr>
<td>P_6</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Very high</td>
<td>Yes</td>
</tr>
</tbody>
</table>
\[ R = \{(P,q) : P,q \text{ has the same value attributes Caugh and Headache} \} \]. Then the attributes 'Cough' and 'Headache' generate the equivalence classes \( \{P_1, P_6, P_7, P_8\}, \{P_2, P_3\}, \{P_4\} \) and \( \{P_5\} \).

\[ R = \{(P,q) : P,q \text{ has the same value attributes Caugh, Headache, Nausea and Temperature} \} \]. Then the equivalence classes for the attributes Cough, Headache, Nausea and Temperature are \( \{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\} \) and \( \{P_7\} \).

**Case 1:**

Let \( \rho_1 \) be a relation which determined from C,H and N. Then
\[
\rho_1 = \{(P_1, P_1), (P_2, P_2), (P_1, P_2), (P_2, P_1), (P_3, P_3), (P_1, P_3), (P_2, P_3), (P_3, P_1), (P_4, P_4), (P_1, P_4), (P_2, P_4), (P_3, P_4), (P_4, P_1), (P_5, P_5), (P_1, P_5), (P_2, P_5), (P_3, P_5), (P_4, P_5), (P_6, P_6), (P_1, P_6), (P_2, P_6), (P_3, P_6), (P_4, P_6), (P_7, P_7), (P_1, P_7), (P_2, P_7), (P_3, P_7), (P_4, P_7), (P_5, P_8), (P_1, P_8), (P_2, P_8), (P_3, P_8), (P_4, P_8), (P_5, P_1)\}.
\]

The topology generated by C,H,N and T has the base \( \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\} \} \).

From \( \rho_1 \):

The topology of all increasing open sets \( \tau^{Inc} \), is
\[
\tau^{Inc} = \{U, \phi, \{P_1\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_7\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\} : P_8 \}\.
\]

The set of all increasing closed sets \( \tau^{Dec} \), is
\[
\tau^{Dec} = \{U, \phi, \{P_2\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\}, \{P_5, P_6, P_8\} : P_8 \} ;
\]

The set of all decreasing closed sets \( \tau^{Dec} \), is
\[
\tau^{Dec} = \{U, \phi, \{P_1\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_7\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\}, \{P_5, P_6, P_8\} : P_8 \} ;
\]

The set of all increasing closed sets \( \tau^{Inc} \), is
\[
\tau^{Inc} = \{U, \phi, \{P_1\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_7\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\}, \{P_5, P_6, P_8\} : P_8 \} ;
\]

The topology of all increasing open sets \( \tau^{Inc} \), is
\[
\tau^{Inc} = \{U, \phi, \{P_1\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_7\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\}, \{P_5, P_6, P_8\} : P_8 \} ;
\]

The set of all increasing closed sets \( \tau^{Dec} \), is
\[
\tau^{Dec} = \{U, \phi, \{P_1\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_7\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\}, \{P_5, P_6, P_8\} : P_8 \} ;
\]

The set of all increasing closed sets \( \tau^{Dec} \), is
\[
\tau^{Dec} = \{U, \phi, \{P_1\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_7\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\}, \{P_5, P_6, P_8\} : P_8 \} ;
\]

Now, we have \( R(X) = \{P_1, P_6, P_8\} \), \( R(X) = \{P_1, P_2, P_3, P_6, P_7, P_8\} \), \( R(X) = \phi \) and \( R(X) = \{P_1, P_2, P_3, P_6, P_7, P_8\} \).

Omitting C:

The topology generated by H,N and T has the base \( \{\{P_1\}, \{P_5\}, \{P_2\}, \{P_6, P_8\}, \{P_7\} \} \).

The topology of all increasing open sets \( \tau^{Inc} \), is
\[
\tau^{Inc} = \{U, \phi, \{P_1\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_7\}, \{P_2, P_3, P_4, P_6, P_8\}, \{P_2, P_5, P_6, P_8\}, \{P_3, P_5, P_6, P_8\}, \{P_4, P_5, P_6, P_8\}, \{P_5, P_6, P_8\} \} ;
\]
The topology of all decreasing open sets $\tau_{\text{Dec}}^c$, is

$$
\tau_{\text{Dec}}^c = \{U, \phi, \{P_1\}, \{P_2, P_3, P_4\}, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_2, P_3, P_4\}, \{P_2, P_4\}, \{P_6, P_7, P_8\}\}.
$$

Since, $\tau_{\text{Dec}}^c \neq \tau_{\text{Inc}}^c$ and $\tau_{\text{Dec}}^c \neq \tau_{\text{Dec}}^t$ then $C$ is not increasing (decreasing) superfluous.

Omitting $H$:

The topology generated by $C, N$ and $T$ has the base \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}.

Then we have $\tau_{\text{Dec}}^H = \tau_{\text{Inc}}^H$ and $\tau_{\text{Dec}}^H = \tau_{\text{Dec}}^H$. Hence $H$ is increasing (decreasing) superfluous.

Omitting $N$:

The topology generated by $C, H$ and $T$ has the base \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}.

Then we have $\tau_{\text{Dec}}^N = \tau_{\text{Inc}}^N$ and $\tau_{\text{Dec}}^N = \tau_{\text{Dec}}^N$. Hence $N$ is increasing (decreasing) superfluous.

Omitted $T$:

The topology of all increasing open sets $\tau_{\text{Inc}}^T$, is

$$
\tau_{\text{Inc}}^T = \{U, \phi, \{P_1\}, \{P_1, P_2\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4, P_6\}, \{P_1, P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4\}, \{P_1, P_2, P_4\}, \{P_1, P_2, P_5\}, \{P_1, P_2, P_6\}, \{P_1, P_2, P_7\}, \{P_1, P_2, P_8\}, \{P_1, P_2, P_9\}\};
$$

and

The topology of all decreasing open sets $\tau_{\text{Dec}}^T$, is

$$
\tau_{\text{Dec}}^T = \{U, \phi, \{P_1\}, \{P_1, P_2\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_3, P_4\}, \{P_2, P_3, P_4\}, \{P_2, P_3, P_4, P_5\}, \{P_2, P_3, P_4, P_6\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_6, P_7\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_5\}, \{P_2, P_3, P_4, P_5, P_6\}, \{P_2, P_3, P_4, P_6\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_6, P_7\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_2, P_3, P_4\}, \{P_2, P_3\}, \{P_2, P_4\}, \{P_2, P_5\}, \{P_2, P_6\}, \{P_2, P_7\}, \{P_2, P_8\}, \{P_2, P_9\}\}.
$$

Since $\tau_{\text{Inc}}^T \neq \tau_{\text{Dec}}^T$ and $\tau_{\text{Dec}}^T \neq \tau_{\text{Dec}}^T$, then $T$ is not increasing (decreasing) superfluous. Therefore increasing (decreasing) core= $\{C, T\}$.

Case 2:

Let $\rho_2$ be a relation which determined from $C, H, N$ and $T$. Then

$$
\rho_2 = \{(P_1, P_1), (P_1, P_2), (P_1, P_3), (P_1, P_4), (P_1, P_5), (P_1, P_6), (P_1, P_7), (P_1, P_8), (P_2, P_1), (P_2, P_2), (P_2, P_3), (P_2, P_4), (P_2, P_5), (P_2, P_6), (P_2, P_7), (P_2, P_8), (P_3, P_1), (P_3, P_2), (P_3, P_3), (P_3, P_4), (P_3, P_5), (P_3, P_6), (P_3, P_7), (P_3, P_8), (P_4, P_1), (P_4, P_2), (P_4, P_3), (P_4, P_4), (P_4, P_5), (P_4, P_6), (P_4, P_7), (P_4, P_8)\}.
$$

The topology generated by $C, H, N$ and $T$ has the base \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}.

From $\rho_2$:

The topology of all increasing open sets $\tau_{\text{Inc}}^T$, is

$$
\tau_{\text{Inc}}^T = \{U, \phi, \{P_1\}, \{P_1, P_2\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4, P_6\}, \{P_1, P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5\}, \{P_1, P_2, P_3, P_4\}, \{P_1, P_2, P_4\}, \{P_1, P_2, P_5\}, \{P_1, P_2, P_6\}, \{P_1, P_2, P_7\}, \{P_1, P_2, P_8\}, \{P_1, P_2, P_9\}\};
$$

and

The set of all decreasing closed sets $\tau_{\text{Dec}}^C$, is

$$
\tau_{\text{Dec}}^C = \{U, \phi, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}, \{P_1, P_2, P_3, P_4, P_5\}.
$$
The topology of all decreasing open sets $\tau^{\text{Dec}}$, is
$\tau^{\text{Dec}} = \{U, \phi, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\},\{P_1, P_2, P_3, P_4, P_5\},\{P_1, P_2, P_3, P_4\},\{P_1, P_2, P_3\},\{P_1, P_2\},\{P_1\}\};$

The set of all increasing closed sets $\tau^{\text{Inc}}$, is
$\tau^{\text{Inc}} = \{U, \phi, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\},\{P_1, P_2, P_3, P_4, P_5\},\{P_1, P_2, P_3, P_4\},\{P_1, P_2, P_3\},\{P_1, P_2\},\{P_1\}\};$

Now, we have $R(X) = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$ and $-\text{Dec} R(X) = \emptyset$.

Omitting C:

The topology generated by H,N and T has the base $\{\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}\}.$

The topology of all decreasing open sets $\tau^{\text{Dec}}$, is
$\tau^{\text{Dec}} = \{U, \phi, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\},\{P_1, P_2, P_3, P_4, P_5\},\{P_1, P_2, P_3, P_4\},\{P_1, P_2, P_3\},\{P_1, P_2\},\{P_1\}\};$

The topology of all decreasing open sets $\tau^{\text{Dec}}$, is
$\tau^{\text{Dec}} = \{U, \phi, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\},\{P_1, P_2, P_3, P_4, P_5\},\{P_1, P_2, P_3, P_4\},\{P_1, P_2, P_3\},\{P_1, P_2\},\{P_1\}\};$

Now, we have $\tau^{\text{Inc}} \neq \tau^{\text{Dec}}$ and $\tau^{\text{Inc}} \neq \tau^{\text{Dec}}$. Hence C is not increasing (decreasing) superfluous.

Omitting H:

The topology generated by C,N and T has the base $\{\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}\}.$

The topology of all decreasing open sets $\tau^{\text{Dec}}$, is
$\tau^{\text{Dec}} = \{U, \phi, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\},\{P_1, P_2, P_3, P_4, P_5\},\{P_1, P_2, P_3, P_4\},\{P_1, P_2, P_3\},\{P_1, P_2\},\{P_1\}\};$

Now, we have $\tau^{\text{Inc}} = \tau^{\text{Dec}}$ and $\tau^{\text{Dec}} = \tau^{\text{Dec}}$. Hence H is increasing (decreasing) superfluous.

Omitting N:

The topology generated by C,H and T has the base $\{\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}\}.$

The topology of all decreasing open sets $\tau^{\text{Dec}}$, is
$\tau^{\text{Dec}} = \{U, \phi, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6\},\{P_1, P_2, P_3, P_4, P_5\},\{P_1, P_2, P_3, P_4\},\{P_1, P_2, P_3\},\{P_1, P_2\},\{P_1\}\};$

Now, we have $\tau^{\text{Inc}} = \tau^{\text{Dec}}$ and $\tau^{\text{Dec}} = \tau^{\text{Dec}}$. Hence N is increasing (decreasing) superfluous.
Omitting $T$:

The topology generated by $C,H$ and $N$ has the base $\{\{P_1\},\{P_2,P_5\},\{P_4\},\{P_5\},\{P_6,P_7,P_8\}\}$. The topology of all increasing open sets $\tau^\text{Inc}_T$, is

$$\tau^\text{Inc}_T = \{U,\phi,\{P_1\}, \{P_1,P_4\}, \{P_1,P_4,P_5\}, \{P_1,P_4,P_5,P_6\}, \{P_1,P_4,P_5,P_6,P_7\}, \{P_1,P_4,P_5,P_6,P_7,P_8\}, \{P_1,P_4,P_5,P_6,P_7,P_8\}\};$$

and The topology of all decreasing open sets $\tau^\text{Inc}_T$, is

$$\tau^\text{Dec}_T = \{U,\phi,\{P_1\}, \{P_1,P_4\}, \{P_1,P_4,P_5\}, \{P_1,P_4,P_5,P_6\}, \{P_1,P_4,P_5,P_6,P_7\}, \{P_1,P_4,P_5,P_6,P_7,P_8\}\}.
$$

Since $\tau^\text{Inc}_T \neq \tau^\text{Dec}_T$ and $\tau^\text{Inc}_T \neq \tau^\text{Dec}_T$, then $T$ is not increasing (decreasing) superfluous. Hence increasing (decreasing) core $= \{C,T\}$.

Case 3:

Let $\rho_3$ be a relation which determined from $C$ and $N$ such that $P_6 = P_7 = P_8$ and $P_2 = P_3$. Then

$$\rho_3 = \{(P_1,P_1),(P_2,P_2),(P_3,P_3),(P_4,P_4),(P_5,P_5),(P_6,P_6),(P_7,P_7),(P_8,P_8)\},$$

The topology generated by $C,H,N$ and $T$ has the base $\{\{P_1\},\{P_2,P_3\},\{P_4\},\{P_5\},\{P_6,P_7,P_8\}\}$. The topology of all increasing open sets $\tau^\text{Inc}_T$, is

$$\tau^\text{Inc}_T = \{U,\phi,\{P_1\}, \{P_1,P_4\}, \{P_1,P_4,P_5\}, \{P_1,P_4,P_5,P_6\}, \{P_1,P_4,P_5,P_6,P_7\}, \{P_1,P_4,P_5,P_6,P_7,P_8\}, \{P_1,P_4,P_5,P_6,P_7,P_8\}\};$$

The set of all decreasing closed sets $\tau^\text{Inc}_C$, is

$$\tau^\text{Inc}_C = \{U,\phi,\{P_2\}, \{P_2,P_3\}, \{P_2,P_3,P_4\}, \{P_2,P_3,P_4,P_5\}, \{P_2,P_3,P_4,P_5,P_6\}, \{P_2,P_3,P_4,P_5,P_6,P_7\}, \{P_2,P_3,P_4,P_5,P_6,P_7,P_8\}\};$$

The topology of all decreasing open sets $\tau^\text{Dec}_T$, is

$$\tau^\text{Dec}_T = \{U,\phi,\{P_2\}, \{P_2,P_3\}, \{P_2,P_3,P_4\}, \{P_2,P_3,P_4,P_5\}, \{P_2,P_3,P_4,P_5,P_6\}, \{P_2,P_3,P_4,P_5,P_6,P_7\}, \{P_2,P_3,P_4,P_5,P_6,P_7,P_8\}\};$$

The set of all increasing closed sets $\tau^\text{Dec}_C$, is

$$\tau^\text{Dec}_C = \{U,\phi,\{P_2\}, \{P_2,P_3\}, \{P_2,P_3,P_4\}, \{P_2,P_3,P_4,P_5\}, \{P_2,P_3,P_4,P_5,P_6\}, \{P_2,P_3,P_4,P_5,P_6,P_7\}, \{P_2,P_3,P_4,P_5,P_6,P_7,P_8\}\};$$

Omitting $C$:

The topology generated by $H,N$ and $T$ has the base $\{\{P_1\},\{P_2,P_3\},\{P_4\},\{P_5\},\{P_6,P_7,P_8\}\}$. The topology of all increasing open sets $\tau^\text{Inc}_C$, is

$$\tau^\text{Inc}_C = \{U,\phi,\{P_1\}, \{P_1,P_4\}, \{P_1,P_4,P_5\}, \{P_1,P_4,P_5,P_6\}, \{P_1,P_4,P_5,P_6,P_7\}, \{P_1,P_4,P_5,P_6,P_7,P_8\}\};$$

Now, we have $R(X) = \{P_1\}$, $\tau^\text{Inc} = \{P_1,P_2,P_3,P_6,P_7,P_8\}$, $R(X) = \phi$ and $R(X) = U$.
The topology of all decreasing open sets $\tau_{C_{Dec}}$ is
$$\tau_{C_{Dec}} = \{U, \phi, \{P_4\}, \{P_2, P_3, P_4\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}\}$$

Now, we have $\tau_{C_{Inc}} \neq \tau_{Inc}$ and $\tau_{C_{Dec}} \neq \tau_{Dec}$. Thus $C$ is not increasing (decreasing) superfluous.

**Omitting $H$:**

The topology generated by $C, N$ and $T$ has the base $\{\{P\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}$.

Then we have $\tau_{H_{Inc}} = \tau_{Inc}$ and $\tau_{H_{Dec}} = \tau_{Dec}$. Hence $H$ is increasing (decreasing) superfluous.

**Omitting $N$:**

The topology generated by $C, H$ and $T$ has the base $\{\{P\}, \{P_2, P_3, P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}$.

Now, we have $\tau_{N_{Inc}} = \tau_{Inc}$ and $\tau_{N_{Dec}} = \tau_{Dec}$. Hence $N$ is increasing (decreasing) superfluous.

**Omitting $T$:**

The topology generated by $C, H$ and $N$ has the base $\{\{P\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_7, P_8\}\}$.

The topology of all increasing open sets $\tau_{T_{Inc}}$ is
$$\tau_{T_{Inc}} = \{U, \phi, \{P_1\}, \{P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_6, P_7, P_8\}\};$$

and

The topology of all decreasing open sets $\tau_{T_{Dec}}$ is
$$\tau_{T_{Dec}} = \{U, \phi, \{P_4\}, \{P_2, P_3, P_4\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_6, P_7, P_8\}\}.$$

Now, we have $\tau_{T_{Inc}} = \tau_{Inc}$ and $\tau_{T_{Dec}} = \tau_{Dec}$. Then $T$ is not increasing (decreasing) superfluous. Therefore the increasing (decreasing) core $= \{C\}$.

**Case 4:**

Let $\rho_4$ be a relation which determined from $C, H, N$ and $T$ such that $P_2 = P_3$ and $P_6 = P_8$. Then
$$\rho_4 = \{(P_1, P_1), (P_2, P_2), (P_3, P_3), (P_4, P_4), (P_5, P_5), (P_6, P_6), (P_7, P_7), (P_8, P_8), (P_2, P_6), (P_3, P_4), (P_3, P_5), (P_5, P_6), (P_3, P_7), (P_5, P_8), (P_3, P_9), (P_7, P_8), (P_7, P_7), (P_9, P_9), (P_9, P_9)\}.$$

The topology generated by $C, H, N$ and $T$ has the base $\{\{P\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_9\}, \{P_7\}\}$.

From $\rho_4$:

The topology of all increasing open sets $\tau_{Inc}$ is
$$\tau_{Inc} = \{U, \phi, \{P_1\}, \{P_4\}, \{P_6, P_8\}, \{P_4, P_4\}, \{P_1, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_4, P_4\}, \{P_1, P_5, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_4, P_4\}, \{P_1, P_5, P_5\}, \{P_1, P_6, P_8\}, \{P_1, P_4, P_4\}, \{P_1, P_5, P_5\}, \{P_1, P_6, P_8\}\};$$

The set of all decreasing closed sets $\tau_{Dec}$ is
$$\tau_{Dec} = \{U, \phi, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}\};$$
The topology of all decreasing open sets $\tau^{\text{Dec}}$, is
\[
\tau^{\text{Dec}} = \{\emptyset, \{P_1, P_4, P_5, P_7\}, \{P_2, P_3, P_5, P_7\}, \{P_2, P_3, P_4, P_5, P_7\}, \{P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9\}\; \\
\text{and}
\]
\[
\tau^{\text{Inc}} = \{\emptyset, \{P_1, P_4, P_5, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9\}\; \\
\text{and}
\]

Now, we have $R(X) = \{P_1, P_6, P_8\}$; $R(X) = \{P_1, P_2, P_3, P_6, P_8\}$, $R(X) = \emptyset$ and
$R(X) = \{P_1, P_2, P_3, P_5, P_6, P_7, P_8\}$.

Omitting C:

The topology generated by H,N and T has the base $\{\{P_1, P_5\}, \{P_2, P_3\}, \{P_4\}, \{P_6, P_8\}, \{P_7\}\}$.

Omitting H:

The topology generated by C,N and T has the base $\{\{P_1, P_5\}, \{P_2, P_3\}, \{P_4\}, \{P_6, P_8\}, \{P_7\}\}$.

Omitting N:

The topology generated by C,H and T has the base $\{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}$.

Omitting T:

The topology generated by C,H and N has the base $\{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_7, P_8\}\}$. 

The topology of all increasing open sets $\tau^\text{Inc}_T$, is
\[
\tau^\text{Inc}_T = \{U, \emptyset, \{P_1\}, \{P_2, P_3\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4, P_5\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}\.
\]

The topology of all decreasing open sets $\tau^\text{Dec}_T$ is
\[
\tau^\text{Dec}_T = \{U, \emptyset, \{P_1\}, \{P_2, P_3\}, \{P_1, P_2, P_3\}, \{P_1, P_2, P_3, P_4\}, \{P_1, P_2, P_3, P_4, P_5\}, \{P_1, P_2, P_3, P_4, P_5, P_6\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}\.
\]

Now, we have $\tau^\text{Inc}_T \neq \tau^\text{Inc}$ and $\tau^\text{Dec}_T \neq \tau^\text{Dec}$. Then $T$ is not increasing (decreasing) superfluous. Therefore the increasing (decreasing) core $\{C, T\}$.

4. Increasing (decreasing) semi approximations from $(U, \tau_R, \rho)$.

**Definition 4.1.**

Let $(U, \tau_R, \rho)$ be aGOTAS and $A \subseteq U$. We introduce the following concepts:

1. $S(A) = A \cap R(R(A))$, is called $R$ - inc semi lower.
2. $S(A) = A \cup R(R(A))$, is called $R$ - inc semi upper.
3. $S(A) = A \cap R(R(A))$, is called $R$ - dec semi lower.
4. $S(A) = A \cup R(R(A))$, is called $R$ - dec semi upper.

**Definition 4.2.**

Let $(U, \tau_R, \rho)$ be a GOTAS and $A \subseteq U$. We define:

1. $A$ is $R$ - inc semi exact if $S(A) = S(A)$, otherwise is $R$ - inc semi rough.
2. $A$ is $R$ - dec semi exact if $S(A) = S(A)$, otherwise is $R$ - dec semi rough.

**Example 4.3:**

From Example 3.3 we have, $X = \{P_1, P_2, P_3, P_6, P_8\}$ isthe set of patients having flu. In Pawlak approximations we have, $R(X) = \{P_1, P_6, P_8\}$ and $R(X) = \{P_1, P_2, P_3, P_6, P_8\}$.

**Case 1:**

From $\rho_1$ we have:

- $R(X) = \{P_1, P_6, P_8\}$ and $R(R(X)) = \{P_1, P_6, P_8\}$ hence $S(X) = \{P_1, P_6, P_8\}$.
- $R(X) = \emptyset$ and $R(R(X)) = \emptyset$ hence $S(X) = \emptyset$.
- $R(X) = \{P_1, P_2, P_3, P_6, P_8\}$ and $R(R(X)) = \{P_1, P_2, P_3, P_6, P_7, P_8\}$ hence...
Case 2:

From $\rho_2$ we have:

\[ R(X) = \{P_1, P_6, P_8\} \quad \text{and} \quad R(R(X)) = \{P_1, P_6, P_8\} \quad \text{hence} \quad S(X) = \{P_1, P_6, P_8\}. \]

\[ R(X) = \emptyset \quad \text{and} \quad R(R(X)) = \emptyset \quad \text{hence} \quad S(X) = \emptyset. \]

\[ R(X) = \{P_1, P_2, P_3, P_5, P_6, P_7, P_8\} \quad \text{and} \quad R(R(X)) = \{P_1, P_2, P_3, P_5, P_6, P_7, P_8\} \quad \text{hence} \quad S(X) = \{P_1, P_2, P_3, P_5, P_6, P_7, P_8\}. \]

Case 3:

From $\rho_3$ we have:

\[ R(X) = \{P_1\} \quad \text{and} \quad R(R(X)) = \{P_1\} \quad \text{hence} \quad S(X) = \{P_1\}. \]

\[ R(X) = \emptyset \quad \text{and} \quad R(R(X)) = \emptyset \quad \text{hence} \quad S(X) = \emptyset. \]

\[ R(X) = \{P_1, P_2, P_3, P_5, P_6, P_7, P_8\} \quad \text{and} \quad R(R(X)) = \{P_1, P_2, P_3, P_5, P_6, P_7, P_8\} \quad \text{hence} \quad S(X) = \{P_1, P_2, P_3, P_5, P_6, P_7, P_8\}. \]

Case 4:

From $\rho_4$ we have:

\[ R(X) = \{P_1, P_6, P_8\} \quad \text{and} \quad R(R(X)) = \{P_1, P_6, P_8\} \quad \text{hence} \quad S(X) = \{P_1, P_6, P_8\}. \]

\[ R(X) = \emptyset \quad \text{and} \quad R(R(X)) = \emptyset \quad \text{hence} \quad S(X) = \emptyset. \]

\[ R(X) = \{P_1, P_2, P_3, P_6, P_7, P_8\} \quad \text{and} \quad R(R(X)) = \{P_1, P_2, P_3, P_6, P_7, P_8\} \quad \text{hence} \quad S(X) = \{P_1, P_2, P_3, P_6, P_7, P_8\}. \]

5- Conclusion:
In this work, we have shown that real world problems can be dealt with ordered topological approximation spaces. We could find that Cough and Temperature are the deciding factors for Flu in case of $\rho_1$, $\rho_2$ and $\rho_4$ but in case of $\rho_3$ Cough is the only deciding factor for Flu. It is also seen that from a clinical point of view, the ordered topological approximation spaces is on par with the medical experts with respect to the disease analyzed here. The rough set model is based on the original data only and does not need any external information, unlike probability in statistics or grade of membership in the fuzzy set theory. It is also a tool suitable for analyzing not only quantitative attributes but also qualitative ones. Thus it is advantageous to use ordered topological approximation spaces in real life situations.

References