Free Vibration of FGM Cylindrical Shell Reinforced With Single-Walled Carbon Nanotubes Using High Order Shear Deformation Theory

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ABSTRACT

In the present work study of the free vibration of FGM thin cylindrical shell reinforced with single-walled carbon nanotubes with non-uniform distribution is presented. Fundamental relations, the equilibrium and stability equations are derived using the third order shear deformation theory. Simply supported boundary condition is considered for both edge of the shell and wave method is used to solve the problem. The effects of different materials and different volume ratio on the natural frequencies are described. The analytical results are compared and validated using the results obtained from the papers is demonstrator the certify equation. The results show that changes in shell geometry, the type of polymer and varying the volume fraction of the carbon nanotube significantly affect on the natural frequencies.

Key words: Cylindrical shell, carbon nanotubes, functional graded materials, third order shear deformation theory

INTRODUCTION

Functional graded material for the first time in Japan was built in 1984 [1] and first were used in heat resistant material to cover the space shuttle and the nuclear reactors that are affected by in high temperatures and very of thermoelastic analysis is done in this area [2,3]. In recent decades a dramatic increase in demand for structures with high resistance to high temperature, energy absorption and light weight, there have been many studies on behavior of FGM cylindrical shells. Buckling of structures made from FGM are subjected to mechanical loads by Brush and Almorth [4] have been investigated. Golterman [5] a relative method for predicting buckling of thin-walled cylinder using a simple and well-known theories presented. He critical elastic buckling of a cylindrical shell completely consistent with the classical theory for the two-mode factor loading estimates and destroyed in accordance with a code of classical stability theory Kuiters be calculated according to [6]. Winterstetter and Schmidt [7] in the context of a comprehensive review and analysis of the axial buckling of cylindrical steel shells under combined loads are being carried out. Vodenticharova and Ansourian [8] analysed the buckling of cylindrical shells under uniform lateral pressure are paid. Pelletier [9], the steady-state response of a thick cylindrical shell subjected to mechanical loading and thermal FGM has been graded, were reviewed and analysed. Boundary conditions considered for the backrest shell is simple and it is assumed that the material properties in the radial direction is changed. Rahimi and colleagues [10] vibrations of a cylindrical shell with functional graded ring intermediate amplifier based on Sanders theory was looked. Using Hamilton's equations of motion and apply the Ritz method is obtained. And material properties vary along the radius. Akhlaghi and Asghari [11] The natural frequencies of a thick -walled cylinder with finite length consisting of a functional graded material properties in the longitudinal and radial directions are changed based on the three-dimensional elasticity equations were investigated. Pradham et al. [12] the vibrational parameters of FGM cylindrical shells with different boundary conditions began. The effect of different boundary conditions and different volume ratios on the natural frequency of the shell. They used Rail methods to solve the problem. Reddy et al. were examined of FGM cylindrical shells under axial harmonic balance dynamics. Their stability, Mathieu-Hill equations to obtain the equations of motion and Bolotin ’s method to solve it began. Sofiyev [14] the balance consisting of ceramic and metal composite cylindrical shell support an axial impact loading was investigated., His love relationships can be used to obtain the equations of Lagrangian Galerkin method for solving the governing equations chose. Ritz method is used to
obtain equations for the effect of volume fraction on natural frequencies can be investigated. Shen nonlinear behavior of composite plates reinforced with nanotubes were investigated under thermal load. Li Zhang and Hui-Shen on the thermal buckling analysis of composite plates reinforced with carbon nanotubes temperature dependent material properties were considered in molecular dynamics method to obtain these properties.

We analyse the free vibration of cylindrical shells reinforced with single-walled carbon nanotubes (SWCNT) in the non-uniform distribution explains, the fact that the reinforcing properties page as graded (FGM) is be. Finally, to investigate the free vibration of cylindrical shell nanocomposite with PMMA and PMPV matrix reinforced with single-walled carbon nanotubes with simply supported boundary condition, explains. Carbon nanotubes properties considered in this study, extracted of paper No.20 For to validation the equations, the results obtained from the numerical solution for a homogeneous cylindrical shell are compared with the results presented in Paper No. 27.

**Modelling of problem**

The figure below shows the micromechanical model and the coordinate system of the page. *Figure 1* show two layers of FGM cylindrical shell is symmetrical relative to the middle level. The length, radius and thickness, respectively is determined with $L$, $R$ and $h$.

![Coordinate system and the geometry of the cylindrical shell](image)

**Fig. 1:** Coordinate system and the geometry of the cylindrical shell [21].

In this study, the modified law of mixtures is used to determine the properties of the FGM shell. Cylindrical shell is orthotropic properties. Volume fraction of nanotubes on the upper and lower plate is considered [19].

$$\frac{4L}{h} V_{\text{CN}} = \frac{V_{\text{CN}}^*}{w_{\text{CN}}}$$

(1)

Such that:

$$V_{\text{CN}}^* = \frac{w_{\text{CN}}}{w_{\text{CN}} + (\rho_{\text{CN}} / \rho_m)(\rho_{\text{CN}} / \rho_m)w_{\text{CN}}}$$

(2)

$w_{\text{CN}}$ is the mass fraction of nanotubes.

$$V_{\text{CN}} + V_m = 1$$

(3)

In the above equations, the index (CN) represents the single-walled nanotubes and index (m) is the matrix. Young’s modulus and shear modulus is defined as follows [14].

$$E_{11} = \eta_1 V_{\text{CN}} E_{11}^{\text{CN}} + V_m E_m$$

(4)

$$\frac{\eta_2}{E_{22}} = \frac{V_{\text{CN}}}{E_{22}^{\text{CN}}} + \frac{V_m}{E_m}$$

(5)

$$\frac{\eta_3}{G_{12}} = \frac{V_{\text{CN}}}{G_{12}^{\text{CN}}} + \frac{V_m}{G_m}$$

(6)
In the above equations E, G, and V are represents the Young’s modulus, shear modulus and volume fraction.

\[ U_1(x, y, z, t) = U(x, y, z, t) + z \phi_1(x, y, z, t) - \frac{4}{3h^3} \left( \frac{\phi_1}{x} + \frac{\partial W}{\partial x} \right) z^3 \]  

\[ U_2(x, y, z, t) = V(x, y, z, t) + z \phi_2(x, y, z, t) - \frac{4}{3h^3} \left( \frac{\phi_2}{R} + \frac{1}{R} \frac{\partial W}{\partial \theta} - \frac{V}{R} \right) z^3 \]  

\[ U_3(x, y, z, t) = W(x, y, t) \]  

**FGM cylindrical shell equations of motion based on third order shear deformation theory:**

Where, U, V and W, are displacements of arbitrary point through the cylindrical shell along coordinate (x, y, z). In the above equations U, V and W, are to change the location of a point on z=0. also \( \phi_1, \phi_2 \) are normal rotation around the transverse axis (y,x).

**Strain-displacement relationships:**

The strain-displacement relationships for a thin shell are (Najafizadeh and Isvandzibaei, 2009).

\[ \varepsilon_{11} = \frac{1}{A_1 \left( 1 + \frac{\alpha_3}{R_1} \right)} \left[ \frac{\partial U_1}{\partial \alpha_1} + \frac{U_2}{A_2} \frac{\partial A_1}{\partial \alpha_2} + U_3 \frac{A_1}{R_1} \right] \]  

\[ \varepsilon_{22} = \frac{1}{A_2 \left( 1 + \frac{\alpha_3}{R_2} \right)} \left[ \frac{\partial U_2}{\partial \alpha_2} + \frac{U_1}{A_1} \frac{\partial A_2}{\partial \alpha_1} + U_3 \frac{A_2}{R_2} \right] \]  

\[ \varepsilon_{33} = \frac{\partial U_3}{\partial \alpha_3} \]  

\[ \varepsilon_{12} = A_1 \left( 1 + \frac{\alpha_3}{R_1} \right) \frac{\partial}{\partial \alpha_2} \left[ \frac{U_1}{A_1 \left( 1 + \frac{\alpha_3}{R_1} \right)} \right] + A_2 \left( 1 + \frac{\alpha_3}{R_2} \right) \frac{\partial}{\partial \alpha_1} \left[ \frac{U_2}{A_2 \left( 1 + \frac{\alpha_3}{R_2} \right)} \right] \]  

\[ \varepsilon_{13} = A_1 \left( 1 + \frac{\alpha_3}{R_1} \right) \frac{\partial}{\partial \alpha_3} \left[ \frac{U_1}{A_1 \left( 1 + \frac{\alpha_3}{R_1} \right)} \right] + \frac{1}{A_1 \left( 1 + \frac{\alpha_3}{R_1} \right)} \frac{\partial U_3}{\partial \alpha_1} \]  

\[ \varepsilon_{23} = A_2 \left( 1 + \frac{\alpha_3}{R_2} \right) \frac{\partial}{\partial \alpha_3} \left[ \frac{U_2}{A_2 \left( 1 + \frac{\alpha_3}{R_2} \right)} \right] + \frac{1}{A_2 \left( 1 + \frac{\alpha_3}{R_2} \right)} \frac{\partial U_3}{\partial \alpha_2} \]  

\[ A_1 = \frac{\partial \varphi}{\partial \alpha_1}, \quad A_2 = \frac{\partial \varphi}{\partial \alpha_2} \]  

That \( A_1 \) and \( A_2 \) are the fundamental form parameters or Lame parameters, \( U_1, U_2, U_3 \) are the displacement at any point \( (\alpha_1, \alpha_2, \alpha_3) \). \( R_1 \) and \( R_2 \) are the radius of curvature related to \( \alpha_1, \alpha_2, \alpha_3 \) respectively.

The third-order theory of Reddy [24] used in the present study is based on the following displacement field,
\[ U_1 = u_1 (a_1, a_2) + a_3 \psi_1 (a_1, a_2) + a_3^2 \varphi_1 (a_1, a_2) + a_3^3 \beta_1 (a_1, a_2) \] 
(17)

\[ U_2 = u_2 (a_1, a_2) + a_3 \psi_2 (a_1, a_2) + a_3^2 \varphi_2 (a_1, a_2) + a_3^3 \beta_2 (a_1, a_2) \] 
(18)

\[ U_3 = u_3 (a_1, a_2) \] 
(19)

These equations can be reduced by satisfying the stress-free conditions on the top and bottom faces of the laminates, which are equivalent to:

\[ \varepsilon_{13} = \varepsilon_{23} = 0 \quad \text{at} \quad z = \pm \frac{h}{2} \] 
(20)

The displacement field can be written as follows:

\[ U_1 = u_1 (a_1, a_2) + a_3 \psi_1 (a_1, a_2) - c_1 a_3^3 \left( - \frac{u_1}{R_1} + \varphi_1 + \frac{1}{A_1} \frac{\partial u_3}{\partial a_1} \right) \] 
(21)

\[ U_2 = u_2 (a_1, a_2) + a_3 \psi_2 (a_1, a_2) - c_1 a_3^3 \left( - \frac{u_2}{R_2} + \varphi_2 + \frac{1}{A_2} \frac{\partial u_3}{\partial a_2} \right) \] 
(22)

\[ U_3 = u_3 (a_1, a_2) \] 
(23)

Necessary parameters for the cylindrical shell are defined as:

\[
\begin{align*}
\alpha_1 &= x, \quad \alpha_2 = \theta, \quad \alpha_3 = z \\
R_2 &= R, \quad R_1 = \infty \\
A_1 &= 1, \quad A_2 = R
\end{align*}
\] 
(24)

Where \( C_1 = \frac{4}{3h^2} \) [25].

**Stress-strain relationships:**

\[
\begin{bmatrix}
\sigma_{x x} \\
\sigma_{\theta \theta} \\
\tau_{x \theta}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{21} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & Q_{55} \\
0 & 0 & 0 & 0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{x x} \\
\varepsilon_{\theta \theta} \\
\varepsilon_{x \theta}
\end{bmatrix}
\] 
(25)

For an orthotropic cylindrical shell the reduced stiffness \( Q_{ij} \) (\( i, j = 1, 2 \) and 6) is defined as

\[ Q_{11} = \frac{E_{11}}{1 - \nu_{12} \nu_{21}} \] 
(26)

\[ Q_{12} = \frac{E_{12}}{1 - \nu_{12} \nu_{21}} \] 
(27)

\[ Q_{22} = \frac{E_{22}}{1 - \nu_{12} \nu_{21}} \] 
(28)

\[ Q_{44} = G_{23} = k_2 G_{12} \] 
(29)

\[ Q_{55} = G_{13} = k_1 G_{12} \] 
(30)

\[ Q_{66} = G_{12} \] 
(31)

The stress resultants:

\[
\begin{bmatrix}
N_{x x} \\
N_{\theta \theta} \\
M_{x x} \\
M_{\theta \theta}
\end{bmatrix} =
\begin{bmatrix}
\sigma_{x x} \\
\sigma_{\theta \theta} \\
\tau_{x \theta}
\end{bmatrix}
\] 
(32)

\[
\begin{bmatrix}
M_{x x} \\
M_{\theta \theta}
\end{bmatrix} =
\begin{bmatrix}
\sigma_{x x} \\
\sigma_{\theta \theta}
\end{bmatrix}
\] 
(33)

Coefficients \( G_{12}, G_{13}, G_{23}, v_{12}, v_{21}, E_{11}, E_{22} \) is calculated from the modified mixtures.
\[
\begin{align*}
\{ P_{x0} \} &= \frac{h}{2} \int \sigma_{x0} z^3 \, dz \\
\{ P_{x\theta} \} &= \frac{h}{2} \int \sigma_{x\theta} t_{x\theta} z^3 \, dz \tag{34} \\
\{ P_{xz} \} &= \frac{h}{2} \int \sigma_{x\theta} \theta z^3 \, dz \tag{37} \\
\{ Q_{xz} \} &= \frac{h}{2} \int \sigma_{xz} z^3 \, dz \tag{35} \\
\{ R_{xz} \} &= \frac{h}{2} \int \sigma_{xz} \theta z^3 \, dz \tag{36} \\
\end{align*}
\]

For Young's modulus, shear modulus, density and moments of inertia of the equation (38) is used:

\[
\begin{align*}
\{ A_{ij} \} &= \begin{bmatrix} 1 \\ z \\ z^2 \\ z^3 \\ z^4 \\ z^5 \end{bmatrix} \\
\{ B_{ij} \} &= \frac{h}{2} \int \theta \theta \theta \theta z^3 \, dz \\
\{ D_{ij} \} &= \frac{h}{2} \int \theta \theta \theta \theta z^3 \, dz \\
\{ E_{ij} \} &= \frac{h}{2} \int \theta \theta \theta \theta z^3 \, dz \\
\{ F_{ij} \} &= \frac{h}{2} \int \theta \theta \theta \theta z^3 \, dz \\
\{ G_{ij} \} &= \frac{h}{2} \int \theta \theta \theta \theta z^3 \, dz \\
\{ H_{ij} \} &= \frac{h}{2} \int \theta \theta \theta \theta z^3 \, dz \\
\end{align*}
\]

Whit Placement in the relationship, the resultant forces and moments will be obtained as:

\[
\begin{align*}
N_{x} &= \frac{1}{1-v_2 2} \left[ (E_{xa} + E_{x1})(e_{xx} + v_2 e_{x0}) + (E_{xb} + E_{xh})(k_{xx} + v_2 k_{x0}) \right] \\
N_{\theta} &= \frac{1}{1-v_2 2} \left[ (E_{xa} + E_{x2})(e_{x0} + v_2 e_{xx}) + (E_{xb} + E_{xh})(k_{x0} + v_2 k_{xx}) \right] \\
N_{x\theta} &= (G_{12a} + G_{12h}) \gamma_{x}^0 + (G_{12b} + G_{12h}) k_{x \theta}^1 + (G_{12d} + G_{12h}) k_{x \theta}^3 \\
M_{x} &= \frac{1}{1-v_2 2} \left[ (E_{xb} + E_{xh})(e_{x0} + v_2 e_{x0}) + (E_{xk} + E_{xl})(k_{x0} + v_2 k_{x0}) \right] \\
M_{\theta} &= \frac{1}{1-v_2 2} \left[ (E_{xa} + E_{x2})(e_{x0} + v_2 e_{xx}) + (E_{xk} + E_{xl})(k_{x0} + v_2 k_{xx}) \right] \\
M_{x\theta} &= (G_{12a} + G_{12h}) \gamma_{x}^0 + (G_{12b} + G_{12h}) k_{x \theta}^1 + (G_{12d} + G_{12h}) k_{x \theta}^3 \\
P_{x} &= \frac{1}{1-v_2 2} \left[ (E_{xa} + E_{x1})(e_{xx} + v_2 e_{x0}) + (E_{xk} + E_{xl})(k_{xx} + v_2 k_{x0}) \right] \\
P_{\theta} &= \frac{1}{1-v_2 2} \left[ (E_{xa} + E_{x2})(e_{x0} + v_2 e_{xx}) + (E_{xk} + E_{xl})(k_{x0} + v_2 k_{xx}) \right] \\
P_{x\theta} &= (G_{12a} + G_{12h}) \gamma_{x}^0 + (G_{12b} + G_{12h}) k_{x \theta}^1 + (G_{12d} + G_{12h}) k_{x \theta}^3 \\
Q_{x} &= k_1 \left[ (G_{12a} + G_{12h}) \gamma_{x}^0 + (G_{12b} + G_{12h}) \gamma_{x}^0 \right] \\
Q_{\theta} &= k_2 \left[ (G_{12a} + G_{12h}) \gamma_{x}^0 + (G_{12b} + G_{12h}) \gamma_{x}^0 \right] \\
R_{x} &= k_1 \left[ (G_{12a} + G_{12h}) \gamma_{x}^0 + (G_{12b} + G_{12h}) \gamma_{x}^0 \right] \\
R_{\theta} &= k_2 \left[ (G_{12a} + G_{12h}) \gamma_{x}^0 + (G_{12b} + G_{12h}) \gamma_{x}^0 \right] \\
\end{align*}
\]
The equations of motion for vibration of a generic shell:

The equations of motion for vibration of a generic shell can be derived by using Hamilton's principle which is described by

\[
\int \left( \delta K + \delta U - \delta V \right) dt = 0
\]

\( \delta U \) is the potential energy, \( \delta V \) virtual work done by external forces and \( \delta K \) is kinetic energy allowed.

Potential energy is obtained as follows:

\[
U = \int \int \sigma_{i j} \delta \epsilon_{i j} R \, dx \, d \theta \, dz
\]

\[
\rightarrow \delta U = \int \int \left[ N_{,xx} \delta \epsilon_{xx}^{(0)} + M_{,xx} \delta k_{xx}^{(1)} + P_{,xx} \delta k_{xx}^{(3)} + N_{,\theta \theta} \delta \epsilon_{\theta \theta}^{(0)} + M_{,\theta \theta} \delta k_{\theta \theta}^{(1)} + P_{,\theta \theta} \delta k_{\theta \theta}^{(3)} + N_{,x \theta} \delta \epsilon_{x \theta}^{(0)} + M_{,x \theta} \delta k_{x \theta}^{(1)} + P_{,x \theta} \delta k_{x \theta}^{(3)} + N_{,z \theta} \delta \epsilon_{z \theta}^{(0)} + M_{,z \theta} \delta k_{z \theta}^{(1)} + P_{,z \theta} \delta k_{z \theta}^{(3)} + Q_{,xx} \delta \gamma_{xx}^{(0)} + R_{,xx} \delta \gamma_{xx}^{(2)} + \frac{1}{R} \delta \varphi_{\theta}^{(0)} + R_{,\theta \theta} \delta \gamma_{\theta \theta}^{(2)} + \frac{1}{R} \delta \varphi_{\theta}^{(3)} \right] R \, dx \, d \theta \, dz
\]

Kinetic energy is calculated from the following equation:

\[
\delta K = \int \int \rho \left( \dot{U}_1 \delta U_1 + \dot{U}_2 \delta U_2 + \dot{U}_3 \delta U_3 \right)
\]

\[
= \int \int \left\{ \left( \dot{\varphi}_x + \frac{4}{3h^2} z^2 \right) \left( \ldots \right) \right\} + \left( \dot{\varphi}_\theta + \frac{4}{3h^2} \right) \left( \ldots \right) + \left( \dot{\varphi}_z + \frac{4}{3h^2} \right) \left( \ldots \right)
\]

Due to the absence of external forces \( \delta V \) (virtual work done by external forces) is zero.

wave method:

Method for solving the wave displacement components are defined as [26].

\[
\begin{align*}
u(x, \theta, t) &= Ae^{i(\theta \omega - k_w x)} \\
w(x, \theta, t) &= Ce^{i(\theta \omega - k_w x)} \\
\varphi_x (x, \theta, t) &= De^{i(\theta \omega - k_w x)} \\
\varphi_\theta (x, \theta, t) &= Ee^{i(\theta \omega - k_w x)}
\end{align*}
\]
Finally, substituting $\delta k$, $\delta U$ in Hamilton, Integral equations of motion and the equations of motion in terms of displacement components are obtained as follows:

$$
A \left\{ k_m^2 \alpha(E_{1a} + E_{1g}) + \frac{1}{R^2} \nabla^2 (G_{12a} + G_{12g}) + I_0 \omega^2 \right\} 
$$

$$
+ B \left\{ \frac{1}{R} \nu_2 k_m \alpha(E_{1a} + E_{1g}) + \frac{4}{3h^2} \nu_2 \mathbf{r} \cdot \mathbf{m} \alpha(E_{1a} + E_{1g}) + \frac{1}{R} \nabla \cdot \mathbf{m} (G_{12a} + G_{12g}) \right\} 
$$

$$
+ \frac{4}{3h^2} \mathbf{r} \cdot \mathbf{m} (G_{12a} + G_{12g}) \right\} 
$$

$$
+ C \left\{ \frac{1}{R} \nu_2 k_m \alpha(E_{1a} + E_{1g}) + \frac{4}{3h^2} \nu_2 \mathbf{r} \cdot \mathbf{m} \alpha(E_{1a} + E_{1g}) + \frac{1}{R} \nabla \cdot \mathbf{m} (G_{12a} + G_{12g}) \right\} 
$$

$$
+ \frac{4}{3h^2} \mathbf{r} \cdot \mathbf{m} (G_{12a} + G_{12g}) \right\} 
$$

$$
+ E \left\{ \frac{1}{R} \nu_2 k_m \alpha(E_{1a} + E_{1g}) + \frac{4}{3h^2} \nu_2 \mathbf{r} \cdot \mathbf{m} \alpha(E_{1a} + E_{1g}) + \frac{1}{R} \nabla \cdot \mathbf{m} (G_{12a} + G_{12g}) \right\} 
$$

$$
+ \frac{4}{3h^2} \mathbf{r} \cdot \mathbf{m} (G_{12a} + G_{12g}) \right\} = 0
$$

(58)
\[\begin{align*} &+ \frac{32}{3h^3R^2} k_2 (G_{12m} + G_{12n}) + \frac{4}{3h^3R} n^2 \alpha(E_{2e} + E_{2k}) + \frac{16}{9h^3R} n^2 \alpha(E_{2f} + E_{2l}) \\
&+ \frac{4}{3h^3R} k_m G_{12e} + G_{12k} + \frac{16}{9h^3R} k_m G_{12f} + G_{12l} + \frac{4}{3h^3R} k_2 (G_{12d} + G_{12l}) \\
&+ \frac{16}{9h^3R^3} k_2 (G_{12f} + G_{12l}) + \left( I_1 + \frac{4}{3h^2} I_3 + \frac{4}{3h^2} I_4 + \frac{16}{9h^4} I_6 \right) \omega^2 \right] = 0 \quad (59) \\
A \left\{ \frac{1}{R} v_{2i} k_m \alpha(E_{2d} + E_{2g}) + \frac{4}{3h^3R^2} \alpha(E_{2d} + E_{2f}) + \frac{4}{3h^2R} v_{2i} k_m \alpha(E_{2d} + E_{2f}) \right\} \\
+ B \left\{ \frac{1}{R^2} \alpha(E_{2a} + E_{2g}) + \frac{4}{3h^3R^3} \alpha(E_{2d} + E_{2f}) + \frac{4}{3h^3R} \alpha(E_{2d} + E_{2f}) \right\} \\
+ C \left\{ \frac{1}{R^2} \alpha(E_{2a} + E_{2g}) + \frac{4}{3h^3R^3} \alpha(E_{2d} + E_{2f}) + \frac{4}{3h^2R} \alpha(E_{2d} + E_{2f}) \right\} \\
+ D \left\{ \frac{1}{R} v_{2i} k_m \alpha(E_{2b} + E_{2h}) + \frac{4}{3h^3R} v_{2i} k_m \alpha(E_{2b} + E_{2h}) \right\} \\
+ E \left\{ \frac{1}{R^2} \alpha(E_{2b} + E_{2h}) + \frac{4}{3h^3R^3} \alpha(E_{2d} + E_{2f}) + \frac{4}{3h^2R} \alpha(E_{2d} + E_{2f}) \right\} \end{align*}\]
+ \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{1b} + E_{1f} \right) + \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right)
+ \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2f} \right) + \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{1d} + E_{1j} \right) + \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2d} + E_{2j} \right) + (I_1 + \frac{4}{3h^2R^2}) \omega^2

(60)

+ \frac{4}{3h^2R^2} n k \left( G_{12e} + G_{12k} \right) + \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{1d} + E_{1j} \right)
+ \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{1d} + E_{1j} \right) + \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{1d} + E_{1j} \right)
+ \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{1d} + E_{1j} \right) + \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right)
+ \frac{8}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right) + \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right)
+ \frac{8}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right) + \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right)
+ \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right) + \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right)
+ \frac{8}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right) + \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right)
+ \frac{4}{3h^2R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right) + \frac{16}{9h^4R^2} \nu_{21} \frac{i k}{m} 2n \alpha \left( E_{2e} + E_{2k} \right)

(61)
The elastic modulus of PMMA / CNT reinforced by nano-walled carbon nanotubes (10, 10) [20] and [19] have been used to reinforce the cylindrical shell. Temperature and size-dependent properties of nanotubes have been considered and studied by molecular dynamics and the results of which are presented below.

### Results And Discussion

**Material properties:**

Single-walled carbon nanotubes (10, 10) have been used to reinforce the cylindrical shell. Temperature and size-dependent properties of nanotubes have been considered and studied by molecular dynamics and the results of which are presented below.

#### Table 1: Properties of Single-walled carbon nanotubes (10, 10) (L=9.26 nm , R=0.68 nm , h=0.067 nm , \( \nu^{GW} = 0.175 \) ) [20].

<table>
<thead>
<tr>
<th>( E_{11}^B ) (TPa)</th>
<th>( E_{22}^B ) (TPa)</th>
<th>( G_{12}^B ) (TPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6466</td>
<td>7.0800</td>
<td>1.9445</td>
</tr>
</tbody>
</table>

Properties of composite materials used in the shell in Tables.2 and Tables.3 have been identified.

### Table 2: The elastic modulus PMPV / CNT used by nano-tubes (10,10) [19].

| \( \nu_m = 0.34 \) , \( \nu_m^{GW} = 2.1 \) GPa |
|---|---|---|---|
| \( \nu^{GW} \) | \( E_{11} \) (GPa) | \( \eta_1 \) | \( E_{22} \) (GPa) | \( \eta_2 \) |
| 0.11 | 94.57 | 0.149 | 2.2 | 0.934 |
| 0.14 | 120.09 | 0.150 | 2.3 | 0.941 |
| 0.17 | 145.08 | 0.149 | 3.5 | 1.381 |

### Table 3: The elastic modulus of PMMA / CNT reinforced by nano-tubes (10,10)[20]

| \( \nu_m = 0.34 \) , \( \nu_m^{GW} = 2.5 \) GPa |
|---|---|---|---|
| \( \nu^{GW} \) | \( E_{11} \) (GPa) | \( \eta_1 \) | \( E_{22} \) (GPa) | \( \eta_2 \) |
| 0.12 | 94.78 | 0.137 | 2.9 | 0.022 |
| 0.17 | 138.69 | 0.142 | 4.9 | 1.626 |
| 0.28 | 224.50 | 0.141 | 5.5 | 1.585 |
Numerical analysis and compare the results of different shell materials with different volume than intended and the natural frequency changes after every $m$, $n$, $h/R$ is obtained. $N$ and $m$ are respectively the wave number of environmental and longitudinal wave number.

Numerical results for the nanocomposites (10,10) with regard to the sex of the shell is presented. $\Omega$ is the dimensionless natural vibration frequency to evaluate the results, the following are considered: [27].

**accuracy of the results:**

Table 4: verify the equations and compared with the reference

<table>
<thead>
<tr>
<th>$n$</th>
<th>(Shah et al., 2009)</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.140641</td>
<td>0.141123</td>
</tr>
<tr>
<td>2</td>
<td>0.054323</td>
<td>0.05434</td>
</tr>
<tr>
<td>3</td>
<td>0.027074</td>
<td>0.027093</td>
</tr>
<tr>
<td>4</td>
<td>0.017776</td>
<td>0.017784</td>
</tr>
<tr>
<td>5</td>
<td>0.017088</td>
<td>0.017092</td>
</tr>
</tbody>
</table>

Table 4 shows the results, there is little error indicates that the equation is true.

Results:

Natural frequencies of FGM cylindrical shells with different volume ratios of nanotubes and polymer PMPV:

Table 5: The values of the dimensionless natural frequencies of FGM cylindrical shell in terms of the variation $m$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\nu_{CN}$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>0.11</td>
<td>0.14</td>
</tr>
<tr>
<td>1</td>
<td>0.269967</td>
<td>0.2784</td>
</tr>
<tr>
<td>2</td>
<td>0.392924</td>
<td>0.401725</td>
</tr>
<tr>
<td>3</td>
<td>0.474443</td>
<td>0.484241</td>
</tr>
<tr>
<td>4</td>
<td>0.536564</td>
<td>0.547427</td>
</tr>
<tr>
<td>5</td>
<td>0.586783</td>
<td>0.598449</td>
</tr>
</tbody>
</table>

Fig. 2: Nondimensional natural frequency $\Omega$ for different volume ratios PMPV in $m$

B) Natural frequencies with different volume ratios of nanotubes and polymer PMPV according to environmental changes in wave number are obtained.

In this case $m = 1$ in all cases we have considered.

The results in Table 6 and Figure 3 can be seen.

Table 6: The values of the dimensionless natural frequencies of FGM cylindrical shell in terms of the variation of $n$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\nu_{CN}$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>0.11</td>
<td>0.14</td>
</tr>
<tr>
<td>1</td>
<td>0.269967</td>
<td>0.2784</td>
</tr>
<tr>
<td>2</td>
<td>0.0340121</td>
<td>0.0366588</td>
</tr>
<tr>
<td>3</td>
<td>0.0160027</td>
<td>0.0175364</td>
</tr>
<tr>
<td>4</td>
<td>0.00101621</td>
<td>0.011105</td>
</tr>
<tr>
<td>5</td>
<td>0.00916664</td>
<td>0.0097687</td>
</tr>
</tbody>
</table>
Fig. 3: Nondimensional natural frequency $\Omega$ for different volume ratio of n times PMPV

C) natural frequencies with different volume ratios of nanotubes and polymer PMPV according to changes in $h/R$ to obtain. In this case, $m$ and $n = 1$

Table 7: The values of the dimensionless natural frequencies of FGM cylindrical shell in terms of the variation of $h/R$

<table>
<thead>
<tr>
<th>$h/R$</th>
<th>$\nu_{CN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td>0.02</td>
<td>0.269972</td>
</tr>
<tr>
<td></td>
<td>0.278405</td>
</tr>
<tr>
<td></td>
<td>0.33843</td>
</tr>
<tr>
<td>0.03</td>
<td>0.270011</td>
</tr>
<tr>
<td></td>
<td>0.278451</td>
</tr>
<tr>
<td></td>
<td>0.338479</td>
</tr>
<tr>
<td>0.04</td>
<td>0.270045</td>
</tr>
<tr>
<td></td>
<td>0.27849</td>
</tr>
<tr>
<td></td>
<td>0.338522</td>
</tr>
<tr>
<td>0.05</td>
<td>0.270089</td>
</tr>
<tr>
<td></td>
<td>0.278541</td>
</tr>
<tr>
<td></td>
<td>0.338577</td>
</tr>
</tbody>
</table>

Fig 4: Nondimensional natural frequency $\Omega$ for different volume ratios PMPV terms of $h/R$

Natural frequencies of FGM cylindrical shells with different volume ratios of polymer nanotubes and PMMA:

A) Natural frequencies with different volume ratios of nanotubes and PMMA polymer according to changes. In this case $n = 1$ in all cases we have considered. The results in Table 8 and Figure 5 can be seen.
Table 8: The values of the dimensionless natural frequencies of FGM cylindrical shell in terms of the change m

<table>
<thead>
<tr>
<th>m</th>
<th>ω_CM/ω</th>
<th>0.12</th>
<th>0.17</th>
<th>0.28</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.304015</td>
<td>0.389414</td>
<td>0.426979</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.448784</td>
<td>0.580311</td>
<td>0.62429</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.543492</td>
<td>0.704236</td>
<td>0.754489</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.615318</td>
<td>0.797838</td>
<td>0.858631</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.673066</td>
<td>0.87295</td>
<td>0.933489</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5: Natural frequency Ω for the different volume ratio of PMMA in m

B) Natural frequencies with different volume ratios of nanotubes and PMMA polymer according to obtain environmental changes in wave number. In this case m = 1 in all cases we have considered. The results in Table 9 and Figure 6 is observed.

Table 9: The values of the dimensionless natural frequencies of FGM cylindrical shell in terms of the variation of n

<table>
<thead>
<tr>
<th>n</th>
<th>ω_CM/ω</th>
<th>0.12</th>
<th>0.17</th>
<th>0.28</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.304015</td>
<td>0.389414</td>
<td>0.426979</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0358979</td>
<td>0.0442234</td>
<td>0.0525729</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0165577</td>
<td>0.0202071</td>
<td>0.0245915</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0106329</td>
<td>0.0131462</td>
<td>0.0158583</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0100424</td>
<td>0.0128542</td>
<td>0.0149211</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6: Natural frequency Ω for the different volume ratio PMMA based on the

C) natural frequencies with different volume ratios of PMMA and nanotubes and polymers based on changes in h / R to obtain. In this case, m and are equal to (1) in all cases we have considered. The results in Table 10 as well as Figure 7 can be seen.
Conclusion:

A study on the vibration of functionally graded cylindrical shell with carbon nanotube support has been presented.

Dimensionless frequencies for each of the polymers with carbon nanotube support in three states has obtained the following results:

a. The first mode frequency in terms of $n$

b. The second mode frequency in terms of $m$

c. The third mode frequency versus $h/R$

The study showed that carbon nanotube support has significant influence on the frequencies, this is because the functionally graded cylindrical shells exhibit interesting frequency characteristics when the constituent volume fractions are varied. In addition, sometimes the frequency of the polymer has been effective. So that the polymer PMMA has better performance.

Finally we can say that increasing the volume percent carbon nanotube (as far as frequency increases are stopped) to increase the frequency that this increase in frequency is good for strengthening the shell.

References


11. Akhlaghi, M., M. Asgari, 2011. mNatural frequency analys is of 2D-FGM thick hollow cylinder based on three-dimensional elasticity.


