Improvement of Boundary Condition for a Shallow Water Model to Extent Predictability of Monsoon

Sunisa Saiuparad and Dusadee Sukawat

Department of Mathematics and Statistics, Faculty of Science and Technology, Rajamangala University of Technology Phra Nakhon, Bangkok, Thailand.
Department of Mathematics, Faculty of Science, King Mongkut’s University of Technology Thonburi, Bangkok, Thailand.

ABSTRACT

Limited area shallow water model is usually considered for simple numerical experiments in meteorology such as predictability of monsoon. The shallow water model has efficiency to predict the monsoon for only 2-3 days due to boundary condition problem. The shallow water model can be improved to extent predictability by implement wall boundary condition. It is found that the wall boundary condition has good efficiency to reduce the problem of boundary condition.

Key words: Boundary Condition, Predictability, Shallow Water Model

INTRODUCTION

All computational fluid dynamics (CFD) problems are defined in terms of initial and boundary conditions. It is important that these correctly and understands their role in the numerical algorithm [1]. Many numerical models have been developed for the problems and have used difference approaches to accommodate the process. The shallow water model is used as the mathematical forecast model. The boundary conditions specified for this model during dynamic initialization consists of cyclic continuity in the zonal direction and open boundary conditions in the southern and northern boundaries [2]. The shallow water model has the boundary condition problems as well. Therefore, the prediction of the model is not efficient. There are various methods to solve a problem of the boundary condition. The first method is outflow boundary condition, I.E. Vignon-Clementel et al. [3] developed and implemented a method to prescribe outflow boundary condition intended for three-dimensional finite element simulations of blood flow based on the Dirichlet-to-Neumann and variational multiscale methods. Outflow boundary condition is derived for any downstream domain where an explicit relationship of pressure as a function of flow rate or velocities can be obtained at the coupling interface. Results have been shown that outflow boundary condition to solve the non-linear three-dimensional equation of blood flow. Furthermore, while the focus of this paper is on the outflow boundary condition the method described could be applied outlet boundary conditions and inlet boundary conditions to include. In [4] regional oceanic models can be developed and used efficiently for the investigation of regional and coastal domains, provided a satisfactory prescription for the open boundary conditions (OBCs) is found.

In addition, A. F. Blumberg et al. [5] presented the application of a new form of radiation condition on the open boundaries. To simulate tidal effected on the circulation a coastal ocean circulation model. The model results are consistent with those of earlier observational and modeling studies. Another method to solve a boundary condition problem is the wall boundary condition. T. Li et al. [6] presented wall boundary condition of gas and solid for accurate predictions flow hydrodynamics. Iztok Tiselj et al. [7] performed direct numerical simulation of fully developed turbulent velocity and temperature fields in a flume based on the wall shear velocity and the height of the flume. To elucidated exactly the role of the wall boundary condition.

The purpose of the work is to solve the boundary condition problem of a shallow water model. In order to model is a more accurate prediction. Due to wall boundary condition is the optimal method. Then this
method can be solved the problem about a boundary condition of the shallow water model. In Section 2, materials and methods are introduced and the experiments to solved the boundary condition problem. Section 3, is results and discussion. Finally, in Section 4, the conclusions to improvement of boundary condition for a shallow water model are discussed.

Materials and Methods

1. Theory and related works:

1.1 The Shallow Water Model:

In this paper the shallow water model [8] is used. The model is described by the following three equations for the 3 unknowns, $u$, $v$, $z$. The equations of motion are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial z}{\partial x} + fv,$$  

(1)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial z}{\partial y} - fu,$$  

(2)

and continuity equation is

$$\frac{\partial z}{\partial t} + u \frac{\partial z}{\partial x} + v \frac{\partial z}{\partial y} = -z \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right).$$  

(3)

The model uses the zonal and meridional components of the wind field ($u$ and $v$) and the height field ($z$) as the predicted variables. The model equations can be written in the form

$$\frac{du}{dt} - fv' + g \frac{\partial z}{\partial x} = 0,$$  

(4)

$$\frac{dv}{dt} + fu' + g \frac{\partial z}{\partial y} = 0,$$  

(5)

$$\frac{dz}{dt} + z \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0,$$  

(6)

where the total time derivative is given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}$$

and $u$ is the $x$ component of the wind vector (m/s).

$v$ is the $y$ component of the wind vector (m/s).

$z$ is the geopotential height (m).

$g$ is the acceleration of gravity (m/s$^2$).

$f$ is the coriolis force $= 2\Omega \sin \phi$, $\phi =$ latitude,

$\Omega =$ angular velocity of the earth. The advective term is done through a semi-Lagrangian approach, while time integration is done through the Matsuno time scheme. In this model, the boundary condition is used to specify the north-south boundaries open. Cyclic boundary condition is assumed in the zonal direction. The values at the southern and northern boundaries are obtained through linear extrapolation.

1.2 Lyapunov Exponent:

The Lyapunov exponent (LE) [9] measures the asymptotic average exponential divergence or convergence of nearby trajectories in phase space and has proven to be a useful diagnostic to detect and quantify chaos [10]. A positive Lyapunov exponent measures the average exponential divergence of two nearby trajectories whereas a negative Lyapunov exponent measures exponential convergence of two nearby trajectories [11]. Consider two divergent trajectories from a 1D flow (i.e., trajectories in continuous time as described by a differential equation), the growth of the difference $d_t$ between the two trajectories over a time period $\Delta t = t_i - t_0$ can be described by

$$d_t = d_0 \exp(\lambda \Delta t)$$  

(7)

The Lyapunov exponent $\lambda$ is given by

$$\lambda(t) = \frac{1}{\Delta t} \ln \left( \frac{d_{t+\Delta t}}{d_t} \right)$$  

(8)

where $d_0$ is the distance between the two trajectories at the initial time and $d_t$ is the distance at the time $t + \Delta t$.

1.3 Finite Time Lyapunov Exponent:

The Finite Time Lyapunov Exponent (FTLE) has been defined for a prescribed finite time interval to study the local dynamics on the attractor. The sensitivity of trajectories over finite time intervals $t$ to perturbations of the initial conditions can be associated with FTLE. The FTLE depends on the initial positions of the trajectories as well as the time of integration of these trajectories [12]. The equation for the FTLE is given by

$$\lambda_{en}(x(t), \delta x(0)) = \frac{1}{\Delta t} \log \left\| \frac{\delta x(t+\Delta t)}{\delta x(t)} \right\|$$  

(9)

where $\delta x(0)$ is the distance between the trajectories with a suitable norm for the initial time, $\delta x(t+\Delta t)$ is the distance between the trajectories at the time $t + \Delta t$ and $\Delta t$ is the time interval. Thus FTLE measures convergence or divergence of nearby trajectories and provides a quantitative measure of a system’s sensitivity to initial condition. Positive FTLE indicates exponential divergence of a nearby trajectory and conversely, negative FTLE indicates exponential convergence [13].

1.4 Wall Boundary Condition:

The wall is the most common boundary encountered in confined fluid flow problem. All
practically relevant flows situations are wall bounded. Near walls the exchange of mass, momentum and scalar quantities is largest [14]. In the velocity field, the fluid velocity components equal the velocity of the wall. The normal and tangential velocity components at an impermeable, non-moving wall are:

\[
\vec{v}_n = v_{wall} = 0 \quad ; \quad \vec{v}_t = 0
\]  

(10)

Mass fluxes are zero and hence convective fluxes are zero.

\[C_{wall} = n \phi = 0\]  

(11)

Diffusive fluxes are non-zero and result in wall-shear stresses.

\[D_{wall} = \int \tau_d n dS\]  

(12)

The specification of wall boundary conditions for the pressure depends on the flow situation. In a parabolic or convection dominated flow a von Neumann boundary condition is used at the wall

\[\frac{\partial p}{\partial n}\bigg|_{wall} = 0\]  

(13)

The wall boundary condition depends on a Reynolds number \(Re = Uy/\nu\) [15]. As \(y\) is decreased to zero, however, a Reynolds number based on \(y\) will also decreased to zero.

Close to the wall the flow is influenced by viscous effects and does not depend on free stream parameters. The mean flow velocity only depends on the distance \(y\) from the wall, fluid density \(\rho\) and viscosity \(\mu\) and the wall shear stress \(\tau_w\). So

\[U = f(y, \rho, \mu, \tau_w)\]  

(14)

Dimensional analysis shows that

\[u^+ = \frac{U}{u^+} = f\left(\frac{\rho u^+ y}{\mu}\right) = f\left(y^+\right)\]  

(15)

Formula (15) is called the law of the wall and contains the definitions of two important dimensionless groups, \(u^+\) and \(y^+\). Note that the appropriate velocity scale is \(u^+ = \sqrt{\tau_w/\rho}\), the so-called friction velocity. The implementation of wall boundary conditions in turbulent flows starts with the evaluation of

\[y^+ = \frac{\Delta y_p}{\nu} \sqrt{\frac{\tau_w}{\rho}}\]  

(16)

where \(\Delta y_p\) is the distance of the near-wall node \(p\) to solid surface \(\nu\) is the kinematic turbulent viscosity (m²/s) \(\tau_w\) is the wall shear stress \(\rho\) is fluid density

If \(y^+ > 11.63\) the flow is turbulent and the wall function approach is used.

2. Experiments detail:

2.1 The Data:

The selected cases for this experiment are the Asian northeast monsoon downscaling for December 2056. The input data are obtained from the Bjerknes Centre for Climate Research (BCCR), University of Bergen, Norway. The global climate model is Bergen Climate Model (BCM) Version 2.0 (BCCR-BCM2.0) from the World Climate Research Programme’s (WCRP’s) Coupled Model Intercomparison Project phase 3 (CMIP3) multi-model data set for the Fourth Assessment Report (AR4) of the Intergovernmental Panel on Climate Change (IPCC) (Intergovernmental Panel on Climate Change - IPCC, 2009) [16]. The BCCR-BCM2.0 model has the resolution of 2.81° long × 2.77° lat, the numerical schemes are semi-lagrangian semi-implicit time integration. The A2 scenario from the BCCR-BCM2.0 is used as the input for the shallow water model. The A2 scenario is based on a high population growth scenario. The zonal wind component (u), meridional wind component (v) and geopotential height (z) at 500 hPa are used as the initial conditions for running the shallow water model as shown in Table 1.

<table>
<thead>
<tr>
<th>Initial Condition</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>02 December 2056</td>
<td>(u), pcmdi.ipcc4.bcc_cpm2_0.sresa2.run1.daily.ua_A2_2056.nc</td>
</tr>
<tr>
<td>0000 UTC</td>
<td>(v), pcmdi.ipcc4.bcc_cpm2_0.sresa2.run1.daily.va_A2_2056.nc</td>
</tr>
<tr>
<td></td>
<td>(z), pcmdi.ipcc4.bcc_cpm2_0.sresa2.run1.daily.zg_A2_2056.nc</td>
</tr>
</tbody>
</table>

The data used as initial condition for running the shallow water model are prepared as shown in the Figure 1.
2.2 Domain:

The domain of the experiment for the shallow water model covers 180°W to 180°E and 40°S to 80°N. The study domain is between longitudes 60°E to 140°E and latitudes 20°S to 60°N, as shown in Figure 2.

Fig. 2: The study domain.

2.3 The Experiment Design:

2.3.1 The Shallow Water Model Forecast:

The model is run with the grid sizes of degree latitude-longitude by the boundary conditions are cyclic in the west-east boundary and open in the north-south boundary in RUNs 1. To improvement the boundary condition with the wall boundary condition in the north-south boundary for the model in RUNs 2. In the control runs (CTRLs), the data in Table 1 are used as the initial condition and the model is run for 7-day forecasts. The perturbations are generated by apply the breeding method [17] to
the original initial geopotential height to obtain small perturbations. These perturbations of geopotential height are then added to the original initial geopotential height to create perturbed geopotential heights for the perturbed runs (PERs). Summary of the predictions are shown in Tables 2 and 3.

### Table 2: Experiment setting for run the shallow water model

<table>
<thead>
<tr>
<th>Description</th>
<th>RUN1</th>
<th>RUN2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Domain</td>
<td>long: 180°W to 180°E, lat: 40°S to 80°N.</td>
<td>long: 180°W to 180°E, lat: 40°S to 80°N.</td>
</tr>
<tr>
<td>Resolution</td>
<td>$\Delta x = \Delta y = 1^\circ$, $\Delta t = 60$ (s)</td>
<td>$\Delta x = \Delta y = 1^\circ$, $\Delta t = 60$ (s)</td>
</tr>
<tr>
<td>Initial Condition</td>
<td>BCCR-BCM2.0 (A2 scenario), 02 Dec 2056, 500 hecto Pascal (hPa)</td>
<td>BCCR-BCM2.0 (A2 scenario), 02 Dec 2056, 500 hecto Pascal (hPa)</td>
</tr>
<tr>
<td>Boundary Condition</td>
<td>Cyclic in the west-east boundary</td>
<td>Cyclic in the west-east boundary</td>
</tr>
<tr>
<td></td>
<td>Open in the north-south boundary</td>
<td>Wall in the north-south boundary</td>
</tr>
<tr>
<td>Forecast Period</td>
<td>7 days</td>
<td>7 days</td>
</tr>
</tbody>
</table>

### Table 3: Experiment setting for testing predictions of the shallow water model

<table>
<thead>
<tr>
<th>Initial Condition</th>
<th>Experiment</th>
<th>Case Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>02 Dec 2056</td>
<td>CTRL1</td>
<td>Control run using BCCR-BCM2.0 model (A2 scenario) data as the initial condition.</td>
</tr>
<tr>
<td></td>
<td>CTRL2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PER1</td>
<td>Similar to PER1 and PER3 but subtract the perturbations generated by the breeding</td>
</tr>
<tr>
<td></td>
<td>PER3</td>
<td>method from the original initial condition.</td>
</tr>
<tr>
<td></td>
<td>PER2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PER4</td>
<td></td>
</tr>
</tbody>
</table>

### 2.3.2 The Breeding Method:

The breeding method has been used to generate initial perturbation for short-to-medium-range atmospheric ensemble forecasting at the National Centers for Environmental Prediction (NCEP). The initial perturbations obtained by breeding are known as bred vectors. The process of the breeding method can be written as flowchart in Figure 3.
Fig. 3: Flow chart showing the steps of breeding method [17].

Results and Discussions

1. The Shallow water model forecast:

The shallow water model is run for the northeast monsoon predictions. The selected cases for this experiment are the Asian northeast monsoon downscaling for December 2056.

1.1 Run 1:

The model is run with the grid size of 1×1 degree latitude-longitude by the boundary conditions are cyclic in the west-east boundary and open in the north-south boundary and the time step is 60 second. The model domain is longitude 180°W to 180°E and latitude 40°S to 80°N. The model is run for 7-day forecasts. The results are shown in Figures 4-7.

Fig. 4: The geopotential heights (m) from BCCR-BCM2.0 model on 02 Dec 2056 0000 UTC with 1×1 degree resolution and the corresponding initial conditions for CTRL1, PER1 and PER2.
Fig. 5: The geopotential height (m) from BCCR-BCM2.0 model on 04 Dec 2056 0000 UTC with 1×1 degree resolution and the corresponding 48-hr forecasts from CTRL1, PER1 and PER2.

Fig. 6: The geopotential height (m) from BCCR-BCM2.0 model on 06 Dec 2056 0000 UTC with 1×1 degree resolution and the corresponding 96-hr forecasts from CTRL1, PER1 and PER2.
Fig. 7: The geopotential height (m) from BCCR-BCM2.0 model on 08 Dec 2056 0000 UTC with 1×1 degree resolution and the corresponding 144-hr forecasts from CTRL1, PER1 and PER2.

Figure 4a shows the geopotential height from BCCR-BCM2.0 model on 02 December 2056 0000 UTC with 1×1 degree resolution. The corresponding initial geopotential heights for CTRL1, PER1 and PER2 are shown in Figures 4b, 4c and 4d, respectively. The geopotential heights from BCCR-BCM2.0 model for 04 December 2056 0000 UTC and the corresponding 48-hr forecasts of CTRL1, PER1 and PER2 are shown in Figures 5a, 5b, 5c and 5d, respectively. Similarly, the geopotential heights from BCCR-BCM2.0 model for 06 December 2056 and 08 December 2056 0000 UTC and the corresponding 96-hours and 144-hour forecasts are shown in Figure 6 and 7.

1.2 Run 2:

The model is run with the grid size of 1×1 degree latitude-longitude by the boundary conditions are cyclic in the west-east boundary and wall in the north-south boundary and the time step is 60 second. The model domain is longitude 180°W to 180°E and latitude 40°S to 80°N. The model is run for 7-day forecasts. The results are shown in Figures 8-11.

Fig. 8: The geopotential heights (m) from BCCR-BCM2.0 model on 02 Dec 2056 0000 UTC with 1×1 degree resolution and the corresponding initial conditions for CTRL2, PER3 and PER4.
Fig. 9: The geopotential height (m) from BCCR-BCM2.0 model on 04 Dec 2056 0000 UTC with 1×1 degree resolution and the corresponding 48-hr forecasts from CTRL2, PER3 and PER4.

Fig. 10: The geopotential height (m) from BCCR-BCM2.0 model on 06 Dec 2056 0000 UTC with 1×1 degree resolution and the corresponding 96-hr forecasts from CTRL2, PER3 and PER4.
Figure 8a shows the geopotential height from BCCR-BCM2.0 model on 02 December 2056 0000 UTC with 1×1 degree resolution. The corresponding initial geopotential heights for CTRL2, PER3 and PER4 are shown in Figures 8b, 8c and 8d, respectively. The geopotential heights from BCCR-BCM2.0 model for 04 December 2056 0000 UTC and the corresponding 48-hr forecasts of CTRL2, PER3 and PER4 are shown in Figures 9a, 9b, 9c and 9d, respectively. Similarly, the geopotential heights from BCCR-BCM2.0 model for 06 December 2056 and 08 December 2056 0000 UTC and the corresponding 96-hours and 144-hour forecasts are shown in Figure 10 and 11.

2. Predictability Measurements for the Shallow Water Model:

The predictability values from LE and FTLE are calculated from the forecast values of the geopotential height in the control runs (CTRLs) and the corresponding values from the perturbed runs (PERs). The experiment cases are defined in Table 3. The predictability values from LE and FTLE for Run 1 by the boundary conditions are cyclic in the west-east boundary and open in the north-south boundary and Run 2 by boundary condition are cyclic in the west-east boundary and wall in the north-south boundary in the northeast monsoon region of Asia (longitude 60°E to 140°E and latitude 20°S to 60°N) are shown in Table 4.

In Table 4, to prepare between Run 1 and Run 2 of the shallow water model forecast. In Run 1, to found that for 24-hr and 48-hr forecasts of PER1 and PER2, the values are positive which indicate divergence of the two nearby trajectories. However, for 120-hr, 144-hr, and 168-hr forecasts of PER1 and PER2 all of the values are negative which indicate convergence of the two nearby trajectories (bold number). Thus, the forecasts from Run1 the PERs start to converge to the forecasts of CTRL1 only after 3 days. In other words, the shallow water model is not sensitive to the initial conditions after 3-day forecast. In Run 2, to found that for 24-hr, 48-hr and 72-hr forecasts of PER1 and PER2, the values are positive which indicate divergence of the two nearby trajectories. However, for 144-hr and 168-hr forecasts of PER1 and PER2 all of the values are negative which indicate convergence of the two nearby trajectories (bold number). Thus, the forecasts from Run2 the PERs start to converge to the forecasts of CTRL2 only after 4 days. In other words, the shallow water model is not sensitive to the initial conditions after 4-day forecast.

Conclusion:

In this paper, improvement the boundary condition of the shallow water model is run for the northeast monsoon predictions. In Run 1, the model is run with the boundary conditions are cyclic in the west-east boundary and open in the north-south boundary. The measurements reveal that the model predictability is about 3 days. In Run 2, the model is run with the boundary conditions are cyclic in the west-east boundary and wall in the north-south
boundary. The measurements reveal that the model predictibility is about 4 days. Therefore, the wall boundary condition suitable for the model. The results in the table 4 be expressed in a compact from in Table 5, which shows the different of predictability on boundary conditions of the model.

### Table 4: Predictability for Run 1 and Run 2

<table>
<thead>
<tr>
<th>Forecast (hour)</th>
<th>method</th>
<th>Run 1</th>
<th></th>
<th>Run 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>LE</td>
<td>1.8304</td>
<td>0.4333</td>
<td>1.2004</td>
<td>1.1768</td>
</tr>
<tr>
<td></td>
<td>FTLE</td>
<td>1.9462</td>
<td>0.4844</td>
<td>2.2365</td>
<td>1.2925</td>
</tr>
<tr>
<td>48</td>
<td>LE</td>
<td>0.8731</td>
<td>0.7484</td>
<td>1.2726</td>
<td>1.1698</td>
</tr>
<tr>
<td></td>
<td>FTLE</td>
<td>1.9995</td>
<td>0.8291</td>
<td>1.3087</td>
<td>0.0570</td>
</tr>
<tr>
<td>72</td>
<td>LE</td>
<td>1.2196</td>
<td>0.9097</td>
<td>2.0728</td>
<td>1.2738</td>
</tr>
<tr>
<td></td>
<td>FTLE</td>
<td>1.3408</td>
<td>0.9904</td>
<td>2.5038</td>
<td>1.5760</td>
</tr>
<tr>
<td>96</td>
<td>LE</td>
<td>1.3009</td>
<td>1.4939</td>
<td>2.9347</td>
<td>2.5394</td>
</tr>
<tr>
<td></td>
<td>FTLE</td>
<td>1.8359</td>
<td>1.6203</td>
<td>3.3657</td>
<td>1.3409</td>
</tr>
<tr>
<td>120</td>
<td>LE</td>
<td>0.3710</td>
<td>0.7467</td>
<td>1.6637</td>
<td>1.5755</td>
</tr>
<tr>
<td></td>
<td>FTLE</td>
<td>0.0939</td>
<td>0.8731</td>
<td>1.7852</td>
<td>1.9045</td>
</tr>
<tr>
<td>144</td>
<td>LE</td>
<td>1.5529</td>
<td>0.9773</td>
<td>1.7309</td>
<td>1.6628</td>
</tr>
<tr>
<td></td>
<td>FTLE</td>
<td>1.7678</td>
<td>1.0984</td>
<td>2.2614</td>
<td>1.5295</td>
</tr>
<tr>
<td>168</td>
<td>LE</td>
<td>1.2132</td>
<td>1.2196</td>
<td>3.4236</td>
<td>1.4390</td>
</tr>
<tr>
<td></td>
<td>FTLE</td>
<td>1.7290</td>
<td>1.3408</td>
<td>1.8589</td>
<td>1.5628</td>
</tr>
</tbody>
</table>

### Table 5: Predictability of the monsoon by the shallow water model.

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Model Predictability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>3 days</td>
</tr>
<tr>
<td>Run 2</td>
<td>4 days</td>
</tr>
</tbody>
</table>

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**References**


