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Research Article

### Dynamics and Survival Time for Entangled Photon Evolution in Four Wave Mixing Process

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#### ABSTRACT

We studied and analyzed quantum correlation time of entangled photon generated by four-wave mixing process within Kerr nonlinear fiber optics ring resonator. Three major parts of Hamiltonian describing the interaction of photon-photon and photon-reservoir were established. Markov approximation was used to eliminate the reservoir operators in order to get the extended master equation of motion of entangled photon for the existence of decoherence. Using photon state  $+P$  representation to transform from photon operators to the corresponding c-number, the dynamics of stochastic equation system was attained under three main factors dependence and the results show that, via graphical approach at thermal equilibrium, their lifetime has long enough to apply to quantum information processing.

*Key words:* Entangled Photon, Quantum Information, Four Wave Mixing, Fiber Optic Ring Resonator

#### INTRODUCTION

Entangled photon, a pair of simultaneous birth of twin photons, plays the heart of quantum information processing such as quantum teleportation, quantum cryptography and quantum computation. Its most amazing property is about simultaneous data transferring between two ports with which each entangled photon stayed, and rendering the highest security, fidelity to communications [1]. There are many methods to generate such a pair of photon. Most of all using the very high intensive pulses of coherent light bombard with a nonlinear  $\chi^{(2)}$  or  $\chi^{(3)}$  materials [23]. The easier and optimum way is by using the nonlinear fiber optics and bombarding with two narrow beams of laser light, calling four wave mixing process (FWM) [4,5,6], in which the downward decay of cascading energy levels emit simultaneously a pair of entangled photon, i.e. signal and idler photons, subjected from the energy and momentum conservation laws. The most prominence of using a pair of photons in quantum processing is the easier preparation and its long lifetime. Actually, entangled photon producing via previous methods must be involved with decoherence phenomena resulting from the interchanging energy between photon system and heat reservoir.

This paper will establish the three types of Hamiltonian to describe theoretically about how to generate entangled photon by using Kerr nonlinear fiber optic in a ring resonator under a four wave mixing process, and analyze the evolution of entangled states in time under the existence of decoherence from a reservoir to search for the optimum and validity of entangled photon lifetime in order to apply in quantum information engineering.

#### Materials and Methods

It is well known that the response to external electric field  $E$  of nonlinear fiber optics at its hollow core will bring about the changing in refractive index, depending on field intensity and electronics properties of the material, as [7]

$$P_i = \epsilon_0 \left( \chi_{i,j}^{(1)} E_j + \chi_{i,j,k}^{(2)} E_j E_k + \chi_{i,j,k,l}^{(3)} E_j E_k E_l + \dots \right), \quad (1)$$

where  $P_i$  stands for an electric polarization of the  $i$ -th order components of a nonlinear fiber optics,  $\epsilon_0$  is the vacuum permittivity and  $\chi^{(n)}$  is the  $n$ -th order component of the electric susceptibility of the medium. From eq. (1), the most contributory to FWM process in this case is the 2-nd term showing the two different electric components to be emitted

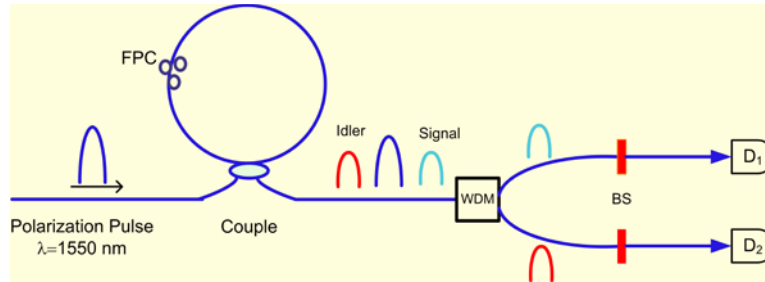
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out from the birefringent nonlinear fiber optic ring resonator. In practically, this is the great choice to generate a pair of entangled photons. If FWM process starts with the pumping electric field in the form  $E = E_0 \cos(\omega t)$ , then the corresponding intensity dependent refractive index of a medium can be written as,

$$n = n_0 + \frac{3\chi^{(3)}}{8n_0} |E_0|^2 = n_0 + n_2 I, \tag{2}$$

where  $n_0 = (1 + \chi^{(1)})^{1/2}$  is a linear refractive index,  $n_2$  is the second-order nonlinear refractive index, and  $I$  is the intensity of the electric field.

A schematic for entangled photon generation in nonlinear fiber optic ring resonator is given in figure 1.



**Fig. 1:** Show a schematic of nonlinear fiber ring resonator that creates entangled photon, i.e. a superposition of signal and idler photons wave function. Having launched into a fiber and travelled many roundtrips within the ring resonator, the outgoing pumping mode of photon pulses are blocked by a wavelength division multiplexing (WDM) apparatus and permit only the signal and idler modes go forward to Detector D1 and D2.

To clarify the concerning FWM process theoretically, it must establish theoretically three types of Hamiltonian in the process. The first Hamiltonian is about nondegenerated pumping ultra-short polarized pulses that launch into nonlinear fiber ring resonator, i.e.

$$H_{\text{pump}} = i\hbar \sum_{k=1,2} \epsilon_k (a_k^\dagger - a_k), \tag{3}$$

where  $a_k^\dagger$  and  $a_k$  are photon creation and annihilation operators respectively. Note that,  $\epsilon_k$  is laser field strength of the pumping pulses of photon modes 1 and 2. The most important part is the second Hamiltonian concerning about photon-photon interaction in the FWM process, i.e.

$$H_{\text{fwm}} = i\hbar \chi a_s^\dagger a_i^\dagger a_1 a_2, \tag{4}$$

where  $i = \sqrt{-1}$ ,  $\hbar = h/2\pi$ ,  $h$  is Planck constant,  $\chi = 3\epsilon_0 \chi^{(2)} \omega^2 k_0^2 / 8\epsilon^2 V_Q$  is an interaction strength within fiber optic ring resonator,  $\epsilon$  is an induced fiber optic dielectric constant,  $k_0$  is a wave number of light in vacuum, and  $V_Q$  is a quantized volume. The commutation relation for photon operators in eq. (3) and (4) are given by  $[a_m^\dagger, a_n] = \delta_{m,n}$  and equal to zero for the others. In this process, at the same time, there must be two conservation laws to be considered, i.e. the

energy,  $\omega_1 + \omega_2 = \omega_s + \omega_i$ , and the momentum,  $k_1 + k_2 = k_s + k_i$ . The last Hamiltonian that must take into account is the photon-reservoir interaction, i.e.

$$V = \hbar \sum_{k=i,s,1,2} (a_k \Gamma^\dagger + a_k^\dagger \Gamma), \tag{5}$$

where, as usual,  $\Gamma^\dagger$  and  $\Gamma$  are the reservoir creation and annihilation operators respectively, and in general  $\Gamma(t) = \sum_l g_l b_l \exp[i(\omega - \omega_l)t]$ , where  $b_l$  is phonon annihilation operator of mode  $l$ ,  $g_l$  is strength constants of mode  $l$  and  $\omega_l$  is phonon frequency of mode  $l$ . Similar to the photon case, the useful boson commutation relations for these phonon or reservoir operators are given by  $[\Gamma_l, \Gamma_{l'}^\dagger] = \delta_{l,l'}$  and equal to zero for the others. It is an appropriate opportunity to establish the total Hamiltonian as

$$H = H_{\text{pump}} + H_{\text{fwm}} + V. \tag{6}$$

At this point, it is convenient to transform eq. (6) into the corresponding Heisenberg equation of motion system. Since the coupling between photon and reservoir operators make the problem more sophisticated to analyze further. Fortunately, the extension of Heisenberg equation of motion incorporated with Markov approximation can eliminate the reservoir operators and left only the system variables. Its form can be written as

$$i\hbar \frac{\partial}{\partial t} \rho_S = [H_{\text{fwm}} + H_{\text{pump}}, \rho_S] + \sum_{k=i,s,1,2} \gamma_k (2a_k \rho_S a_k^\dagger - a_k^\dagger a_k \rho_S - \rho_S a_k^\dagger a_k) + \sum_{k=i,s,1,2} 2n_k^{\text{th}} \gamma_k (a_k \rho_S a_k^\dagger - \rho_S a_k a_k^\dagger - a_k^\dagger \rho_S a_k + a_k^\dagger \rho_S a_k), \tag{7}$$

where  $\rho_S$  is a system density operator or density matrix,  $\gamma_k$  is a damping rate of mode  $k$ ,  $n_k^{\text{th}} = 1 / (e^{\hbar\omega_k / k_B T} - 1)$  is the Bose - Einstein distribution function at thermal equilibrium in absolute temperature  $T$ , and  $k_B$  is Boltzmann constant.

However, eq. (7) is actually still hard and complicated to solve for  $\rho_S$  as function of time, for the reason of being in operator language, so the next strategy is to cast all operators into c-number. This can be achieved by applying the photon operator  $+P$  representation [8] and then eq. (7) is easily reduced into Fokker- Planck equation as,

$$\begin{aligned} \frac{\partial P(\alpha, t)}{\partial t} = & \left\{ \frac{\partial}{\partial \alpha_1} (\chi \alpha_s \alpha_i \alpha_2^+ - \epsilon_1 + \gamma_1 \alpha_1) + \frac{\partial}{\partial \alpha_1^+} (\chi \alpha_s^+ \alpha_i^+ \alpha_2 - \epsilon_1 + \gamma_1 \alpha_1^+) \right. \\ & + \frac{\partial}{\partial \alpha_2} (\chi \alpha_s \alpha_i \alpha_1^+ - \epsilon_2 + \gamma_2 \alpha_2) + \frac{\partial}{\partial \alpha_2^+} (\chi \alpha_s^+ \alpha_i^+ \alpha_1 - \epsilon_2 + \gamma_2 \alpha_2^+) \\ & - \frac{\partial}{\partial \alpha_i} (\chi \alpha_s^+ \alpha_2 \alpha_1 + \gamma_i \alpha_i) - \frac{\partial}{\partial \alpha_i^+} (\chi \alpha_s \alpha_2^+ \alpha_1^+ + \gamma_i \alpha_i^+) \\ & - \frac{\partial}{\partial \alpha_s} (\chi \alpha_i^+ \alpha_2 \alpha_1 + \gamma_s \alpha_s) - \frac{\partial}{\partial \alpha_s^+} (\chi \alpha_i \alpha_2^+ \alpha_1^+ + \gamma_s \alpha_s^+) \\ & \left. - \frac{\partial^2}{\partial \alpha_2 \partial \alpha_1} \chi \alpha_s \alpha_i - \frac{\partial^2}{\partial \alpha_s \partial \alpha_i} \chi \alpha_2 \alpha_1 + \frac{\partial^2}{\partial \alpha_s^+ \partial \alpha_i^+} \chi \alpha_2^+ \alpha_1^+ - \frac{\partial^2}{\partial \alpha_2^+ \partial \alpha_1^+} \chi \alpha_s^+ \alpha_i^+ \right\} P(\alpha, t), \tag{8} \end{aligned}$$

This equation can be decomposed into the drift and the diffusion terms to get a corresponding stochastic equation.

where the correlations of noise variables are given by

**Results and Discussions**

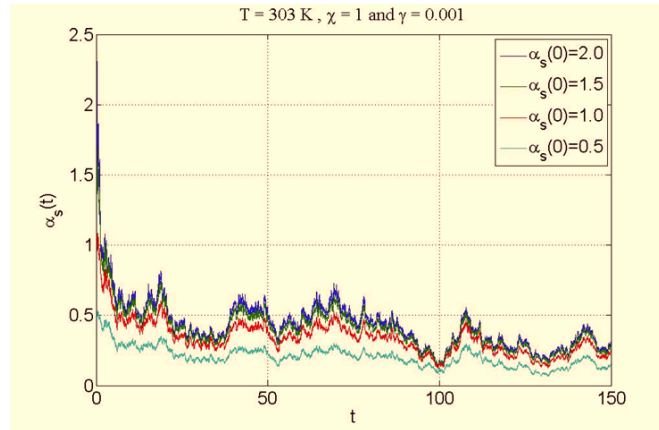
$$\begin{aligned} \langle \eta_k(t) \rangle &= \langle \eta_k^\dagger(t) \rangle = 0, \\ \langle \eta_j(t) \eta_k(t') \rangle &= \langle \eta_j^\dagger(t) \eta_k^\dagger(t') \rangle = \delta_{jk} \delta(t - t'), \end{aligned} \tag{10}$$

The Kramers-Moyal expansion is applied to eq. (8) to render the corresponding stochastic equation system for all four kinds of photons, i.e. [9]

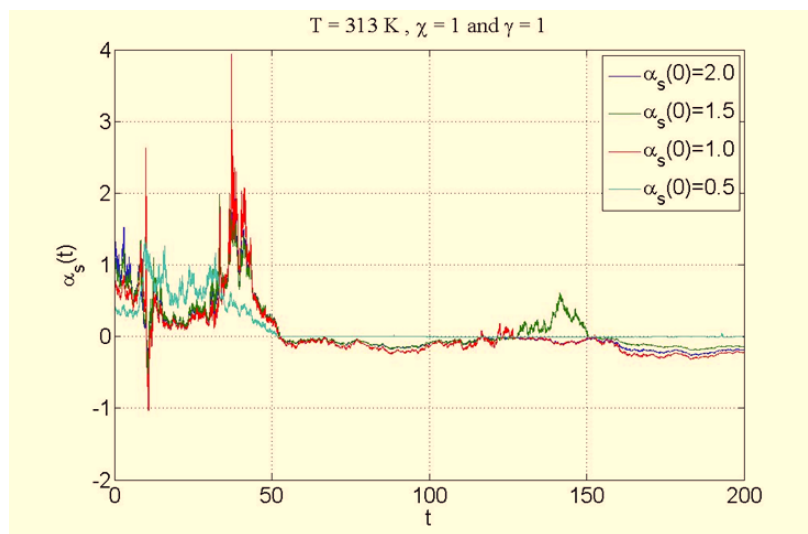
and  $j, k = 1, 2, s, i$  are indexing the photon's modes.

$$\left. \begin{aligned} \frac{d}{dt} \alpha_1 &= -\chi \alpha_s \alpha_i \alpha_2^+ + \epsilon_1 - \gamma_1 \alpha_1 + \sqrt{2n_1 + \chi |\alpha_s|^2} \eta_1, \\ \frac{d}{dt} \alpha_2 &= -\chi \alpha_s \alpha_i \alpha_1^+ + \epsilon_2 - \gamma_2 \alpha_2 + \sqrt{2n_2 + \chi |\alpha_i|^2} \eta_2, \\ \frac{d}{dt} \alpha_i &= \chi \alpha_1 \alpha_2 \alpha_s^+ - \gamma_i \alpha_i + \sqrt{2n_i + \chi |\alpha_i|^2} \eta_i, \\ \frac{d}{dt} \alpha_s &= \chi \alpha_1 \alpha_2 \alpha_i^+ - \gamma_s \alpha_s + \sqrt{2n_s + \chi |\alpha_2|^2} \eta_s, \end{aligned} \right\}, \tag{9}$$

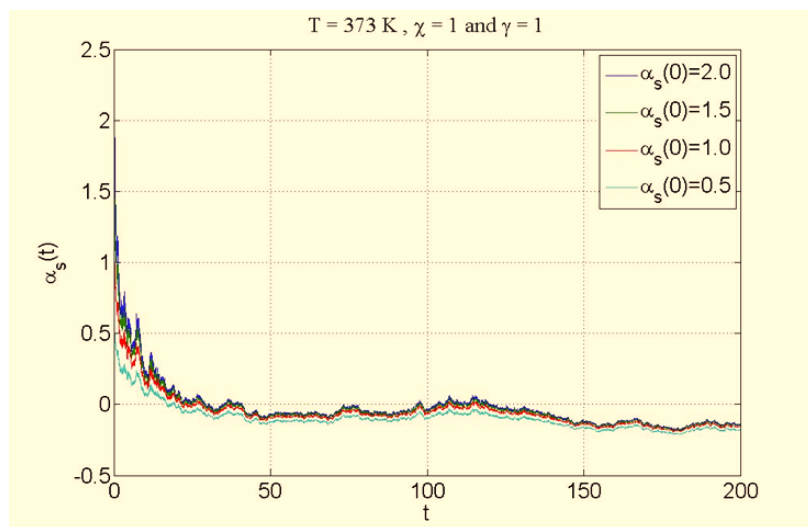
Numerical results of eq. (9) are provided by graphical approach as shown in fig. 3-5 with the effect of three main factors dependence: temperature  $T$  in Kelvin, nonlinear susceptibility coupling constant  $\chi$  and damping rate  $\gamma$  to be assigned and tested. Note that, the horizontal axis in the graphs stand for time in unit of second and the vertical axis means signal photon state amplitude. The various high temperature effects affect the evolution and validity of entangled photon state lifetime.



**Fig. 3:** Show the survival time of the signal photon states at temperature  $T = 303$  K with nonlinear susceptibility  $\chi = 1$  and damping rate strength  $\gamma = 0.001$ .



**Fig. 4:** Show the evolution and survival time of the signal photon states with the assigned initial values at temperature  $T = 313$  K, nonlinear susceptibility  $\chi = 1$  and damping rate strength  $\gamma = 1$ .



**Fig. 5:** Show the survival time of the signal photon states at temperature  $T = 373$  K with nonlinear susceptibility  $\chi = 1$  and damping rate strength  $\gamma = 1$ .

It is found from fig. 3-5 that the system encounter noises [10], and the entangled photon lifetime especially in fig. 3, at the lowest given temperature, is longer than the others. The highest abnormal ultra-short upward and downward peaks in fig. 4 occur temporarily from the resonance effect, from the interference between the systems' and reservoirs' photons. The highest upper peaks of the signal states are accounted for the unmatching between the electronics energy levels of reservoir and the signal photon energy, and the lowest peak is the photon absorption of reservoir, i.e. the reservoir energy width be almost equal to photon energy. Eventually, the reservoir injects energy into the system again and gains the photon energy via the constructive interference to restore the lifetime of signal state to survive again. This suggests that the temperature dependence of the reservoir plays the central role to entangled photon lifetime.

#### *Conclusion:*

Three types of Hamiltonian: the pumping ultra-short pulses, photon-photon interaction via FWM process and photon – reservoir coupling part have been established. From the stochastic equation system, the main factor from environment inducing the dissipation that affect to the entangled state lifetime. However, this lifetime can survive by selecting some appropriate reservoir temperatures. Fig. 3 and 4 suggest that long lifetime of the signal state exists at low temperature and can avoid for the short lifetime effect by considering the unmatching between the reservoir electronic energy width and the signal photons' energy.

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