ORIGINAL ARTICLES

**A New Model Describes the Simultaneous Effect of the Incident Ion Energy and Angle on the Sputtering Yield**


**ABSTRACT**

No equation exists combining the sputtering yield as a function of two variables, i.e. the ion incident energy and angle. The conicoidal modeling has been constructed to offer such equation. The pictorial methodology trial suggests the conicoidal dimensions to evaluate the equation's parameters and their corresponding projected components on the target surface at various incident energies and angles. Also, the pictorial methodology solves the counteracting effects acting upon the behavior of the sputtering yield values with the continuous increasing of the sputtering efficiency with increasing ion incident angles. The solution was interpreted through the decreasing effect of the escape depth $\lambda_{\text{esp.dep}}(\theta)$, the decreasing effect of the constant effective energy $E_{\text{eff}}$, and the increasing effect of the encounter collision numbers $\omega$ parameters. We found that the power ratio $(J/h)$ plays a role in controlling the height and position of the maximum sputtering yield $S_{y\text{-max}}$ of the present theoretical data to be shifted more towards the corresponding available experimental data.

**Key words:** Nanoscale materials, nanoparticles. Theories and models: sputtering phenomenon, Atomic, molecular and ion impact and interaction with surfaces.

**Introduction**

The interaction of particles with solid surfaces causes several processes. Sputtering is one from the important effects in this regime. For both physical understanding of the collision processes and the erosion of a target surface, sputtering of atoms from solid surfaces under ion bombardment has been studied for several decades. Sputtering plays a substantial role in research and various practical interests in several technological fields, e.g. cleaning, sputtered thin film deposition (in large scale for technical applications), as tool for high resolution depth profiling (Benninghoven, A., 1984) and in plasma wall interaction problems (Langley, R.A., et al., 1984). Due to the obvious advantages and frequent applications, much theoretical and experimental works have already been done. Several surveys were collected by Anderson and Bay (1981), Jackson (1980), Yamamura and Tawara (1996), Behrisch and Eckstein (2007), Matsunami et al., (1984). Also, different authors have described the experimental data of the sputtering yield by a simple geometrical models for example, the ellipsoidal statistical modeling by Schwarz et al., (1979), Monte Carlo programs based on the binary collision approximation by Biersack and Eckstein (1984), a semi-empirical formula by Yamamura (1984), an empirical equation by Wetz et al., (1997) and the conicoidal statistical modeling by Grais et al., (2010). A number of unsolved problems or untested hypotheses, on quantitative analysis, have been mentioned in various theories and modeling. Some of these difficulties arise from the approximations done to some parameters presented in the previous theories or from the necessity to choose other values for some parameters, to force fit the calculated sputtering yield data to the corresponding experimental data. The advantage of the conicoidal modeling is that the visualization could solve some of the above denoted difficulties by applying the pictorial methodology (Grais, Kh. I., et al., 2010) to evaluate the present equation's parameters, without subjected them to any approximations. In the present paper, such parameters, as the radius of the imaginary existing statistical sphere $r_{0}(\theta)$, the atom-atom mean free path in the solid target, $\lambda_{\text{a-a,mfp}}(\theta)$, the escape depth $\lambda_{\text{esp.dep}}(\theta)$ and the ion range $R(\theta)$, at normal incidence, are re-directed towards the target surface so that their components could be pictorially evaluated, at various incident angle ($\theta$), relative to the target normal (r.t.T.N), at a constant ion energy. The pictorial methodology evaluation of the sputtering yield versus ion incident angles casts some light on the geometrical effects of the conicoidal construction. The present obtained theoretical data of the sputtering yield $S_{y}(\theta)$ has be confronted with those of TRIM.SP of Biersack and Eckstein's data for $H^{+}$/Ni combination, with the semi-empirical of Wetz data for $Xe^{+}$/Ni combination and with their available experimental data (Behrisch, R., & W. Eckstein, 2007; Biersack, J.P., & W. Eckstein, 1984; Wetz, P., et al., 1997), to illustrate the **Corresponding Author:** Grais Kh.I., Spectroscopy department, Phys.Dept., National Research Centre, Dokki,Giza,Cairo,Egypt

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interesting features of the conicoidal visualization on angular and energy dependences of the sputtering yield, and also to give favorable evidence on the pictorial methodology idea arising from the conicoidal model construction.

Theoretical Treatment:

Previous outlines:

Grais et al’s equation (Grais, Kh. I., et al., 2010), arisen from the geometrical construction of the conicoidal model in Fig.1, for the sputtering yield $S_y(0^\circ)$ as a function of ion energies $E_i$ at normal incidences is given by:

$$S_y = \left(\frac{\pi B}{k_{ne}^2} \right) \left(\frac{3}{8\pi \psi \Delta H_a} \right)^{\frac{2}{3}} \left( \frac{1.2 \beta E_i^{1/3}}{1.2 + 0.8 \beta E_i^{1/3}} \right)^2 \left( 1 - \left( \frac{E_a}{E_i} \right) \right)^{\frac{1}{3}} $$

(1)

Where $\beta = k_{ne}(3\pi \psi \Delta H_a)^{1/3} / 2$, $k_{ne} = R/2E_i$ (in angstrom per electron volt) is the energy loss rate given in (Schwarz, and Helms, 1979); $R$ is the ion range; $\Delta H_a$ is the number of displaced atoms per unit surface area and $A$ the fraction of the displaced atoms in the statistical region; $\psi \Delta H_a$ as the number of the heat of atomization per atom $\Delta H_a$ and $B$ as the fraction of displaced atoms at the surface that actually escape.

Fig.1a represents a prime fixed conicoidal construction (PFCC) of volume $V_{PFCC}$ with the long axis $2h(1\rightarrow7) = 241.88$ A$^o$, together with a corresponding imaginary existing sphere (IES) of volume $V_{IES}$ with the radius $r_S(0^o)(2\rightarrow4C) = 60.47$ A$^o$. They are equal in size, i.e. $\frac{4}{3}\pi r_S^3(0^o) = \frac{2}{3}\pi r_S^2(0^o)h$ (Bronstein, and Semendjajew, 1964). So, in figure 1 a and b we can see that $h(1\rightarrow4C) = 2r_S(0^o) = 120.94$ A$^o$. $R(0^o)(2\rightarrow6) = 120.94$ A$^o$ is the ion range, $\lambda_{esp.dep}(0^o)(3\rightarrow4C) = 48.38$ A$^o$ is the escape depth of atoms in the solid, $b(0^o)(8\rightarrow9) = 72.56$ A$^o$ is the diameter of the intersected statistical affected area on the target surface, at normal incidence and $\Delta r_s(0^o)(2\rightarrow3) = 12.09$ A$^o$ is the height of the protruded spherical zone (PSZ). The symbols in Fig.1a hold for Fig. 1b. Also, they have been defined along the text and the figure captions of 1a and 1b.

**Fig. 1a:** The geometric construction employed in the present model: $R(0^o)(2\rightarrow6)$ is range of the primary incident ion, $\lambda_{esp.dep}(0^o)(3\rightarrow4C)$ is the escape depth of the sputtered atom, $r_s(0^o)(2\rightarrow4C)$ is radius of the imaginary existing sphere at normal incidence, $h(1\rightarrow4C)$ is the height of the protruded spherical zone (PSZ). The illustrated dimensions occur at normal incidence on 100keV Xe+/Ni combination, $r_s(0^o) = 60.47$ A$^o$, $\lambda_{esp.dep}(0^o)(3\rightarrow4C) = 48.38$ A$^o$, $\Delta r_s(0^o)(2\rightarrow3) = 12.09$ A$^o$, $b(0^o)(8\rightarrow9) = 36.28$ A$^o$ and $h = 120.94$ A$^o$ are dimensionally estimated by the pictorial methodology.
Present outlines:

The course of formulating a single equation for the sputtering yield $S_y(\theta)$ as a function of both incident energies $E_i$ and angles $\theta$ variables assumes similar arguments (Grais, Kh. I., et al., 2010), with addition series of modifications, to render Eq. (1) to the desired single equation of both variables.

Pictorial methodology evaluation of the normalized sputtering yields at incident angles:

The expression for the normalized sputtering yield $S_y(\theta')/S_y(0)$ at various incident angles could be pictorially obtained from Fig. 1b, where two identical (black and magenta) conicoidal constructions overlapping each other at an angle at $\theta' = 45^\circ$ (for instance), relative to the target normal incidence (r.t. T N) of the fixed black construction, from the common impact point (3), on the target surface, for 100 keV Xe$^+$/Ni combination. The magenta constriction is considered the movable one forming the desired incident angle $\theta'$ under investigation. The symbols in Fig. 1b are the same as in Fig. 1a, and they have been defined along the text and the figure 1a and 1b captions.

The $S_y(\theta)$, at normal incidence, is assumed to be proportional to the statistical affected surface area (Schwarz, S.A. & C.R. Helms, 1979; Grais, Kh. I., et al., 2010) $A_{sta}(0^\circ) = \pi b^2(0^\circ) = 4.1351E3A^2$, where $2b(0^\circ)(8\rightarrow9) = 72.56 A^\prime$, is the diameter of $A_{sta}(0^\circ)$. So, $S_y(45^\circ)$ would be proportional to the statistical affected surface area $A_{sta}(45^\circ) = \pi b^2(45^\circ) = 1.0211E4 A^2$, where $2b(45^\circ) = 114.02 A^\prime$ is the diameter of the statistical affected surface area $A_{sta}(45^\circ)$ at the incident angle 45$^\circ$. $A_{sta}(45^\circ)$ is evaluated by extrapolating the normal $2b(0^\circ)$ diameter, located on the target surface, to intersect the magenta lateral branches of the envelope surrounding the magenta secondary identical movable conicoidal construction (SIMCC) of volume $V_{SIMCC}$, to be $2b(45^\circ) = 114.02 A^\prime$, at incident angle $\theta' = 45^\circ$. So, the normalized sputtering yield $S_y(\theta')/S_y(0)$ would be equal to 2.469, calculated at incident angle $\theta' = 45^\circ$. Other values for $S_y(\theta')/S_y(0)$ at various incident angles $\theta'$ have been computed, following the above procedure, and drawn in Fig. 2-up.

**Fig. 1b:** Pictorial representation of the sputtering yields angular dependence, at incident angle ($\theta'$) = 45$^\circ$. The normal-to-glancing incidence normalized ratios of $R(\theta') / R(0) = 76.97/108.85 = 0.707$, $\lambda_{esp.dep}(\theta') / \lambda_{esp.dep}(0) = 34.21 / 48.38 = 0.707$, $r_s(\theta') / r_s(0) = 42.13 / 60.47 = 0.7$ (Fig. 3.), while $b(\theta') / b(0) = 57.01/36.28 = 1.57$, where $2b(\theta')$ is diameter of the affected surface area at incident angle ($\theta'$), presented in Fig. 1b as the extension of the normal $2b(0)$ diameter ($8\rightarrow9$) on the surface to cut the magenta lateral branches boundaries of the movable conicoidal model, and $\Delta r_s(\theta') / \Delta r_s(0) = 26.26 / 12.09 = 2.27$ (Fig. 5.). The symbols and their meanings hold fixed as shown in Fig. 1a.
Figure 2-up shows the double logarithmic represents $S_y(\theta^o)/S_y(0^o)$ versus sec($\theta^o$). Each pictorially calculated normalized sputtering yield point represents two overlapped yield values for 1 and 100 kev Xe+ /Ni combination, showing that pictorially calculating points are independent of the energy, ion and target in use. Also, these normalized sputtered values scatter abruptly that they failed to form a smooth straight line, but give three or more connecting straight lines with reducing slopes, expecting the continues decreasing effect in the sputtering yield values with increasing incident angles. Each line occupies a certain incident angle region, having an empirical equation of the form:

$$S_y(\theta^o)/S_y(0^o) = G(\sec(\theta^o))^\xi$$  \hspace{1cm} (2)

Eq. (2) could describe the whole normalized yield points providing that the couple parameters (G,ξ) should also vary with incident angles region. Fig. 2-up shows that the parameters (G,ξ) are (1.3), (2,0.73) and (3,0.41) at the incident angle region (0°,40°), (40°,75°) and (75°,85°) respectively. Thus, Fig.2-up assumes that the angular dependence of the yield changes with changing angle regions under study. Such behavior could be affected by the structural illustration of the conicoidal model. However, Fig. 2-up shows also that the pictorially evaluated normalized yields seem to scatter better around a logarithmic curve, covering the whole ion incident angles giving a format trend line for the double logarithmic representation as following:

![Fig. 2: (up and down) The normalized sputtering yield $[S_y(\theta^o)/ S_y(0^o)]$ and Sputtering efficiency (Sp.Eff) respectively versus the ion incident angles ($\theta^o$).](image)

![Fig. 3: (a). ln $[\lambda_{esp.+dep}(\theta^o)/ \lambda_{esp.+dep}(0^o)]$, (b) ln $[(r_s(\theta^o) + \lambda_{esp.+dep}(\theta^o)) / (r_s(0^o) + \lambda_{esp.+dep}(0^o))]$, and (c) ln $[r_s(\theta^o)/ r_s(0^o)]$ versus ln (sec. $\theta$).](image)
\[ S_y(\theta^o)/S_0(0^o) = 2.8366 \ln(\sec(\theta^o)) + 1.201 \]  

(3)

Eq. (3) assumes that each evaluated yield value could have its own couple \((G, \xi)\) parameter, which could be evaluated from Eq.(3) as follow:

Differentiating Eq. (3) w.r.t \(\sec(\theta^o)\), \(\xi\) could be found as:

\[ \xi = \frac{2.8366}{\sec(\theta^o)} \]  

(4)

\(\xi\) values are substituted in Eq. (2) with the corresponding pictorially determined \(S_y(\theta^o)/S_0(0^o)\) values, at various incident angles, to give the corresponding \(G\) values.

The \((G, \xi)\) computed values are the same for 1 and 100 Kev Xe\(^+\)/Ni combinations, and listed in the following Table 1:

<table>
<thead>
<tr>
<th>(\theta^o)</th>
<th>(\xi)</th>
<th>(\theta^o)</th>
<th>(G)</th>
<th>(\theta^o)</th>
<th>(\xi)</th>
<th>(\theta^o)</th>
<th>(G)</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>2.48</td>
<td>2.96</td>
<td>1</td>
<td>45</td>
<td>2.01</td>
<td>2.4</td>
<td>1.23</td>
</tr>
<tr>
<td>5</td>
<td>2.83</td>
<td>2.91</td>
<td>1.001</td>
<td>50</td>
<td>1.28</td>
<td>2.27</td>
<td>1.18</td>
</tr>
<tr>
<td>10</td>
<td>2.79</td>
<td>2.89</td>
<td>1.004</td>
<td>55</td>
<td>1.63</td>
<td>2.11</td>
<td>1.172</td>
</tr>
<tr>
<td>15</td>
<td>2.74</td>
<td>2.86</td>
<td>1.05</td>
<td>60</td>
<td>1.42</td>
<td>1.93</td>
<td>1.193</td>
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<td>2.82</td>
<td>1.03</td>
<td>65</td>
<td>1.2</td>
<td>1.71</td>
<td>1.298</td>
</tr>
<tr>
<td>25</td>
<td>2.57</td>
<td>2.77</td>
<td>1.08</td>
<td>70</td>
<td>0.97</td>
<td>1.47</td>
<td>1.489</td>
</tr>
<tr>
<td>35</td>
<td>2.32</td>
<td>2.62</td>
<td>1.165</td>
<td>80</td>
<td>0.493</td>
<td>0.846</td>
<td>2.64</td>
</tr>
<tr>
<td>40</td>
<td>2.173</td>
<td>2.52</td>
<td>1.289</td>
<td>85</td>
<td>0.247</td>
<td>0.457</td>
<td>4.44</td>
</tr>
</tbody>
</table>

Fig. 4 shows the parameter \(G\) versus sec(\(\theta^o\)) and the corresponding expected polynomial equation for low and heavy incident \(H^+\) and Xe\(^+\)ion, impacting the Ni target:

\[ G = (\delta)(\sec(\theta^o))^2 + (\sigma)(\sec(\theta^o)) + (\phi) \]  

(5)

The treble constants \((\delta, \sigma, \phi)\) varies as \((-0.0166, 0.5324, 0.4039)\) respectively.

Confirmation to the above analysis was provided by other investigators (Rol, P.K., et al., 1960; Bader, M., et al., 1960; Almen, O., & G. Bruce, 1961; Ramer, C.E., et al., 1964), who gave for \(\xi\) the values 2, 1.5 and 1, in the energy regions (20-60 keV), (60-110 keV) and 25 keV for various ion/target combinations. While the present work offers for \(\xi\) different values, ranging from 3 to 0.25 (Table 1), depending also on the incident angle under study, irrespective of the ion energy and ions/targets combination. It appears therefore, from the above analysis that the angular dependence of the yield changes with the angle- as well as the energy-region under study (Carter, G., & J.S. Colligon, 1968) for a certain ion/target combination, with the expectation that the couple \((G, \xi)\) could also be angle dependent.

![Fig. 4: The factor (G) versus sec (\(\theta^o\)).](image)
The method of developing a single equation combining two effects of both ion incident energy and angle on the sputtering yield could be followed in Appendix 1. The derived equation is:

\[
S_y(\theta) = \frac{\eta}{\pi} \frac{\Delta H \eta_n}{\Delta H_\text{a}} \left( \frac{1}{A(\theta) \eta_n} \right)^{1/3} \left( 1.2 \beta E_{\text{eff}}^{-1/3} (\sec(\theta))^1/3 \right) / \left( 1.2 + 0.8 \beta E_{\text{eff}}^{-1/3} (\sec(\theta))^{1/3} \right)
\]

(6)

Results and Discussions

The course of computation:

The operational parameters \(A(0^\circ), \ k_n(0^\circ), \ R(0^\circ), \ r_{\text{se}},(0^\circ), \ k_{\text{se},\text{dep}}(0^\circ), \ \beta, \ \text{and} \ \gamma\) presented above in various equations, and their corresponding values at various incident angles \(\theta\), have been determined by the pictorial methodology, which depends on the conoidal model construction. The parameters \(k_n, \ k_e, \ \text{and} \ \kappa_n\) of nuclear, electronic and their product stopping powers respectively are determined by Eqs. (27, 29) in Ref. (Schwarz, S.A. & C.R. Helms, 1979). Their computed values for \(k_n = k_n^*k_e\) are 1.167343, and 2.9318676E-4 for \(H^+/\text{Ni}\), and \(Xe^+/\text{Ni}\) combinations respectively. It should be noticed that the chosen product \((k_n^*k_e)\) is considered after Eq.(32) in Ref. (Schwarz, S.A. & C.R. Helms, 1979), which shows a possible connection form between \(k_n\) and \(k_e\). The factor \(\eta\) takes different values depending on incident energy, ion/target combination, despite the obvious difference between the theoretical value of \(A(0^\circ)/A(0^\circ) = \eta E^{-16}\) for the light incident ions \(H^+\) ion, obtained by Eq.(16) in Ref. (Grais, Kh. I., et al., 2010) and that of the present used value \((\eta E^{-11})\). The fraction of the displaced atoms \(A(0^\circ)\) and the reciprocal of the energy loss \(k_n(0^\circ)\) are assumed constant for the present used values \((\eta E^{-11})\) and 1.167343 respectively for the light incident ions \(H^+\) ion, and \((\eta E^{-3})\) and 2.9318676E-4 respectively for heavy incident ions \(Xe^+\) ion, irrespective of the incident angle \(\theta\) values. \(A(0^\circ)\) and its corresponding \(A(0^\circ)\) values as well as \(k_n(0^\circ)\) and its corresponding \(k_n(0^\circ)\) values, are small enough and exist in the expressions \(\beta = k_n(0^\circ)^{-1/3}/\Delta H_nA(0^\circ)^{-1/3}\) and \((\eta E^{-11}) = \eta E^{-11}k_n(0^\circ) = 1.167343\) for the light incident ions \(H^+\) ion, and \((\eta E^{-3}) = 2.9318676E-4\) for heavy incident ions \(Xe^+\) ion, irrespective of the expected small variations in the parameters \(A(0^\circ)\) and \(k_n(0^\circ)\) values at various incident angles \(\theta\), to have little effect on computing the \(S_y\) value. Eq.(6) was obtained after Eq. (6), raising to different powers. We have found that, in all comparisons to experiment, good results are obtained with \(A(0^\circ) = \eta E^{-11}, k_n(0^\circ) = 1.167343\) for the light incident ions \(H^+\) ion, and \((\eta E^{-3}) = 2.9318676E-4\) for heavy incident ions \(Xe^+\) ion, irrespective of the expected small variations in the parameters \(A(0^\circ)\) and \(k_n(0^\circ)\) values at various incident angles \(\theta\), to have little effect on computing the \(S_y\) value. Eq.(6) was obtained to be relatively insensitive to any variations in \(A(0^\circ)\), and \(k_n(0^\circ)\) values at various ion incident angles. The factor \(\psi\) of the heat of atomization, presented in the displaced energy \(E_{\text{dep}} = \psi \Delta H_a\), is assumed to be 3.5 for \(H^+/\text{Ni}\), and 2 for \(Xe^+/\text{Ni}\) combinations. The parameters \(\beta = 1.281597E-5, \text{ and} \ 7.696869E-5\) were obtained for \(H^+/\text{Ni}, \ Xe^+/\text{Ni}\) combinations respectively, using Eq. (15) or (16) in Refs. ((Schwarz, S.A. & C.R. Helms, 1979; Grais, Kh. I., et al., 2010) respectively, with the other parameters \(B, \Delta H_a, n, \psi\) which have the values 0.147, 3.9442ev, 0.09126 for the Ni-target.

The physical meaning of the exponent parameter \(\omega\):

The number of displaced atoms per unit area \(A(0^\circ)\) and the corresponding reciprocal energy loss \(k_n(0^\circ)\) in \(A^\circ\text{ev}^{-1}\) would be inherent inclusion in the whole intersected statistical affected surface area \(A_{\text{ss}}(0^\circ)\) and the whole range \(R(0^\circ)\) respectively. So, \(A(0^\circ)\) and \(k_n(0^\circ)\) have been multiplied by \((\sec(\theta))^{1/3}\) and \((\cos(\theta))^{1/3}\) respectively, due to increasing of \(A_{\text{ss}}(0^\circ)\) or \(S_y(0^\circ)\) and decreasing of \(R(0^\circ)\) by re-directed them towards the target surface respectively, after a series of collisions encounters with increasing incident angle. Thus, the factor \(\omega\) is assumed to be the number of collision encounters in the present model, when the primary displaced atom
(1) causes a series of collisions and deflections before its final sputtering process. At the last encounter position, atom (I) would be deflected towards the target surface, carrying the last remained energy \((E^*-E_d)\), which should be in excess of \((10^6\text{eV})\) (Seitz, E., & J.S. Koehler, 1956) to be sputtered, where \((E_d)\) is the displaced energy. Part of this remained energy would be used to overcome the binding surface energy and refract the target surface with an emission angle as sputtering particle. It should be mentioned that the last encountered atom has, in some way to impart an impulse force to the primary atom (I), with a component directed towards the target surface, to be sputtered. \(\omega\) has been computed by Eq. (7), where the factor 2 is replaced by 3, and the parameter \(\xi\) is calculated by Eq. (4). Their computed values are the same for 1 and 100 KeV Xe\(^+/\)Ni combination, and listed in Table 1. Inspecting the table, it could be grouped into four angle stages, depending on the ion incident angle under study. The first region starts at the incident angle zero and ends to \(40^o\), with the approximated average value of \(\omega\) equal 3, suggesting the three encounters collision approximation between the atoms, through collision cascades. The second region covers the incident angles ranging from \(45^o\) to \(60^o\), to describe the two collision encounters approximation between the atoms through collision cascades. The third and the fourth regions cover the incident angles between \(65^o\) to \(80^o\) and \(\theta^o > 80^o\) respectively; describing one and zero collision encounters approximation between the atoms through collision cascades. The fourth region could be described as the region where no displaced atoms are created by the incident ions, but instead the incident ions would be totally reflected before depositing its energy at the target surface. Here, it should be worthy mentioned that Biersack and Eckstein (1984) investigated for the energy and angular distributions of the sputtered yield, for various ions including \(H\) and \(Xe\) on \(Ni\), using Monte Carlo Program extension TRIM.SP, to describe sputtering by a sequence of independent binary collisions between the atoms. However, the present model predicts four possible collision encounters approximation between the atoms, i.e. 3, 2, 1 and 0 depending on the angle region under study, i.e. at the incident angles zero to \(40^o\), \(45^o\) to \(60^o\), \(65^o\le \theta^o \le 80^o\) and \(\theta^o > 80^o\) respectively.

**The geometrical effect on the parameters \(\xi\) and \(G\) in Eq. 2 to 5:**

The extraordinary behavior of the Eq. (2), shown in Fig. 2-up, could be affected by the geometrical representation of the conicoidal model construction. Visualizing Fig. 1a and 1b, the upper half of the whole \(V_{IES}\) volume is presented within the escape depth length \(\lambda_{esp, dep}(\theta^o)\) region, which is responsible for the disturbed atoms to leave the target surface as sputtered atoms. Fig. 1b shows the process of obtaining a certain incident angle \(\theta^o\) under investigation. The magenta movable construction is tilted so that its magenta long axis 2h would make the desired incident angle \((\theta^o) = 45^o\) relative to the black fixed long axis 2h of the black conicoidal construction, from the common impact point (3), on the target surface. It is seen that \(\lambda_{esp, dep}(\theta^o)\) decreases with increasing angle \(\theta^o\), following by continuous sputtering reduction and simultaneous increasing incident ion reflection, from the surface, with increasing incident angle, until \(\lambda_{esp, dep}(\theta^o)\) disappears. Also, previous work (Grais, Kh. I., et al., 2010) assumed that the sputtering yield \(S_{y}(\theta^o)\) depends on the whole imaginary existing spherical embedded volume \(V_{IES}\), as well as the whole incident ion energy \(E^{eff}\). Figure 1a and 1b shows that a portion of the whole volume \(V_{IES}\), i.e. protruded spherical zone volume \(v_{psz}\) does not contribute to the whole \(V_{IES}\). So, it should be subtracted from \(V_{IES}\) to obtain both effective embedded volume \(V^{eff}\) and effective incident energy \(E^{eff}\), which are actually contributing in the sputtering process. Moreover, the sputtering efficiency fraction (SEF), i.e. the fraction of energy not deposited in the target (Schwarz, S.A., 1988), could be approximated to \(v_{psz} / V_{IES}\). The protruded spherical zone (PSZ) volume, \(v_{psz}\), above the target surface, represents the fraction of energy not deposited in the target (Schwarz, S.A., 1988), so that the fraction \(v_{psz} / V_{IES}\) could be defined as the SEF of the whole deposited incident energy in the target surfac.

The \(v_{psz}\) volume has been calculated at various incident angles \(\theta^o\) from Ref. (Bronstein, I.N., & K.A. Semendjajew, 1964) page 152, where:

\[
v_{psz} = \pi / 3 \Delta r^3_s(\theta^o)(3r_s(0^o) - \Delta r_s(\theta^o))
\]

\(\Delta r_s(\theta^o)\) is the height of the protruded spherical zone volume \(v_{psz}\). It has been pictorially evaluated from Figs. 1a and 1b at various ion incident angles \(\theta^o\), at a constant ion energy, bearing in mind that \(r_s(\theta^o)\), at zero incident angle, holds fixed for the unchangeable \(V_{IES}\), by changing ion incident angles \(\theta^o\). For instance, 100 kev Xe\(^+/\)Ni combination has black \(r_s(\theta^o)(2\rightarrow 4C) = 60.47\) \(A^o\) using Eqs. (6,11), in Refs. (Schwarz, S.A. & C.R. Helms, 1979; Grais, Kh. I., et al., 2010) respectively. The magenta \(\Delta r_s(\theta^o)(2\rightarrow 3)\) and the black \(\Delta r_s(\theta^o)(2\rightarrow 3)\) are 26.26 \(A^o\) and 12.09 \(A^o\) respectively (using the pictorial methodology), showing that \(\Delta r_s(\theta^o)\) increases with increasing incident angles \(\theta^o\).

Fig. 5 shows the normalized \(\Delta r_s(\theta^o) / \Delta r_s(0^o)\) versus the angle \(\theta^o\) varies as the following equation:

\[
\Delta r_s(\theta^o) = \Delta r_s(0^o) \exp(\mu \theta^o)
\]
μ is 0.018 holding also for 1 kev Xe+/Ni combination, and for other energies ions/targets combinations. The pictorial calculated $S_y(θ^0)/S_y(0^0)$ and $t_{psz}/V_{IES}$ versus $sec(θ^0)$ and their curves trend hold fixed irrespective of the incident energies and ions/targets combinations under study. Fig. 2 (up and down) includes the two functions $F1(S_y(θ^0)/S_y(0^0))$ and $F2(t_{psz}/V_{IES})$ versus $sec(θ^0)$, (up and down) respectively, in a double logarithmic representation. It could be seen, that the two functions increase with increasing incident angle, but with different increasing rates. F1 curve seems to approach a maximum point at a certain incident angle $θ_{max}$, expecting that $S_y(θ^0)/S_y(0^0)$ decreases with increasing $sec(θ^0)$, while $F2$, after that expected $θ_{max}$, continues increasing with increasing incident angles. Thus, at angles very close or equal to 90°, it is expected that $F1$ of the sputtering yield would approach the zero value, and the sputtering efficiency $F2$ would be at its highest efficient value. So, a resultant expectation is that the incident ions would be totally reflected without depositing their energies into the target surface. So, $F1$ curve could be extrapolated, after the maximum point $(S_y(θ^0)/S_y(0^0))_{max}$, towards the assumed presented available experimental yield data. Such assumed presented experimental yield data are hard to get from the literatures, as the incident angles approaching the angle region between $θ_{max}$, $θ_{max} ≤ θ < 90°$. It appears therefore, from the above analysis that the angular dependence of the yield changes with the angle region as well as the energy region under study (Carter, G., & J.S. Colligon, 1968; Rol, P.K., et al., 1960; Bader, M., et al., 1960; Almen, O., & G. Bruce, 1961; Ramer, C.E., et al., 1964) for a certain ion/target combination. Therefore it is expected that the couple $G$ and $ξ$ could also be angle dependent. Thus, the apparent contradiction of Eq. (2) could be solved by replacing its apparent straight line characteristic by an increasing logarithmic curve having the form as in Eq. (3). This form describes the whole curve of the function $ln |S_y(θ^0)/S_y(0^0)|$ versus $ln sec(θ^0)$, presented in Fig. 2-up, providing that the factors $ξ$ and $G$ in Eq. 2, should also vary with $sec(θ^0)$ in different forms, with the resultant Eqs. 3, 4 and 5.

![Fig. 5: $ln|Δr(θ^0)/Δr(0^0)|$ versus the incident angles ($θ^0$) for 1 Kev Xe+/Ni combination and 100 Kev Xe+/Ni combination.](image)

The sputtering yield as a function of incident ion angle and energy:

Figs. 6(a,b,c,d) and 7(a,b) show the sputtering yields $S_y(θ^0)$ versus incident ion angles $θ^0$, at a decreasing effective incident energy $E_{i,eff}$ (while the incident energy $E_i$ is constant). For H⁺ and Xe⁺ bombardment on Ni, various incident ion energies, ranging from 0.5 to 200 KeV, were studied for the angular dependence of the sputtering yields at various incident angles, ranging from 0° to 88°, with special interest on the interval around the maximum yield. For comparison, available experimental sputtering yield data for light H⁺ and heavy Xe⁺ ions (Behrisch, R., & W. Eckstein, 2007; Biersack, J.P., & W. Eckstein, 1984; Wetz, P., et al., 1997) are included in these figures and presented by (open triangles), which are used as references to be matched by the present theoretical data. Also, they include the present theoretical data shown by (plus signs), Biersack and Eckstein (1984) theoretical data and Wetz et al., (1997) theoretical data, presented by (open circles). Here, it is worthy to mention that the experimental and theoretical data of Wetz et al., (1997), for 200 keV Xe⁺/Ni combination, were presented there by the normalized $Sy(θ^0)/Sy(0^0)$ values. These values have been multiplied by the estimated $S_y(0^0) = 7.15$ (Biersack, J.P., & W. Eckstein, 1984), to obtain the absolute $S_y(θ^0)$ values at various incident angles. Also, for 1 kev Xe⁺/Ni combination, the theoretical data were taken from Fig.(15a), Ref. (Biersack, J.P., & W. Eckstein, 1984), where no available experimental data is found. So, the present theoretical data will be compared with theoretical data of 1 kev Xe⁺/Ni combination (Biersack, J.P., & W. Eckstein, 1984). All Figs. (6 and 7) show that the yield grows slowly with a flat slope around the angle region 0°-50°, then it increases considerably by steeper slope and goes through a maximum yield between 65°and 86°, depending on $E_{i,eff}$ ions/targets combinations, followed by a rapid decrease at larger incident angles. The height of the
maximum yield \( S_{y,\text{max}} \) at the angle \( \theta_{\text{max}} \) position, is specific for the ion effective energy \( E_{i,\text{eff}} \) and the ion/target combination (Chini, T., et al., 1992; Bay, H.L., & I. Bohdansky, 1979) in use. Figs. (6 and 7) show that the curves have maximum yields for the light and heavy ions, where the maximum yield value increases with increasing incident effective energy \( E_{i,\text{eff}} \) (while the normal incident ion energy is constant) accompanied by shifting of the corresponding \( \theta'_{\text{max}} \) towards larger \( \theta' \) value, in agreement with Y. Yamamura (1984), Biersack and Eckstein (1984) and Behrisch and Eckstein (2007) (p.107), findings. Also, inspection of Figs (6 and 7) show that, at higher energies, \( H^+ \) ion impact on Ni is more effective at glancing incident angles, where its yield peak comes closer to 90° than \( Xe^+ \) ion impact on Ni. Moreover, it is essential also to mention that the pre-existing reasonable available experimental \( S_y(\theta') \) values helped in determining the ratio \( J/h \) (by solving Eq.27 in Appendix 1 for \( \text{sec}(\theta'_{\text{max}})J \)) using \( \theta'_{\text{max}} \) value corresponding to the experimental \( S_{y,\text{max}} \) value.

![Fig. 6: Sputtering yield against incident angles for the (a: 450 ev, b: 1 KeV , c: 4Kev and d: 50 Kev) H+/Ni combination.† The data are from Ref (Behrisch, R., & Eckstein W.)](image)

The obtained power parameters \( J \) and \( h \) values have been applied to determine the present obtained theoretical maximum yield \( S_{y,\text{max}} \) value by varying \( h \) and \( J \) of \( \sec(\theta') \), presented in the nominator and denominator of Eq.6, to match the available experimental data. For 1 and 200 keV \( Xe^+/Ni \) combinations, the couples \((h,J)\) were evaluated to be \((0.748208, 5.190075), \text{ and } (0.806969, 1.472593), \text{ i.e } h/J=6.937 \text{, and } 1.825 \text{ or } \sim 2, \text{ respectively.} \) Also, for the 0.5, 1, 4, and 50 kev \( H^+/Ni \), the couples \((h,J)\) were evaluated to be \((0.523485, 4.388088), (0.389, 3.504), (0.541624, 2.8975), \text{ and } (0.912919, 1.865838) \text{ respectively, with } J/h= 8.3977, 9.01, 5.37, \text{ and } 2.0438 \text{ respectively.} \) It is noticed that the parameter ratio \( J/h \) decreases with increasing incident effective energy \( E_{i,\text{eff}} \) (at a constant normal incident ion energy \( E_i \)) irrespective of ion/target combination.
On the other hand, at incident angles $\theta > \theta_o$ the second angle region extended from $90^\circ$ to $525 \text{ kev}$, Fig. 1a and 1b, approaching the zero value for each $V (IES)$ effect, with increasing incident angle $\theta$, and the second region extending from $\theta_{\text{max}}$ up to angles $\theta^\prime$ very close to $90^\circ$, $S_y(\theta^\prime)$ values face the decreasing effect, with increasing incident angle $\theta^\prime$, which results from the continuous decreasing of $\lambda_{\text{esp,dep}}(\theta^\prime)$ values (see Fig. 1a and 1b), approaching the zero-value at $\theta^\prime = 90^\circ$. A reduction in creating displaced atoms and increasing of ions loss through the free surface, takes place simultaneously, due to the continuous reduction in $\lambda_{\text{esp,dep}}(\theta^\prime)$. As a consequence this would lead to decreasing effect on sputtering yield values over all the incident angles. At the same time, the range of the cascade collisions would be longer than the range of the first displaced target atom, expecting the enhancement effect on the sputtering yield values. This enhancement effect predominates at the incident angles $\leq \theta_{\text{max}}$, where the cascade collision numbers are 3 and 2, compensating the decreasing effect of $\lambda_{\text{esp,dep}}(\theta^\prime)$ at these incident angles, (see Table 1), leaving the sputtering yield values to continue their increase. On the other hand, at incident angles $> \theta_{\text{max}}$, the enhancement effect on the sputtering yield values will be negligible, as the cascade collision numbers are 1 and 0, and the reduction effect on the sputtering yields will continue at this angle region.

Also, inspection of Fig. 2 (up and down), it could be seen that the normalized sputtering yield $S_y(\theta^\prime)/S_y(0)$ increases, in the first angle region extended from zero to $\theta_{\text{max}}$. After $\theta_{\text{max}}$, it is expected that $S_y(\theta^\prime)$ values, in the second angle region extended from $\theta_{\text{max}}$ up to angels $\theta^\prime$ very close to $90^\circ$, begin to decrease with the continuous increasing of (SEF), in the angle region above $\theta_{\text{max}}$ up to angels $\theta^\prime$ very close to $90^\circ$, (not extrapolated in Fig. 2-up, see also the previous section). The counteracting effects are noticed through the increasing in the normalized sputtering yield, in the first angle region, with the continuous increasing SEF until $\theta_{\text{max}}$ is reached. This happen to face the decreasing effect arising from the reduction of both the whole statistical spherical volume $V(IES)$ to the effective volume $V_{\text{eff}} = V(IES) - v_{\text{psz}}$, and the whole incident energy $E_i$ to the effective energy $E_i,\text{eff} = (V_{\text{eff}} / V(IES))E_i$, as mentioned before. By the incident angles $\theta^\prime$ increasing, reductions occur by the continuous increasing of the volume $v_{\text{psz}}$, (representing the fraction of energy not deposited in the target (Schwarz, S.A., 1988)), in addition the continuous decreasing of the effective incident energy $E_i,\text{eff}$ happens (while the incident energy is constant). For example for 1 kev and 200 kev Xe$^+$/Ni combinations the $E_i,\text{eff}$ changes (for instance) from 972 to 525 ev, and from 194.4 to 105 kev respectively. These reductions would affect the magnitude of the sputtering yield $S_y(\theta^\prime)$ values, decreasing them with increasing incident angles, in the first angle region. But, the enhancement effect opposes these reduction effects by the cascade collision numbers 3 and 2 predominating at incident angles $\theta_{\text{max}}$ and compensates the decreasing effects on $S_y(\theta^\prime)/S_y(0)$ values, keeping them to increase with increasing incident angles in first angle region, (see Table 1). Also, by analyzing Fig. 2-up, it is expected that the normalized sputtering yield $S_y(\theta^\prime)/S_y(0)$ values would decrease with increasing SEF in the angle region above $\theta_{\text{max}}$ to angels $\theta^\prime$ very close to $90^\circ$, due to the continuous decrease in the effected incident energy $E_i,\text{eff}$ (at a constant normal incident ion energy $E_i$) from 972 to 525 ev, and from 194.4 to 105 Kev for 1 kev and 200 kev Xe$^+$/Ni combinations respectively, for instance, as the incident angle increases from $\theta^\prime = 0$ to angels $\theta^\prime$ very close to $90^\circ$. Also, when the protruded zone volume

**Fig. 7:** Sputtering yield against ion incident angles for (a: 1 kev and b: 200 kev ) Xe$^+$/Ni combination.

* The data adopted from Ref (Biersack, J.P., & Eckstein, W. and Wetz, P. et al.)
$v_{\text{esc}}$ becomes half the whole volume $V_{\text{IES}}$, the fraction of energy not deposited in the target $\text{SEF}$ would be at its maximum value, while the normalized sputtering yield $S_y(\theta^o)/S_y(0)$ values would be at its lowest value. Thus, at such increased sputtering efficiency, with increasing incident angles from $\theta^o_{\text{max}}$ to angles very close to $90^o$, the ion reflection becomes more important and is dominant at grazing incidence, while sputtering yield decreases steeply to zero value at angles very close to $90^o$. At glancing angles, the ions escape from the target surface before depositing its energy. Thus, extremely low sputtering yield are encountered at such reduced $E_{i,\text{eff}}$ and $V_{\text{eff}}$. Therefore, the sputtering yield would be easy and detectable at incident angle $\theta^o$ away from $\theta^o = 90^o$, providing that the deposited energy should be enough to create displaced atoms in the escape depth region, where they could gain outward impulsive force to activate the sputtering process.

The disagreement between the present calculated sputtering yields, by the pictorial methodology of the conicoidal construction visualization, and those arising from Biersack and Eckstein's Monte Carlo model (Biersack, J.P., & W. Eckstein, 1984) and Wetz et al.'s semi-empirical formula (Wetz, P., et al., 1997) could be due to some alterations in several aspects of the computation. Among them are applying unreasonable stopping powers or approximating their values, and the parameters in other models could be adjusted to force fit their theoretical data to the experimental data. Our confidence on the pictorial methodology evaluation of sputtering yield data could rest upon the overall reasonable agreement between the present theoretical data and the available experimental, if one remembers that no adjustment has been made to the parameters values presented in Eq. 6, while the same available experimental data deviate obviously from the corresponding theoretical data of Biersack and Eckstein (2007, 1984) and Wetz et al., (1997).

This good agreement between the experimental and the conicoidal model data reflects also evidence that Eq. 6 could be of value in analytical application in plasma glow discharge and ion impact processes, since its theoretical yield values could match the corresponding available experimental data at the whole incident angles ranging from $0^o$ to $88^o$, for light and heavy ion masses, at constant incident ion energies.

**Conclusions:**

Eq. (6), of the statistical conicoidal model, describes a single equation for two functions, i.e. $S_y(\theta^o)$ at a decreasing effective incident energy $E_{i,\text{eff}}$ (while the incident energy $E_i$ is constant) ($F1$) and $S_y(E)$ at constant angle $\theta^o (F2)$ at various ion incident angles $0^o$ to $88^o$ and energies $0.5$ to $200$ Kev dependences of the sputtering yield $S_y(\theta^o)$ for light $H^+$ and heavy $Xe^+$ ion. The conicoidal construction offers the pictorial methodology trial, to evaluate dimensionally all existing parameters and their projected components presented in Eq 6. So, no approximations have been done to these dimensionally evaluated parameters to force fit the available experimental data for comparison. The pictorial methodology solved also the counteracting effects, acting upon the behavior of the sputtering yield $S_y(\theta^o)$ values, with respect to the continuous increasing of the sputtering efficiency fraction, with increasing ion incident angles $\theta^o$. The solution was interpreted through the decreasing effect of the escape depth $\lambda_{\text{emp}}(\theta^o)$, the decreasing effect of the effective energy $E_{i,\text{eff}}$ (at a constant normal incident ion energy $E_i$) with increasing incident angles, and the increasing effect of the encounter collision numbers $\omega$ parameters, which are angle-region dependences. We found that the power ratio $\omega/h$ of sec($\theta^o$) in Eq (6), plays a role in controlling the height and position of the maximum sputtering yield $S_{y,\text{max}}$ of the present theoretical data to be shifted more towards the corresponding available experimental data (Biersack and Eckstein 2007; 1984; Wetz, P., et al., 1997), leading to realistic sputtering yield data, at the angle region from $0^o$ to $88^o$ for low and high incident energies $0.5$ to $200$ Kev. These results give evidence for the validity application of the present proposed pictorial methodology.

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**Appendix 1:**

**Method of developing the general equation:**

Starting with the basic Eq.(2 or 12) in Refs. (Schwarz, S.A. & C.R. Helms, 1979; Grais, Kh. I., et al., 2010) respectively, at normal incidence $\theta^o = 0$, the $S_y(0^o)$ will be:

$$S_y(0^o) = B(A(0^o)n_o)^{2/3}A_{\text{sput}}(0^o) \tag{10}$$

Where $A(0^o)$ is the fraction of atoms in the statistical region which are displaced. $B$ is the fraction of displaced atoms at the statistical affected surface area $A_{\text{sput}}(0^o)$ that actually escape, and $(A(0^o)n_o)^{2/3}$ is the
number of displaced atoms per unit surface area (Schwarz, S.A. & C.R. Helms, 1979; Grais, Kh. I., et al., 2010).

Ref. (Grais, Kh. I., et al., 2010) and Eq. (12) showed for the statistical affected surface area $A_{sasa}(0^o)$ the following equation:

$$A_{sasa}(0^o) = \pi (1.2r_s^2 (0^o)/1.2r_s (0^o) + \lambda_{exp.dep}(0^o))^2$$  \hspace{1cm} (11)

Where $r_s(0^o)$ and $\lambda_{exp.dep}(0^o)$ are the radius of the $V_{IES}$ and the escape depth respectively (Grais, Kh. I., et al., 2010).

Substituting Eq. (11) in Eq. (10), then

$$S_r (0^o) = \pi B(A(0^o)n_o)^{2/3} (1.2r_s^2 (0^o)/1.2r_s (0^o) + \lambda_{exp.dep}(0^o))^2$$  \hspace{1cm} (12)

Substituting eq. (12) in eq. (2) we obtain:

$$S_r (\theta^o) = G \pi B(A(0^o)n_o)^{2/3} \lambda_{exp.dep}(0^o) (sec(\theta^o))^{2} (sec(\theta^o))^{2} \lambda_{exp.dep}(0^o)$$  \hspace{1cm} (13)

Dependence of $A(0^o)$, $k_{nd}(0^o)$, $r_s(0^o)$ and $\lambda_{exp.dep}(0^o)$ parameters on the incident angles:

The operational parameters $A(0^o)$ and the nuclear-electronic stopping power $k_{nd}(0^o)$ should be changed to $A(\theta^o)$ and $k_{nd}(\theta^o)$ at various incident angle $\theta^o$. $A_{sasa}(0^o)$ in Eqs. (1.3,6,10-13), is the intersected affected statistical area on the target surface, at normal incidence. Thus, the factor $[A(0^o)n_o]^{2/3} A_{sasa}(0^o)$ gives the whole number of displaced atoms in a disc layer of area $A_{sasa}(0^o)$ (Schwarz, S.A. & C.R. Helms, 1979; Grais, Kh. I., et al., 2010), which would contribute in sputtering process. Also, the pictorial determined area $A_{sasa}(0^o)$ value, from Fig.1a and 1b shows that it increases with increasing incident angle $\theta^o$. So, the number of displaced atoms would increase with increasing incident angles $\theta^o$, suggesting that the parameter $A(0^o)$ should be multiplied with $sec(\theta^o)$, as done with the area $A_{sasa}(0^o)$ in Eq. (2), where $S_r(0^o)$ is proportional to $A_{sasa}(0^o)$. Also, the parameter $k_{nd}(0^o)$ (in A / (eV)), would represent the ion range $R(0^o)$ when it is multiplied by the whole incident energy $E_i$. So, the parameter $k_{nd}(0^o)$ is inherent inclusion in the whole range $R(0^o)$, and $k_{nd}(0^o)$ should be treated in the same way as the range decreases with increasing incident angles and multiplied by $(cos(\theta^o))^m$. The insertion of $\omega$ is essential to represent the number of collision encounters by the first displaced atom before its sputtering.

Figs.1a and 1b showed that $r_s(0^o)(2 \rightarrow 4C) =60.47 A^o$, $R(0^o)(2 \rightarrow 6) =120.94 A^o$ and $\lambda_{exp.dep}(0^o)(3 \rightarrow 4C) = 48.38 A^o$ and their projected values, at the incident angle $\theta^o$= 45°, (for instance), are: $r_s(0^o) = 42.76 A^o$, which can be calculated by subtracting the projected magenta line $3 \rightarrow 6 = 76.97 A^o$ (of the magenta line $r_s(0^o)+\lambda_{exp.dep}(0^o)$) - 34.21 A° from the projected magenta line $\lambda_{exp.dep}(0^o)$ $3 \rightarrow 4C$, $R(0^o)$ is approximately the projection of the magenta line $r_s(0^o)+\lambda_{exp.dep}(0^o)$ i.e $3 \rightarrow 6 = 76.97 A^o$, and $\lambda_{exp.dep}(0^o)$ is the projection of the magenta line $\lambda_{exp.dep}(0^o)$, i.e $3 \rightarrow 4C$ = 34.21 A°. It is noticed that the projected parameters decrease with increasing incident angles $\theta^o$, suggesting that their values at normal incidence should be multiplied by $cos(\theta^o)$ to re-direct them towards the target surface. Other projected values, for $\lambda_{exp.dep}(\theta^o)$, $R(\theta^o)$ and $r_s(\theta^o)$, are pictorially determined from Figs.1a,b at various ion incident angles $\theta^o$. Their normalized values, $\lambda_{exp.dep}(\theta^o)/ \lambda_{exp.dep}(0^o)$, $R(\theta^o)/ R(0^o)$ and $r_s(\theta^o)/r_s(0^o)$ versus $sec(\theta^o)$ are shown in Fig. 3, where their straight lines, characterized by the symbols (open circles, crosses, and closed rectangles respectively) overlap each other with equal slope exponent factor $m = 1$, as follows:

$$\lambda_{exp.dep}(\theta^o) = \lambda_{exp.dep}(0^o) sec^m(\theta^o)$$

$$R(\theta^o) = R(0^o) sec^m(\theta^o)$$

$$r_s(\theta^o) = r_s(0^o) sec^m(\theta^o)$$  \hspace{1cm} (14)

Substituting Eq. (14) for $\lambda_{exp.dep}(\theta^o)$, $r_s(\theta^o)$ and $A(\theta^o)$ in Eq. (13) we obtain:

$$S_r(\theta^o) = G \pi B(A(\theta^o)(sec(\theta^o))^{m} n_o)^{2/3} \lambda_{exp.dep}(\theta^o) (sec(\theta^o))^m + \lambda_{exp.dep}(\theta^o)(sec(\theta^o))^m (sec(\theta^o))^2$$  \hspace{1cm} (15)

Conversion of expressions containing some other parameters at normal angles and their correspondences at various incident angles:
Refs. (Schwarz, S.A. & C.R. Helms, 1979; Grais, Kh. I., et al., 2010) gave for \( r_s(0^o) \) and \( \beta(0^o) \) the expressions:

\[
rs(0o) = \left[ \frac{3E_i}{8\pi \eta_0 A(0o) \Delta H_n} \right]^{1/3}
\]

and

\[
r_s(\theta^o)(\sec(\theta^o))^m = E_i \left[ \left( \frac{3E_i}{8\pi \eta_0 A(\theta^o) \sec(\theta^o)^{\eta_0} \Delta H_n} \right)^{1/3} \right]^{1/3} = E_i \left[ \left[ \frac{E_i}{\eta_0 A(\theta^o) \sec(\theta^o)^{\eta_0} \Delta H_n} \right]^{1/3} \right]^{1/3} \]

\[
\beta = k_{ne}(\theta^o) \left( \frac{2\pi \eta_0 A(0o) \Delta H_n}{3} \right)^{1/3}
\]

respectively, which would be affected by incident angles since they contain parameters \( A(\theta^o) \) and \( k_{ne}(\theta^o) \) as following:

\[
\lambda_{esp. dep}(0^o) = 0.8 \left[ \frac{3E_i}{8\pi \eta_0 A(0o) \Delta H_n} \right]^{1/3} \left( \frac{3E_i}{8\pi \eta_0 A(\theta^o) \sec(\theta^o)^{\eta_0} \Delta H_n} \right)^{1/3} + 0.8k_{ne}(\theta^o)\gamma
\]

Where \( \gamma = \left( \frac{8\pi \eta_0}{3\pi \Delta H_n A(\theta^o)} \right) \)

Also, it is assumed that the nuclear energy loss \( k_{n}(dE_i/dR)_n \) is independent of the ion energy in the range from 30ev up to 200kev under investigation. The electronic energy loss \( k_{e}(dE_i/dR)_e \) of the incident ion, despite its small value, is assumed to be connected to the total energy loss \( k_{ne} \approx (k_n k_e) \) (Schwarz, S.A. & C.R. Helms, 1979). Thus, the ion total range \( R(\theta^o) \) could be calculated from:

\[
R(\theta^o) = 2rs(\theta^o), \text{ thus from Eq. (18), } rs(\theta^o) = kne(\theta^o)E_i, \text{ and Eq. (24)}
\]

But, according to the pictorial methodology, \( R(\theta^o) = 2r_s(\theta^o) \), thus from Eq. (18), \( r_s(\theta^o) = k_{ne}(\theta^o)E_i \) and

\[
r_s(\theta^o)(\sec(\theta^o))^m = k_{ne}(\theta^o)(\sec(\theta^o))^{\eta_0} E_i \text{, (Grais, et al., 2010) and Eq.(14)}
\]

And, \( \lambda_{esp. dep}(\theta^o) = 0.8 r_s(\theta^o) = 0.8k_{ne}(\theta^o)E_i \). \text{ (Grais, et al., 2010) and Eq.(14)}

Then, \( \lambda_{esp. dep}(\theta^o)(\sec(\theta^o))^m = 0.8k_{ne}(\theta^o)(\sec(\theta^o))^{\eta_0} E_i, \) \text{ (Grais, et al., 2010) and Eq.(14)}

Substituting Eqs. (22) to (16) in Eq. (23) we get:

\[
S_y = \frac{\pi \eta_0 (A(\theta^o)\eta_0)^{2/3}}{(\sec(\theta^o))^{2/3}} \left[ \left( \frac{3E_i}{8\pi \eta_0 A(\theta^o) \sec(\theta^o)^{\eta_0} \Delta H_n} \right)^{1/3} + 0.8k_{ne}(\theta^o)\sec(\theta^o)^{\eta_0} E_i \right] \]

Introducing \( \gamma = \left( \frac{8\pi \eta_0}{3\pi \Delta H_n A(\theta^o)} \right) \) in Eq.(21), Eq. (25) is obtained:

\[
S_y = \pi \eta_0 \left[ \frac{A(\theta^o)\eta_0}{(\sec(\theta^o))^{2/3}} \left( \frac{3E_i}{8\pi \eta_0 A(\theta^o) \sec(\theta^o)^{\eta_0} \Delta H_n} \right)^{1/3} \right]^{2/3}
\]

Introducing also the factor \( \beta \), by multiplying both the nominator and the dominator inside the large brackets of Eq. (25), by \( k_{ne}(\theta^o)^{\eta_0} \), Eq.(1) is obtained:

\[
S_y = \frac{\pi \eta_0 (A(\theta^o)\eta_0)^{2/3} / k_{ne}(\theta^o)^{\eta_0}}{(\sec(\theta^o))^{4/3}} \left( \frac{3E_i}{8\pi \eta_0 A(\theta^o) \sec(\theta^o)^{\eta_0} \Delta H_n} \right)^{1/3} \left( 1.2 + 0.8k_{ne}(\theta^o)\sec(\theta^o)^{\eta_0} E_i \right)
\]

The final form Eq.(24), of the sputtering yield-incident angles dependence, will be obtained after re-arranging Eq.(23) as follows:

\[
S_y = \left( \frac{\pi \eta_0}{k_{ne}(\theta^o)^{\eta_0}} \right)^{4/3} \left( \frac{3E_i}{8\pi \Delta H_n} \right)^{1/3} \left( \frac{A(\theta^o)\eta_0}{(\sec(\theta^o))^{2/3}} \right)^{4/3} \left( 1.2 + 0.8k_{ne}(\theta^o)\sec(\theta^o)^{\eta_0} E_i \right)
\]
Where the factors $\frac{\zeta}{2} = h$ and $m\omega + ao\zeta /3 = J$ (25)

To determine the incident ion energy and the incident ion angle at maximum sputtering yield, i.e. $(E_{\text{i, max}}, \theta_{\text{max}}, S_{\text{max}})$, Eq.(24) should be reduced to an expression for $E_{\text{i, max}}$ and $\theta_{\text{max}}$, by setting $\partial S / \partial E_{\text{i}} = 0$ and $\partial S / \partial \sec \theta = 0$, for the peak yield, respectively. As results, it may be easily shown that:

$$E_{\text{i, max}} = 1.84 \left[ \frac{1}{\beta (\sec (\theta_0))^{3/2}} \right], \text{ at constant } \theta_0$$

and

$$\theta_{\text{max}} = \left[ \frac{1.2 \ h}{0.8(\beta - h) \beta E_{\text{i, eff}}^{2/3}} \right]^{1/2}, \text{ at constant } (E_{\text{i, eff}})$$

To describe the sputtering yield at low and high energies at various incident ion angles and energies, a correction term $\left[ 1 - (E_{\text{th}}/E_{\text{i, eff}})^{1/2} \right]$ with $\Omega = 0.8$ for both $H^+$/Ni and $Xe^+$/Ni combinations is added to Eq.(20) and $E_{\text{th}}$ is the threshold energy. Also, $E_{\text{i}}$ would be replaced by the effective energy $E_{\text{i, eff}}$, so that the following Eq.(28) could be written as follow:

$$S_y \left( \theta^0 \right) = \pi B G / k^2 (\theta^0)^3 \left( 8 \pi \mathcal{H}_0 \right)^{4/3} (1 / A(\theta^0) n_0)^{2/3} *$$

$$\left[ 1.2 \beta E_{\text{i, eff}}^{1/3} (\sec (\theta^0))^{3/2} / 1.2 + 0.8 \beta E_{\text{i, eff}}^{2/3} (\sec (\theta^0))^{1/2} \right]^{1/2} \left[ 1 - (E_{\text{th}}/E_{\text{i, eff}})^{1/2} \right]$$

(28)

which resembles the above mentioned equation (6) in the text.

References


