ORIGINAL ARTICLES


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ABSTRACT

We have proposed a novel Flexible Cross Correlation (FCC) code for Spectral Amplitude Coding - Optical Code Division Multiple Access (SAC-OCDMA) system. The FCC code possesses various advantages; including easy code construction, shorter code length and the important properties of this code is the flexible cross correlation which means that the Phase Induced Intensity Noise (PIIN) is reduced, thus minimizing the correlation effects. Bit Error Rate (BER) performance of FCC code is compared with other SAC-OCDMA codes such as MDW, MFH and EDW respectively through the theoretical analysis. The results revealed that our proposed code not only concealed the PIIN noise but also allows larger cardinality (number of users) as compared to other SAC-OCDMA codes such as MDW, MFH and EDW respectively.

Key words: Flexible Cross Correlation (FCC) Code, Phase Induced Intensity Noise, Cardinality, SAC-OCDMA, Optical Communication

Introduction

The principles of spread-spectrum CDMA system was implemented in optical signal is to spread the energy over the frequency band that is much wider than the minimum bandwidth required to send a bit of information (Begovic, A., N. Behlilovic, 2007). It has been widely explored as a technology for military, government and modern optical access networks (Hong Xi Yin,). In OCDMA system, each user transmits an assigned code whenever a bit of “1” is to be transmitted and does not transmit anything whenever a bit of “0” is to be transmitted. In OCDMA, the most important consideration is the code design; improperly code designed and higher number of simultaneous users can be seriously degraded the system performance due to existing of Multi Access Interference (MAI) (Anuar, M.S., 2009). MAI is the interference from other users transmitting at the same time, which will limit the effective error probability with the presence of noise in the overall system. In OCDMA system, Phase Induced Intensity Noise (PIIN) is deeply related to MAI due to overlapping spectra from different users (Wei, Z. and H. Ghafouri-Shiraz, 2002). Therefore, the key to an effective optical CDMA system is the choice of efficient address code sequences with desirable cross correlation properties. Inappropriate cross correlation among the address sequences will cause PIIN between code sequences increased. PIIN depends on the number of interfering users and cannot be improved by increasing the transmitted power (Anuar, M.S., 2007). One of the effective solutions for alleviating the PIIN is decreasing the number of interferences between the signals of different users. Most codes have been proposed in OCDMA systems such as Modified Frequency Hopping (MFH), Modified Quadratic Congruence (MQC) and Modified Double Weight (MDW) (Wei, Z., 2001; Wei, Z., 2001; Aljunid, S.A., 2005) codes. However, these codes have several limitations such as the code is either too long (e.g. Optical Orthogonal Code and Prime Code), construction is complicated (e.g. MFH code), or poorer cross correlation (e.g. Hadamard) and fixed an even natural number for Modified Double Weight (MDW) code. Finally, the longer code length had limited the flexibility of the codes since it will require wide bandwidth source (e.g. Prime code).

In this paper, a new code called the Flexible Cross Correlation (FCC) code has been developed. It has been assumed that the in phase cross correlation value can be flexible which ensures that each codeword can be easily distinguished from every other address sequence. The code is optimum in the sense that the code length is shorter for a given in phase cross correlation function. The FCC code can be constructed with simple tridiagonal matrix property and any given number of users and weights. Finally, we analyze the proposed code with the mathematical theoretically. It has been shown that, the system performance significantly improved with the proposed code instead of Prime, Hadamard, Modified Frequency Hopping (MFH), Optical Orthogonal and Modified Double Weight (MDW) codes in term of cardinality, power receive, bit rate and fiber transmission.

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2. Development of the Proposed Code:

2.1 Essential of optical CDMA code:

Optical codes are family of K (for K users) binary \([0, 1]\) sequences of length \(N\), code weight \(W\) (the number of “1” in each codeword) and the maximum cross-correlation, \(\lambda_{\text{max}}\). The optimum code set is one having any desired cross-correlation properties to support the maximum number of users with minimum code length. This ensures guaranteed quality of services with least error probabilities for giving number of users K at least for the short haul optical networking (Anuar, M.S., 2006). The auto and cross- correlation functions of these sequences are defined, respectively, by

\[
\begin{align*}
\lambda_x(\tau) &= \sum_{i=1}^{N} x_i x_{i+\tau} = W \quad \text{for } \tau = 0 \\
\lambda_{xy}(\tau) &= \sum_{i=1}^{N} x_i y_{i+\tau} = 0 \quad \text{for } \tau = 0
\end{align*}
\]

It shows that, major bottleneck in the successful implementation of all optical networks is basically when all the users try to transmit their data simultaneously. It can be conquered by designing coding sequences such that they may cause least overlapping between data chips (Hilal Adnan Fadhil, 2011).

2.2 Mathematical Preliminaries:

Let \(A = \{a_n\}\) and \(B = \{b_n\}\) be the sequences of length \(N\) such that:

\[
\begin{align*}
\{a_n\} &= '0' \text{ or } '1', i = 0, \ldots, N - 1 \\
\{b_n\} &= '0' \text{ or } '1', i = 0, \ldots, N - 1
\end{align*}
\]

Since \(a_n\) is a \([0, 1]\) binary sequence, the maximum value of \(\lambda_x(\tau)\) in Equation (1) is for \(\tau = 0\) and is equal to \(W\), the code weight of the sequence can be expressed as:

\[
\lambda_x(0) = W
\]

If \(\lambda_{\text{axm}}\) & \(\lambda_{\text{sym}}\) denote the maximum out of phase auto correlation and cross correlation values respectively, then an optical code of length \(N\) and code weight \(W\) can be written as \((N, W, \lambda_{\text{axm}}, \lambda_{\text{sym}})\). A \((N, W, \lambda_{\text{axm}}, \lambda_{\text{sym}})\) - FCC code is called the constant- weight symmetric when \(\lambda_x = \lambda_y = \lambda_{\text{max}}\). We used the shorthand notation of \((N, W, \lambda_{\text{max}})\) and \(\Phi(N, W, \lambda_{\text{max}})\) for the largest possible cardinality. It may also be noted that for an optical code \(a_n\) with code weight ‘\(W\)’ for auto correlation can be written as follows:

\[
\lambda_{\text{axm}}(0) = \lambda_x(0) = \sum_{N=0}^{N-1} x_i x_j = W
\]

In practice for \(K\) users, it is required to have a \(K\) number of codes in a set for given values of \((N, W, \lambda_{\text{max}})\). The codes described by Equation (3) can also be represented in vector form as:

\[
\begin{align*}
A &= \{a_i\} \text{ for } i = 0, 1 \ldots N - 1 \\
B &= \{b_i\} \text{ for } i = 0, 1 \ldots N - 1
\end{align*}
\]

Where, \(A\) and \(B\) are vectors of length \(N\) with elements as defined by Equation (6).

In terms of the vectors \(A\) and \(B\), Equation (1) and Equation (2) can be written as,
\[
\begin{align*}
\lambda_{A(0)} &= AA^T = W \\
\lambda_{AB(0)} &= AB^T 
\end{align*}
\] (7)

Where \(A^T\) and \(B^T\) denote the transpose of vectors \(A\) and \(B\), respectively.

3. Algorithm for FCC Code Development:

In fiber optic CDMA system to allow receivers to distinguish each of the possible users, to reduce channel interference and to accommodate large number of users, optical codes should have large values of \(W\) and the size \(K\).

**Step 1:**

We consider a set of \(K\), \((N, W, \lambda_{\text{max}})\) FCC code for \(K\) users. This set of codes is then represented by \(K \times N\) code matrix, where the \(K \times N\) code matrix, can be expressed by Equation (8).

\[
A^W_K = \begin{bmatrix}
    a_{11} & b_{12} & c_{13} \\
    0 & a_{24} & c_{25} \\
    0 & 0 & c_{36} \\
    0 & 0 & 0 \\
\end{bmatrix}
\] (8)

where,

\[
\begin{align*}
A_1 &= a_{11}, b_{12}, ..., d_{1N} \\
A_2 &= d_{14}, a_{24}, ..., d_{27} \\
A_3 &= c_{23}, d_{24}, ..., a_{37} \\
A_K &= 0, 0, ..., 0, a_{kN}
\end{align*}
\]

The \(K \times N\) code matrix, is here called the Tridiagonal Code Matrix, whose elements \(a_{ij}\) of \(K \times N\) code matrix, is the binary sequence \([0, 1]\) and can be written as in Equation (9).

\[
A^W_K = a_{ij} = '0' \text{ or '1' for } i=1,2,..K, \quad j=1,2,..N
\] (9)

The rows of \(A_1, A_2\) and \(A_k\) represent the \(K\) codeword and it is assumed that, the code weight of each of the \(K\) codeword is to be \(W\).

**Step 2:**

After the \(K\) codes represented by the \(K\) rows of the \(K \times N\) code matrix, in Equation (8), are to represent a valid set of \(K\) codeword with in phase cross correlations \(\lambda_{\text{max}}\) and code weight \(W\), it must satisfy the following conditions:

1. The elements \(\{a_{ij}\}\) of must have values “0” or “1”

\[
a_{ij} = "0" \text{ or "1" for } i=1,2,..K, j=1,2,..N
\] (10)

2. The code weight of each codeword should be equal to \(W\) where,

\[
\sum_{j=1}^{N} a_{ij} = W, \quad i=1,2,..K
\] (11)

3. The in phase cross correlation \(\lambda_{\text{max}}\), between any of the \(K\) code words (\(K\) rows of the matrix \(A^W_K\)) should not exceed code weight \(W\). That is,

\[
A_iA_j^T = \begin{cases}
    \leq \lambda_{\text{max}} & \text{for } i \neq j \\
    = W & \text{for } i = j
\end{cases}
\] (12)
4. From Equation (12), it is seen that the $W$ is the in-phase auto-correlation function of codes. $A_i A_i^T$ is the out of phase correlation between the $i^{th}$ and the $j^{th}$ codes. It follows that $A_i A_i^T$ should be greater than $A_i A_j^T$. In other words, $W > \lambda_{\text{max}}$ (13)

5. All $K$ rows of $A_K^W$ should be linearly independent because each codeword must be uniquely different from other words. That is to say the rank of the $KxN$ code matrix, $A_K^W$ should be $K$. Moreover, for $A_K^W$ to have rank $K$, then;

$$N \geq K$$ (14)

Step 3:

From the five conditions above in Step 2, one of the matrices binary sequences from Equation (8) in Step 1 that has been generated, whose the first $i^{th}$ row for the first $K$ user is given by,

$$A_i = \begin{pmatrix} r(i-1) \\ 0...0 \\ 11...1 \\ 10...0 \end{pmatrix}$$ (15)

The length $N$ of the codes which is the length of the rows of the matrix is given by,

$$N = WK - \lambda_{\text{max}} (K-1)$$ (16)

It can be seen that the length $N$ is minimum under the assumed conditions.

Step 4:

On the basis of the above discussions, the construction of an optical code having a value of $K$, $N$, $W$ and $\lambda_{\text{max}}$ consists of the following steps:

1. For a given number of users $K$, and code weight $W$, forms a set of in phase cross correlation code with a minimum length as is given by Equation (15).
2. The length $N$ of $KxN$ code matrix, $A_K^W$ has defined by the Equation (16)
3. The $K$ rows of the $KxN$ code matrix, $A_K^W$ that gives the $K$ optical CDMA codes having in phase cross correlation, code weight $W$ and minimum code length $N$.

Example:

This procedure will now be explained with the help of an example:-

It is desired to generate a set of minimum length with in phase cross correlation optical code. Table 1 shows the FCC code for a given number of users, code weight $W$ and cross correlation $\lambda_{\text{max}}$. Here $K=4$, $W=3$, $\lambda_{\text{max}} \leq 1$, the $KxN$ code matrix, for the parameters given are;

$$A_4^3 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$
Table 1: The FCC code with flexible $\lambda_{\text{max}} \leq 1$, $K = 4$, $W = 3$ and $N=9$.

<table>
<thead>
<tr>
<th>Active Users</th>
<th>Codeword</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>1 1 1 0 0 0 0 0 0</td>
</tr>
<tr>
<td>$K_2$</td>
<td>0 0 1 1 1 0 0 0 0</td>
</tr>
<tr>
<td>$K_3$</td>
<td>0 0 0 0 1 1 1 0 0</td>
</tr>
<tr>
<td>$K_4$</td>
<td>0 0 0 0 0 0 1 1 1</td>
</tr>
</tbody>
</table>

Table 2 shows the code length $N$, code weight $W$, and the cross-correlation $\lambda_{\text{max}}$ for 30 number of users for various OCDMA codes used in the SAC-OCDMA system. The code length of the codes is an important parameter in SAC-OCDMA system. It is clearly shown that, the FCC code had a shorter code length which are 31, 42, 35, 56, 144 and 90 as compared with SAC-OCDMA codes such as MFH, RD, MQC, KS and MDW respectively. It is shown that in Table 2, the FCC code revealed better flexible cross correlation property of ($\lambda_{\text{max}} \leq 1$) and the FCC code can be designed with smaller code weight equal to two, ($W = 2$) as compared with other SAC-OCDMA codes such as MFH, RD, MQC, KS and MDW respectively.

Table 2: Comparison of Various Codes Used in Families of SAC-OCDMA System.

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>CODES</th>
<th>NUMBER OF USERS (K)</th>
<th>WEIGHT (W)</th>
<th>CODE LENGTH (N)</th>
<th>CROSS CORRELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MFH</td>
<td>30</td>
<td>7</td>
<td>42</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>RD</td>
<td>30</td>
<td>4</td>
<td>35</td>
<td>Variable $\lambda_{\text{c}}$</td>
</tr>
<tr>
<td>3</td>
<td>MQC</td>
<td>30</td>
<td>8</td>
<td>56</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>KS</td>
<td>30</td>
<td>4</td>
<td>144</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>MDW</td>
<td>30</td>
<td>4</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>FCC</td>
<td>30</td>
<td>2</td>
<td>31</td>
<td>$\lambda_{\text{max}} \leq 1$</td>
</tr>
</tbody>
</table>

4. System Performance Analysis:

The FCC code has been used NAND subtraction technique (Nasim Ahmed, 2012) for receiver detection scheme. The NAND technique can be best illustrated in Figure 1. This technique is competent of eliminating the PIIN, less complexity of the encoder and decoder design, and improved the system performance. From Figure 1, a spectral amplitude signal at the receiver side is divided into two divisions. The first division is the signal from a user $X$ related to cross-correlation between $X$ and $Y$, and the second division a result from the NAND detection scheme between $X$ and $Y$, which have the same cross-correlation amplitude which is related to the signal in the first division.
The effect of the presence of noise in our analyses considered only shot noise \( \langle i_{\text{shot}} \rangle \), incoherent intensity noise \( \langle i_{\text{PIIN}} \rangle \) and thermal noise \( \langle i_{\text{thermal}} \rangle \) to evaluate the system performance. The proposed system is based on the NAND subtraction detection (Nasim Ahmed, 2012) with Fiber Bragg Grating (FBG) followed by a photodetector. Signal to Noise Ratio (SNR) is calculated at the receiver side and only one photodiode accommodated for each user and \( I \) is denoted as the current flow through the photodiode. The SNR is defined as the average of signal to noise power, 
\[
\text{SNR} = \frac{\|I^2\|}{\sigma^2}
\]

where \( \sigma^2 \) is the average power of noise which is given by;

\[
\sigma^2 = 2eBI + I^2B\tau_c + \frac{4K_BT_eB}{R_L}
\]

Equation (17) can be expressed as;

\[
\sigma^2 = \langle i_{\text{shot}}^2 \rangle + \langle i_{\text{PIIN}}^2 \rangle + \langle i_{\text{thermal}}^2 \rangle
\]

where, \( e \) is the electron charge, \( I \) is the average photocurrent, \( I^2 \) is the power spectral density for \( I \), \( B \) is the noise equivalent of electrical bandwidth, \( K_b \) is the Boltzmann constant, \( T_e \) is the absolute receiver noise temperature and \( R_L \) is the receiver load resistor. From Equation (17) it has been assumed that the optical bandwidth is much larger than the maximum electrical bandwidth. The first term, second and third from Equation (18) can be denoted as the shot noise component, an intensity noise and the thermal noise respectively. The coherence source time \( \tau_c \) is given as (Nasim Ahmed, 2012):

\[
\tau_c = \frac{\int_0^\infty G^2(v)dv}{\int_0^\infty [G(v)dv]^2},
\]

where, \( G(v) \) denotes as the single sideband source power of spectral density (PSD). Noticed, the effect of receiver’s dark current has been neglected in this proposed system analysis. The broadband pulse coming thru to the FBG as an incoherent light field is mixed and incident upon a photo-detector while the phase noise of the fields causes an intensity noise in term of the photo-detector output.

Here we consider the FCC codeword as shown in Table 1 with flexible cross correlation optical code families. The autocorrelation of each codeword and the cross correlation between any two distinct codeword and are satisfy as mentioned in Equation (1) and Equation (2). Hence, according to the properties of FCC codeword in Table 1, the \( X_i \) and \( Y_j \) denoted the \( i \)th element of the \( K \)th row and \( j \)th element of \( N \) column for FCC codeword sequences which can be written as:

\[
\sum_{j=1}^N \sum_{i=1}^K X_{ij}Y_{ij} = \begin{cases} W, & \text{For } i = j, \\ 1, & \text{For } i \neq j \end{cases}
\]

For NAND operation;

\[
\sum_{j=1}^N \sum_{i=1}^K X_{ij}Y_{ij} (\overline{X_{ij} \cdot Y_{ij}}) = \begin{cases} 1, & \text{for } i \neq j, \\ 1, & \text{for } i = j \end{cases}
\]

Therefore, from Equation (21), the NAND operation of \((\overline{X_{ij} \cdot Y_{ij}})\) is valid for \(i \neq j\). Equation (20) shows the cross correlation for is equal to one for \(i = j\). Thus, MAI can be fully relegated when the subtraction is valid for \((\overline{X_{ij} \cdot Y_{ij}})\) with when \(i \neq j\). Thus, the properties of subtraction can be expressed as follows:
The proposed system was analyzed with transmitter and receiver (refer Figure 1), we used the same assumption that used in (Nasim Ahmed, 2012) and those are important for mathematical preliminaries simplicity. Since, to analyze the proposed system, we assume the following assumptions:

- Each unpolarized source PSD and its spectrum is flat over the system bandwidth of $[v_0,\Delta v/2]$ with amplitude $P_{sr}/\Delta v$, $v_0$ is the central optical frequency and $\Delta v$ is the optical source bandwidth expressed in Hertz.
- Each user has equal power at receiver.
- Each bit stream from each user is synchronized.
- Each power spectral component has an identical spectral width.

Based on the above assumptions, the proposed systems can easily analyze using the Gaussian approximation. The power spectral density of the received optical signals can be written as (Elwyn, D.J. Smith, 1998):

$$G(v) = \frac{P_{sr}}{\Delta v} \sum_{i=j, i\neq j}^{N} dK \sum_{j=1}^{N} \sum_{i=1}^{K} X_{ij} Y_{ij} [\prod(i)],$$

(23)

where, $P_{sr}$ is the received power from a single source, $\Delta v$ can be assumed as a perfect rectangular unit step function and can be illustrated in Figure 2.

Here $\prod(i)$ is equal to;

$$\prod(i) = \{u[v-v_0-\frac{\Delta v}{2N}(-N+2i-2)] - u[v-v_0-\frac{\Delta v}{2N}(-N+2i)]\}$$

(24)

From Equation (24), the PSD of the system can be written as follows:

$$G(v) = \frac{P_{sr}}{\Delta v} \sum_{i=j, i\neq j}^{N} dK \sum_{j=1}^{N} \sum_{i=1}^{K} X_{ij} Y_{ij} \left[u[v-v_0-\frac{\Delta v}{2N}(-N+2i-2)] - u[v-v_0-\frac{\Delta v}{2N}(-N+2i)]\right]$$

(25)

where, the \{0, 1\} is an element of unit step function given as follows:

$$1, \text{for } v \geq 0$$
$$0, \text{for } v < 0$$

$$\prod(i)$$

(26)

Equation (25) is the total incident powers at the input of PIN 1 and PIN 2 for NAND detection as shown in Figure 1 and can be simplified as follows:
\[ \int_0^\infty G_1(v) \, dv = \frac{P_{sr}}{AV} \sum_{i,j,K} dk \sum_{j=1}^K X_{ij} Y_{ij} \left\{ u[\frac{\Delta v}{N}] \right\} dv \]
\[ = \frac{P_{sr}}{N} + \frac{P_{sr}}{N} \sum_{i,j,K} dk \]
(27)

\[ \int_0^\infty G_2(v) \, dv = \frac{P_{sr}}{AV} \sum_{i,j,K} dk \sum_{j=1}^K X_{ij} Y_{ij} (X_{ij} \cdot Y_{ij}) \left\{ u[\frac{\Delta v}{N}] \right\} dv \]
\[ = \frac{P_{sr}}{N} + \frac{P_{sr}}{N} \sum_{i,j,K} dk \]
(28)

Only one PSD spectrum will be calculated and the photodiode current \( I \) can be written as follows:
\[ I = \Re \int_0^\infty G(v) \, dv \quad (29) \]
\( \Re \) represents as the responsivity of the photo-detectors and is given by \( \Re = (\eta e)/(hv_c) \), where \( \eta \) is the quantum efficiency, \( e \) is the electron charge, \( h \) is the Planck’s constant, and \( v_c \) is the central frequency of the original broadband optical pulse (Nasim Ahmed, 2012). In the above equations, \( dK \) is the data bit of the Kth user that carries the value of either “1” or “0”. Consequently, the photocurrent \( I \) can be expressed as:
\[ I = \Re \left[ \frac{P_{sr}}{N} \right] \quad (30) \]

The power of shot noise can be written as:
\[ \langle I_{shot}^2 \rangle = 2eB(I_1 + I_2) = 2eB\Re \left[ \int_0^\infty G_1(v) \, dv + \int_0^\infty G_2(v) \, dv \right] \]
\[ = 2eB\Re \left[ \frac{P_{sr}}{N} + \frac{P_{sr}}{N} \sum_{i,j,K} dk + \frac{P_{sr}}{N} + \frac{P_{sr}}{N} \sum_{i,j,K} dk \right] \]

Thus, the average total noise power can be written as follows:
\[ I_{shot} = 2eB\Re \left[ \frac{P_{sr}}{N} \right] [W + 3] \quad (31) \]

We assume that, the intensity noise will dominate the broadband sources. Hence, with power spectral density from each user is the same; therefore we calculate the receiver intensity noise directly from the total power spectral density of each photodiode. From Equation (18), the intensity noise at the receiver output is given by (Elwyn, D.J. Smith, 1998):
\[ \langle I_{PIN}^2 \rangle = B(I_1^2 \tau_{c1} + I_2^2 \tau_{c2}) = I^2 \tau_c * B \quad (32) \]

Where \( I_1 \) and \( I_2 \) are the average photodiode currents, and \( \tau_c \) are the coherence times of the light incident on each photodiode as shown in Equation (19). From Equation (29), the power spectral density of \( I^2 \) can be described as follows:
\[ \langle I_{PIN}^2 \rangle = B\Re^2 \left[ \int_0^\infty G_1^2(v) \, dv + \int_0^\infty G_2^2(v) \, dv \right] \]
By using Equation (23), approximating the summation of the variance of the receiver photocurrent can be expressed as:

\[
\langle I_{PIN}^2 \rangle \approx \frac{BR^2}{N\Delta v} \sum_{j=1}^{N} \left\{ Y_j \cdot \left[ \sum_{i=j}^{K} X_{j,i} \right] \times \left[ \sum_{m=1}^{K} d_mC_m(i) \right] \right\}
\]

\[
+ \frac{BR^2}{N\Delta v} \sum_{j=1}^{N} \left\{ X_{j}Y_{j} \cdot \left[ \sum_{i=j}^{K} X_{j,i} \right] \times \left[ \sum_{m=1}^{K} d_mC_m(i) \right] \right\}
\]

\[
\langle I_{PIN}^2 \rangle \approx \frac{BR^2}{N\Delta v} \left[ \frac{KW}{N} \right] \times \sum_{j=1}^{N} \sum_{i=1}^{K} X_{j}Y_{j}
\]

\[
+ \left[ \frac{KW}{N} \right] \sum_{j=1}^{N} \sum_{i=1}^{K} X_{j}Y_{j} \left( X_{j} \cdot Y_{j} \right)
\]

\[
\langle I_{PIN}^2 \rangle \approx \frac{BR^2}{N\Delta v} \left[ \frac{KW}{N} \right] \times \sum_{j=1}^{N} \sum_{i=1}^{K} X_{j}Y_{j}
\]

\[
+ \frac{BR^2}{N\Delta v} \left[ \frac{KW}{N} \right] \sum_{j=1}^{N} \sum_{i=1}^{K} X_{j}Y_{j} \left( X_{j} \cdot Y_{j} \right)
\]

\[
\langle I_{PIN}^2 \rangle \approx \frac{BR^2}{N\Delta v} \left[ \frac{KW}{N} \right] \left[ W \right] \sum_{j=1}^{N} \sum_{i=1}^{K} X_{j}Y_{j} + \sum_{j=1}^{N} \sum_{i=1}^{K} X_{j}Y_{j} \left( X_{j} \cdot Y_{j} \right)
\]

\[
I_{PIN} = \frac{BR^2KW}{N^2\Delta v} \left[ W + 3 \right]
\]

Since, from Equation (10) until Equation (14), shown the properties of FCC code is unique and independent of each other, Equation (33) is also independent of the active users' data, consequently proposed coding systems does not depend on the timing of transitions in the user data and it applied to the asynchronous systems. Thermal noise is given as (Bartolo, R.E., 2012);

\[
I_{TN} = \frac{4K_BT_0B}{R_L}
\]
\[
SNR = \left[ \frac{9RP_{sr}W}{N} \right]^2 \left[ \frac{2eBRP_{sr}}{N} \right][W + 3] + B9R^2 \left[ \frac{P_{sr}^2KW}{N^2\Delta V} \right][W + 3] + \frac{4K_wT_wB}{R_L} \]  \tag{35}
\]

Since, there is no pulses are sent for the data bit ‘0’ and assuming that the noise distribution is Gaussian, thus; the corresponding Bit Error Rate (BER) can be obtained as follows (Nasim Ahmed, 2012):

\[
P_e = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{SNR}{8}} \right) \tag{36}
\]

Finally, Equation (35) and Equation (36) will be used for the theoretical calculation for an evaluation of the proposed coding system using FCC code.

**Result and Discussion**

5.1 Relationship Between Received Power and PIIN Noise:

PIIN is strongly related to MAI due to the overlapping of spectra from different users. Figure 3 shows the relations between PIIN noise and received power ($P_{sr}$). The value of $P_{sr}$ is varied from -30dBm to 20dBm. When the received power increases the PIIN noise for MFH, EDW, MDW and FCC codes increases linearly. From figure 3 shows, the PIIN noise in FCC code is not as much of others code, thus the PIIN noise can be effectively suppressed using FCC code due to flexible cross correlation as discussed in section three. Moreover, FCC code can be designed with smaller code weight equal to two than other OCDMA codes such as MFH, EDW and MDW respectively.

![Fig. 3: PIIN Noise versus Received Power ($P_{sr}$) for the same number of users (K=9).](image)

5.2 Relationship Between Number of Users and PIIN Noise:

Figure 4 shows the relation between number of user and PIIN noise (where the effects of shot and thermal noise are neglected). The figure clearly shows that, the PIIN noise in FCC code is remains constant at $10^{-13}$ for K=9 even the number of users increasing simultaneously. In contrast, for MFH codes the PIIN noise increase when the number of simultaneous user increases. This is because of an algorithm property of the code design and code construction (as mentioned is section three).
5.3 Effect of Number of Users on System Performance by Considering PIIN Only:

Figure 5 shows the plot between the number of simultaneous users and the system performance by considering only PIIN noise (where the effects of shot and thermal noise are neglected). It clearly shows that FCC code has a better BER than MDW and MFH codes. It shows that the BER increases as the number of the user increases. If PIIN noise were considered, then SNR is given as follows:-

\[ SNR = \frac{W \Delta V}{BK[W + 3]} \]  

In Equation (38), SNR depends on code weight \( W \), number of user \( K \), \( \Delta V \) and \( B \). However, the values of \( B \), and \( \Delta V \) are fixed (ie. \( B = 311 \text{MHz} \) and \( \Delta V = 3.75 \text{THz} \)) but \( K \) varied from 10 to 130 users. At BER 10\(^{-9}\), system performance using the FCC code can support much higher numbers of users than EDW and MFH codes. This is truly proved with properties and arrangement of FCC code, thus PIIN noise can be suppressed.

5.4 Effects of Received Power on Shot Noise:

Figure 6 shows the relation between shot noise and received power, \( P_{sr} \) at the receiver. From Equation (35), shot noise is given by:
\[ \text{Shot noise} = \left[ \frac{2eBPR_s}{N} \right] [W + 3] \] (38)

The shot noise increases linearly with \( P_s \). The result shows that shot noise can be reduced by increasing the code weight \( W \) for MFH codes. However, for the FCC code increasing the code weight, it will not significantly affect the shot noise. Compared between MDW and MFH codes, it is clear that FCC code has the lowest shot noise effect.

![Fig. 6: Shot Noise versus \( P_s \) for Various OCDMA Codes.](image)

6. Conclusion:

The FCC OCDMA coding system has been derived by considering the effect of the shot, intensity and thermal noise average power respectively. Based on the equations, the results of the system performance are presented. We found that, the PIIN noise for FCC code still remains at \( 10^{-13} \) as compared with other SAC-OCDMA codes such as MDW, MQC and MFH; even the number of users is increasing simultaneously. This will give an opportunity in SAC-OCDMA system for better quality of service in optical access networks for future generation’s usage. It is desirable to have code with flexible cross correlation to enhance the performance of SAC-OCDMA systems, minimizing correlation effects and allows large number of cardinality.

References


