An Optimized Algorithm assisted PTS scheme for PAPR reduction in OFDM Systems

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ABSTRACT

Partial Transmit Sequence (PTS) is an attractive and pragmatic technique for reduction of Peak to Average Power Ratio (PAPR) in Orthogonal Frequency Division Multiplexing (OFDM) system; however it demands an exhaustive search over all possible combinations of allowable phase factors, resulting in larger computational complexity. In this paper the search for optimal phase factor is expressed as a combinatorial optimization problem and is effectively solved using the proposed algorithm assisted PTS method. The proposed algorithm performs a parallel direct search and generates candidates comparable to that of Optimal – PTS (O-PTS) but with reduced search complexity. The simulation performed on a OFDM signal with 1024 sub carriers justifies that the proposed algorithm with reduced computational complexity when compared to O-PTS and achieves significant PAPR reduction performance.

Key words:

Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a widely used multicarrier modulation scheme for high data rate communication systems due to its ability to combat multipath fading and inter symbol interference (Cimini Jr., 1985). Inspite of its wide spread application in various wireless standards including Digital Audio Broadcasting (DAB), Digital Video Broadcasting – Terrestrial (DVB – T), Wireless Microwave Access (WiMAX), Wireless Local Area Networks (WLANs), Wireless Metropolitan Area Networks (WMANs) and 4G Wireless Systems, the multicarrier nature of an OFDM signal can result in large PAPR when added in phase (Jiang and Wu, 2008). Large PAPR demands highly expensive power amplifier to overcome the effects of in-band and out-of-band radiation.

Several solutions have been proposed to reduce PAPR of an OFDM signal including Clipping techniques, Selective Mapping (SLM) method, PTS, companding, Tone Reservation (TR) and Tone Injection (TI). Among these approaches, PTS is a distortion less phase optimization scheme which subdivides the input data into disjoint sub blocks and applies a phase rotation vector to each sub block to achieve significant PAPR reduction with a small amount of redundancy in OFDM systems. In PTS scheme, the exhaustive search complexity for optimal phase factors increases with the number of sub blocks.

Certain optimization solutions (Cimini and Sollenberger, 2000) were proposed but they demand more side information to be transmitted to the receiver. A low complexity PTS (LC – PTS) scheme by Xiao et al., 2007 achieves suboptimal PAPR reduction performance compared to conventional PTS method. Very recently algorithm assisted PTS scheme with reduced complexity is employed to find the optimal phase rotation factors such that PAPR reduction is achieved. The search for optimal phase factors in PTS approach rewrites the PAPR problem as (i) a stochastic approximation problem and is solved by using cross – entropy method to reduce PAPR (Chen, 2009). (ii) a combinatorial optimization problem which is effectively solved using Quantum Inspired Evolutionary Algorithm (Chen, 2010). (iii) a global optimization problem and is solved using a population based search method called Electromagnetism - like (EM) Algorithm (Chen, 2011). (iv) iterative heuristic search method which eliminates the iterations to visit the solution obtained recently called Parallel Tabu search Algorithm is used in conjunction with PTS (TS-PTS) to optimize the PAPR statistics (Taspinar et al., 2011). In 2010, Wang et al., proposed a sub optimal method with three control parameters by combining a numeric function optimization algorithm called Artificial Bee Colony Algorithm with PTS (ABC-PTS) to reduce the search complexity of the allowable phase factors, such that PAPR reduction is achieved.

The outline of the paper is given as follows. Section 1II presents the OFDM system model and problem formulation. Section 2 discusses the problem statement of PTS scheme. Section 3 introduces the proposed algorithm based PTS method. Section 4 discusses the Numerical and Simulation results. Performance analysis
of the proposed method with existing methods is discussed in Section 5 and conclusions are drawn in Section 6.

2. Problem Statement:

Most of the solutions provided are based on PTS scheme demands either large amount of side information to be transmitted or suboptimal PAPR reduction performance. Hence it is mandatory to employ PTS technique with reduced computational complexity and lesser amount of side information to be transmitted with reduced search complexity to find optimal phase rotation vectors so that PAPR reduction is achieved.

3. OFDM System model and PAPR problem formulation:

Inverse Fast Fourier Transform (IFFT) is used for modulating the multiple sub band signals in OFDM systems. The envelope of an OFDM signal $x = [x_0, x_1, ..., x_{N-1}]$ with $N$ sub carriers to be transmitted is given by Muller and Huber as

$$x_n = IFFT\{X\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi nk}{N}}, \quad (1)$$

Where $X = [X_0, X_1, ..., X_{N-1}]$ is the sub band signal in frequency domain. The transmitted signal will have large peak value when added in phase and demands the OFDM system to use expensive HP amplifier in order to perform linear amplification of the OFDM signal. The PAPR of the transmitted OFDM signal is given as

$$PAPR = 10\log_{10} \frac{\max\{\{|x_n|^2\}\}}{E[\{|x_n|^2\}]} \quad (dB) \quad (2)$$

In PTS method, the OFDM signal with $N$ subcarriers $X_K$ is partitioned into $V$ disjoint sub blocks as

$$X_K = \sum_{m=1}^{V} X_{(m)}_K \quad \text{where all the other sub blocks in the sub carriers are assigned to zero.}$$

An optimization parameter called rotation factor or phase factor $b_\mu^{(i)} = e^{j\phi_\mu^{(i)}}, \phi_\mu^{(i)} \in \{0,2\pi\}$ is introduced to each sub block $i$ and the peak power optimized PTS in frequency domain is written as

$$\tilde{x}_\mu^{(i)} = \sum_{i=1}^{V} b_\mu^{(i)} X_{(i)}^{(m)} \quad (3)$$

The phase rotated time domain signal is obtained by performing IFFT operation on Eqn. (3)

$$\tilde{x}_\mu^{(i)} = IFFT\left\{\sum_{i=1}^{V} b_\mu^{(i)} X_{(i)}^{(m)}\right\} \quad (4)$$

The goal of PTS method is to find the optimal phase factor that produces a candidate signal with minimum PAPR (Chen, 2010). Since the search for a global optimal phase factor is a complex task the phase factor selection is limited to a set of finite number of elements, $V$. Since the transmitted OFDM signal undergoes a generation process, the phase factors are transmitted as side information to the receiver in order to recover the original signal.

4. Proposed Algorithm Assisted Pts Scheme:

A computationally efficient algorithm which performs a parallel direct search to find the optimal phase factors is proposed and it is employed with the PTS method to achieve a significant PAPR reduction performance. Block diagram of the proposed algorithm assisted PTS scheme is given in Fig. 1.

(1) Define the upper and lower limit for each OFDM symbol, $0 \leq x_{(i)}^{(m)} \leq x_{\mu m}$.
(2) Randomly select the initial parameters uniformly in the interval $[0, x_{\mu m}]$.
(3) For a given OFDM symbol $x_{(i)}^{(m)}$, select three random phase rotation factors $\mu_p$, $\mu_q$ and $\mu_r$, such that the indices $p$, $q$, and $r$ are distinct.
(4) Add the weighted difference of two phase rotation factors to the third factor to produce a trial phase factor \( v_{\mu,c+1}^{(i)} = x_{\mu p} + F(x_{\mu q} - x_{\mu p}) \) where \( F \) is a constant from \([0, 2]\).

(5) The candidate phase factor \( b_{\mu,c+1}^{(i)} \) is developed from the elements of OFDM symbol \( x_{\mu,c}^{(i)} \) and the elements of trial phase factor \( v_{\mu,c+1}^{(i)} \).

(6) Elements of trial phase factor \( v_{\mu,c+1}^{(i)} \) is created based on the candidate phase factor \( b_{\mu,j,c+1}^{(i)} \) with probability \( CR \).

\[
\begin{align*}
    b_{\mu,j,c+1}^{(i)} = \begin{cases} 
        v_{\mu,j,c+1}^{(i)}, & \text{if } \text{rand}_{ji} \leq CR \text{ or } j = I_{\text{rand}} \\
        x_{\mu,c}^{(i)}, & \text{if } \text{rand}_{ji} \leq CR \text{ and } j \neq I_{\text{rand}} 
    \end{cases}
\end{align*}
\]

Where

\[
    i = 1, 2, \ldots N,
\]

\[
    j = 1, 2, \ldots D,
\]

\[
    \text{rand}_{ji} \sim U[0, 1],
\]

\[
    I_{\text{rand}} = [1, 2, \ldots D],
\]

\[
    x_{\mu,c}^{(i)} \neq x_{\mu,c}^{(i)}.
\]

(7) \( x_{\mu,c+1}^{(i)} \) is compared with the trial phase factor \( v_{\mu,c+1}^{(i)} \) and the one with lowest PAPR value is selected for transmission

\[
    x_{\mu,c+1}^{(i)} = \begin{cases} 
        b_{\mu,c+1}^{(i)}, & \text{if } f(b_{\mu,c+1}^{(i)}) \leq f(x_{\mu,c}^{(i)}), \\
        x_{\mu,c}^{(i)}, & \text{otherwise}
    \end{cases}
\]

(8) Repeat from step (3) to step (7) until all the OFDM symbols equivalent candidates with reduced PAPR are selected for transmission.

5. Numerical Simulation:

To analyze and compare the performance of the proposed algorithm, MATLAB simulation is performed on a WiMAX based OFDM system with IEEE 802.16 – 2004 standard. The simulation uses \( 10^5 \) random OFDM symbols to generate Complementary Cumulative Distributive Function (CCDF) of the PAPR of OFDM signal with number of sub carriers, \( N = 1024 \), CCDF = Pr\{PAPR > PAPR0\} to compare the performance of the proposed algorithm based PTS, conventional PTS (Muller and Huber, 1997) and
Transformation of PTS (T-PTS) (Zhu et al., 2008). The parameters and its values used for the performance analysis of the proposed algorithm by simulation are presented in Table 1.

Table 1: Details of OFDM Parameters used for simulation.

<table>
<thead>
<tr>
<th>Description of parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Sub carriers, N</td>
<td>1024</td>
</tr>
<tr>
<td>Modulation type</td>
<td>QPSK</td>
</tr>
<tr>
<td>Bandwidth, BW</td>
<td>1 MHz</td>
</tr>
<tr>
<td>Interpolation factor, L</td>
<td>4</td>
</tr>
<tr>
<td>FFT size</td>
<td>4096</td>
</tr>
</tbody>
</table>

The CCDF of the PAPR of a QPSK modulated OFDM signal based on conventional PTS, T – PTS and proposed algorithm with their respective number of sub blocks, M is presented in Fig. 2. Solid State Power Amplifier (SSPA) model with input back off 0, 1, … 12dB and smoothness factor 0.5, 2 is employed for the simulation of BER vs. Signal to noise ratio plot. The BER performance of the proposed method is compared with that of OPTS is given is Fig. 3.

Fig. 2: Comparison of PAPR CCDF of various PTS schemes with the proposed method.

Fig. 3: BER performance comparison of proposed method and OPTS scheme.

Phase factor search complexity and PAPR reduction performance comparison of different PTS based PAPR reduction methods with number of sub blocks M = 16, phase factors W = 2, Population size P = S = 30, and number of iterations K = G = 30 is presented in Table 2, while the computational complexity involved in various PTS based PAPR reduction methods are presented in Table 3. The amount of side information that needs to be transmitted for recovering the original data block at the receiver is an important factor to estimate the performance of PAPR reduction method. In conventional PTS (Muller and Huber, 1997) the amount of
side information bits required for transmission is \( \log_2(W^{M-1}) \), while the proposed algorithm requires \( \log_2(NW) \).

### Table 2: Search complexity comparison for various PTS Schemes.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Search Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-PTS</td>
<td>( W^{M-1} = 2^{15} = 32768 )</td>
</tr>
<tr>
<td>ABC PTS</td>
<td>( PK = 30 \times 30 = 900 )</td>
</tr>
<tr>
<td>TS PTS</td>
<td>( LN = 4 \times 256 = 1024 )</td>
</tr>
<tr>
<td>Proposed method</td>
<td>((K \times M) + (M - 1)W = 510)</td>
</tr>
</tbody>
</table>

### Table 3: Computational complexity comparison for various PTS schemes.

<table>
<thead>
<tr>
<th>Method</th>
<th>Complex Additions</th>
<th>Complex Multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTS</td>
<td>( LN(M - 1) )</td>
<td>( LN(M + 1) )</td>
</tr>
<tr>
<td>LC - PTS</td>
<td>( LN(M - 1) \left( \frac{N-P}{W} + P \right) )</td>
<td>( LN(M + 1) \left( \frac{N-P}{W} + P \right) )</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>( \frac{N}{2M} \log_2(NW) + N )</td>
<td>( \frac{N}{M} \log_2(NW) + N \log_2 \frac{N}{2M} )</td>
</tr>
</tbody>
</table>

### 6. Conclusion:

This paper presented a novel algorithm to find the optimal phase factor in PTS method so that PAPR reduction is achieved with reduced search complexity and no compromise in BER performance when compared to O – PTS method. The optimal phase factor search is rewritten as a convex optimization problem which is effectively solved by the proposed algorithm. The simulation result shows that the proposed algorithm achieves significant PAPR reduction and BER performance.

### References


