Optimization of the base frequency of cylindrical stiffened composite shells with internal fluid

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ABSTRACT

The free vibration of cylindrical composite shell with internal liquid is studied in this paper. The shell composed of several layers and stiffeners which are rings and stringers. The first order shear theory was used for shell and stiffeners. Stiffeners were used in equations as discrete elements. The Reily-Ritz method was used for solving the problem. This method is based on minimum potential energy principle. The potential and kinetic energy of shell and each stiffeners and kinetic energy of liquid are substituted in the functional of energy. The section shape of each stiffener is rectangle. The liquid is ideal and sloshing was neglected. For catching the best and biggest natural base frequencies, shell and stiffeners were optimized by genetic algorithm. Using genetic algorithm, the best fiber angles of each layer of composite were obtained to reach the maximum base frequency, also the best ratio of height to width was obtained, and eventually the number of rings and stringers and their height to width ratios were obtained, in order to reach the optimized stiffened shell with the highest base frequency.

Key words: Stiffener, ring, stringer, genetic algorithm.

Introduction

Dynamics of thin-walled cylindrical shells has been widely studied in recent decades. In the past years, these studies have been based on the classic theory of shells. Many of the engineering applications, including petrochemical industries, chemical process equipments, energy production devices, water transmission lines, and etc. need tanks and pipes for storage and transportation of fluids, hence hereby the importance of studying the cylindrical tanks and shells is stressed for these applications.

In the past, the classical theory of plates and shells was used, but by the introduction and application of composite materials in industrial scales, it was perceived that the application of classical theory for plates and shells composed of composite materials might be accompanied by very erroneous results. The above stated problems made the researchers use the first order and other higher order theories. In this research, the first order theory has been used. In comparison to the classic theory and higher order theories, the first order shear theory is a combination of more accuracy with respect to the classic theory, and also needs less computation than higher order theories.

One of the other problems considered these days, is achievement of the best vibration and buckling state in shells, i.e. finding the optimum state for a certain desired design. Hence, different optimization methods were developed, wherein the genetic algorithm was a simple, but effective method and considerable attention is paid to it in terms of optimization, thanks to the speed of advanced computers now commercially available.

Zhi Pan, Xuebin Li, and Janjun Ma have investigated the free vibrations of a cylindrical shell reinforced with rings, under arbitrary boundary conditions. In 2006, A.A. Jafari, and M. Bagheri studied the free vibrations of a thin cylindrical shell reinforced with rings, in which the distances between the rings and also their offsets were variable. In the same year, Rong-Tyai Wang et al. studied the vibrations of a composite cylindrical shell reinforced with rings, wherein the rings were homogenous and the distances were equal, but the first order shear theory was used then.

In 2010, M. Bagheri, A.A. Jafari studied the free vibrations of a cylindrical shell reinforced with rings, having unequal distances and offsets; in order to reach the best optimized vibration state. In 1979, Amabili M. studied the vibrations of a vessel with internal non-viscous incompressible fluid, where the effects of vibrations of the free surface of the fluid and the hydrostatic pressure were ignored, and Reily-Ritz method was used to calculate the mode shapes.

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Material and Methods

Shell energy:

Displacement equations of a composite cylindrical shell have been derived, using the first order shear theory. In this theory, the displacement of the middle surface of the shell is taken as reference and the displacements of the other points are related to the middle surface as follows:

\[ u = u_0(x, \theta) + z\psi_x(x, \theta), \quad v = v_0(x, \theta) + z\psi_\theta(x, \theta), \quad w = w_0(x, \theta) \]

(1)

Fig. 1: A schematic of the cylindrical shell and displacement variables.

In relation 1, \( u_0, v_0, \) and \( w_0 \) are displacements of the middle surface along the \( x, \theta, \) and \( z \) directions, respectively. \( \psi_x, \) and \( \psi_\theta \) are the rotations of the middle surface around \( x, \) and \( \theta \) directions. Also, \( z \) is the distance of each point of the shell from the middle surface.

Using the equations of displacement-strain in cylindrical coordinate system, and ignoring the second order terms, the following relations are achieved, wherein \( R \) is the radius of the middle surface of the shell:

\[ \varepsilon_r = \frac{\partial u}{\partial x}, \quad \varepsilon_\theta = \frac{1}{R} \left( \frac{\partial v}{\partial x} + w \right), \quad \varepsilon_z = \frac{\partial w}{\partial x}, \]

\[ \gamma_{r\theta} = \frac{\partial v}{\partial z} + \frac{1}{R} \left( \frac{\partial w}{\partial \theta} \right) \frac{v_0}{R}, \quad \gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{\theta z} = \frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} \]

(2)

Also, based on the thickness variable, the strains can be stated:

\[ \{\varepsilon\} = \{\varepsilon^0\} + z \{\varepsilon^1\} \]

(3)

Strain vector is considered as:

\[ \varepsilon^T = \{\varepsilon_x^0, \varepsilon_\theta^0, \gamma_{r\theta}^0, k_x, k_\theta, k_{x\theta}, \gamma_{r z}^0, \gamma_{z z}^0\} \]

(4)

In the above relations, \( \varepsilon_x^0, \) and \( \varepsilon_\theta^0 \) are the strains of the mid-surface, \( k_x, k_\theta, \) and \( k_{x\theta} \) are the curvedness of mid-surface, and \( \gamma_{r\theta}^0 \) and \( \gamma_{r z}^0 \) are the transverse shear values. By substituting relation 1 in relations 2, the strain matrix components can be achieved as following:

\[ \varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad \varepsilon_\theta^0 = \frac{1}{R} \left( \frac{\partial v_0}{\partial x} + w_0 \right), \quad \gamma_{r\theta}^0 = \frac{1}{R} \left( \frac{\partial u_0}{\partial \theta} \right) + \frac{\partial w_0}{\partial x}, \quad k_x = \frac{\partial \psi_x}{\partial x}, \quad k_\theta = \frac{1}{R} \left( \frac{\partial \psi_\theta}{\partial \theta} \right), \]

\[ k_{x\theta} = \frac{1}{R} \left( \frac{\partial \psi_\theta}{\partial \theta} \right), \quad \gamma_{r z}^0 = \psi_x + \frac{\partial w_0}{\partial x}, \quad \gamma_{z z}^0 = \psi_\theta + \frac{\partial v_0}{\partial \theta} = \frac{v_0}{R} \]

(5)

Generally, the stiffness matrix for an orthotropic material is:
Wherein, the components are:

\[
A_k = \sum_{k=1}^{N} \overline{Q}_k (z_k - z_{k-1}), \\
B_y = \frac{1}{2} \sum_{k=1}^{N} \overline{Q}_k (z_k^2 - z_{k-1}^2), \\
D_y = \frac{1}{3} \sum_{k=1}^{N} \overline{Q}_k (z_k' - z_{k-1}'^2), \\
H_y = k_0 \sum_{k=1}^{N} \overline{Q}_k (z_k - z_{k-1}),
\]

(7)

Where, \(z_k\) and \(z_{k-1}\) show the distances of the middle surface from the outer and inner surfaces of the \(k\)-th layer, as indicated in Figure 2. \(N\) indicates the number of layers and \(\overline{Q}_k\) is the transformed stiffness matrix for the \(k\)-th layer. Also, \(k_0\) is the shear correction factor.

![Fig. 2: Number of layers and their distance from the mid-surface.](image)

\(\overline{Q}_k\) is defined as:

\[
[\overline{Q}] = [T']^{-1} [Q] [T']^T
\]

(8)

Where \(Q\) is the reduced stiffness matrix for the orthotropic material, and \(T\) is the rotation matrix. Eventually, the potential energy of the shell is calculated from the following relation:

\[
U_{shell} = \frac{1}{2} \int_0^l \int_0^{2\pi} \epsilon^T [S] \epsilon R d\theta dx
\]

(9)

Where, \(l\) is the length of the cylinder, \(\epsilon\) is the strain vector, and \(S\) is the stiffness matrix. Kinematic energy of the shell is also found by:

\[
T_{shell} = \frac{1}{2} \int_0^{2\pi} \int_0^l \left[ \frac{\partial u}{\partial \theta} \left( \frac{\partial u}{\partial \theta} \right)^2 + \frac{\partial v}{\partial \theta} \left( \frac{\partial v}{\partial \theta} \right)^2 \right] + \left[ \frac{\partial u}{\partial \theta} \frac{\partial \psi}{\partial \theta} \right] + \left[ \frac{\partial v}{\partial \theta} \frac{\partial \psi}{\partial \theta} \right] + R d\theta d\theta
\]

(10)
Wherein:

\[
\bar{\rho} = \sum_{k=1}^{n} \rho^k \left( z_{k} - z_{k-1} \right), \quad Q = \frac{1}{2} \sum_{k=1}^{n} \rho^k \left( z_{k}^2 - z_{k-1}^2 \right), \quad I = \frac{1}{3} \sum_{k=1}^{n} \rho^k \left( z_{k}^3 - z_{k-1}^3 \right)
\]

(11)

And, \( \rho^k \) is the density of the \( k \)-th layer, and \( n \) is the number of layers.

**Calculation of the energy of stiffeners:**

**Fig. 3:** Cylindrical reinforced shell.

In figure 3, the cylindrical reinforced shell is shown. The offset of stiffeners, which is the distance of the middle surface of the shell from the middle surface of the stiffener, have been shown by \( z_s \) and \( z_r \), which indicate stringer and ring, respectively. The stiffeners used in the present research are all of rectangular cross-section, and their height and thickness are shown by \( d \) and \( b \), respectively (Figure 4). These values with the subscript of \( r \) indicate rings, and with the subscript of \( s \) indicate stringers.

**Fig. 4:** Cross-section of a stiffener.

**Energy of the rings:**

First, the following are defined:

\[
I_{z_{ri}} = \frac{b_{ri} d_{ri}^3}{12}, \quad I_{x_{ri}} = \frac{b_{ri}^3 d_{ri}}{12}, \quad A_{ri} = b_{ri} d_{ri},
\]

\[
z_{ri} = \frac{h + d_{ri}}{2}, \quad J_{ri} = \frac{1}{3} \left[ 1 - \frac{192 b_{ri}}{\pi^2 d_{ri}} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n^3} \tanh \left( \frac{n \pi d_{ri}}{2b_{ri}} \right) \right] b_{ri}^3 d_{ri}
\]

(12)

In relation 12, the index \( ri \) indicates the \( i \)-th ring. \( I_{x_{ri}}, I_{z_{ri}} \) are the second moments of the ring cross-section around axes, passing through the center of the ring cross-section, respectively, which are parallel with the x, and z axes. \( A_{ri} \) is the area of the cross-section and \( J_{ri} \) is the flexural stiffness of the ring. Eventually, \( z_{ri} \) is the offset
of the ring, whose value is positive for the external ring, and negative for the internal ring. The potential energy
of the ring is achieved from the following relation:

\[
U_n = \int_0^{2\pi} \left\{ \frac{E_n I_{zz}}{2(R + z_n)} \left[ \frac{\partial w_n}{\partial x} + \frac{1}{(R + z_n)} \frac{\partial^2 u_n}{\partial \theta^2} \right]^2 + \frac{E_n I_{xy}}{2(R + z_n)} \left[ \frac{\partial w_n}{\partial \theta} - w_n \right]^2 + \frac{G_n J_{zz}}{2(R + z_n)} \left[ \frac{\partial^2 w_n}{\partial x \partial \theta} + \frac{1}{(R + z_n)} \frac{\partial u_n}{\partial \theta} \right]^2 \right\} d\theta
\]

wherein:

\[ u_{ri} = u_0 + z_{ri} \varphi_x , \quad v_{ri} = v_0 \left( 1 + \frac{z_{ri}}{R} \right) + z_{ri} \varphi_\theta , \]

\[ w_{ri} = w_0 \]

The kinetic energy of the ring is calculated as follows:

\[
T_n = \frac{\rho_n z_n}{2} \left\{ A_n \left[ \left( \frac{\partial u}{\partial t} \right)^2 + \left( \frac{\partial v}{\partial t} \right)^2 + \left( \frac{\partial w}{\partial t} \right)^2 \right] \right\} (R + z_n) d\theta
\]

Energy of the stringers:

First, the following must be defined:

\[
I_{yzj} (x) = \frac{b_j d_j^3}{12} , \quad I_{zsj} (x) = \frac{b_j d_j^3}{12}
\]

\[
A_{sj} (x) = b_j d_j (x) , \quad \overline{z}_{sj} (x) = \pm \frac{h + d_j (x)}{2}
\]

\[ J_{yj} (x) = \frac{1}{3} \left[ 1 - \frac{192 y_j}{\pi d_y (x) n^2} \sum_{n=1,3,5,...} \frac{1}{n^2} \tanh \left( \frac{n \pi d_y (x)}{2 y_j} \right) \right] b_j^3 d_y (x) \]

In relations 16, the index \( sj \) indicates the \( j \)-th stringer. \( I_{yzj}, I_{zsj} \) are the second moments of the cross-
section around the two axes perpendicular to each other, respectively. \( A_{sj} (x) \) is the area of the cross-section,
and \( J_{yj} (x) \) is the flexural stiffness of the stringer. Eventually, \( \overline{z}_{sj} (x) \) is the offset of the stringer, whose
value is positive for the external stringer, and negative for the internal stringer.
The potential strain energy of the stringer is achieved as follows:
The kinetic energy of the stringer is achieved as follows:

\[ T_{sj} = \frac{\rho_s}{2} \int_0^l \left[ \frac{\partial u_{sj}}{\partial r} \right]^2 + \frac{\partial v_{sj}}{\partial \theta} \frac{\partial u_{sj}}{\partial r} + \frac{\partial w_{sj}}{\partial \theta} \frac{\partial u_{sj}}{\partial r} \right] \times R + z_{sj} \right] d\theta \]  \tag{19}

Investigation of the effect of internal fluid on vibrations of cylindrical shell:

In order to investigate the effect of internal fluid on vibrations of the cylindrical shell, one should analyze the interaction between the solid and fluid states at their interface, using a mathematical model. Assumptions of the mathematical model are as follows:
- Flow of the fluid is a potential flow.
- Fluid is ideal, i.e. non-viscous and incompressible.
- Displacements are small, so that the linear theory can be used.
- Velocity of the fluid along the cylinder axis is zero.
- Effects of the surface waves are ignored.

Regarding the first assumption of potential flow, the potential flow function in the cylindrical coordinate system can be written as:

\[ \nabla^2 \Phi = 0 \]

\[ \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial x^2} = 0 \]  \tag{20}

wherein, \( \Phi \) is the velocity potential function of the fluid, and \( x, \theta, \) and \( r \) are axial, circumferential, and radial components in the cylindrical coordinate system, respectively.

Components of the velocity of fluid flow are determined as follows:
\[ V_x = \frac{\partial \Phi}{\partial x} \]
\[ V_\theta = \frac{1}{R} \frac{\partial \Phi}{\partial \theta} \]
\[ V_r = \frac{\partial \Phi}{\partial r} \]

\(V_r, V_\theta, V_x\) are components of fluid velocity along the axial, circumferential, and radial directions, respectively. In order to find the effects of the fluid, one should consider the boundary conditions caused by the interaction between fluid and solid, and apply them in the differential equations of the fluid. Hence, regarding the fact that the fluid does not penetrate into the shell, there is always a constant touch between the outer fluid layer and the internal wall of the shell, and the radial velocity component of the fluid is the same constant as that of the shell at their interface. These assumptions can be defined through the following equation:

\[ V_r = \left. \frac{\partial \Phi}{\partial r} \right|_{r=R} = \left( \frac{\partial w}{\partial t} \right)_{r=R} \]  

(22)

In order to solve the differential equation of the velocity potential function, the method of separation of the variables can be used. Therefore, the velocity potential is considered as the product of the two functions, as below:

\[ \Phi(x, \theta, r, t) = R(r)S(x, \theta, t) \]

(23)

So as to solve find the \(S(x, \theta, t)\) function, one can apply the boundary conditions of relation 22. By application of boundary conditions and substituting the above relation into the boundary condition relation, equation 24 will be achieved:

\[ \Phi(x, \theta, r, t) = \left( \frac{\partial R(r)}{\partial r} \right)_{r=R} \frac{\partial w(x, \theta, t)}{\partial t} \]

(24)

Where, \(w\) is the displacement of the shell in its radial direction.

Substituting relation 24 in the flow potential function (relation 20), and performing the mathematical simplifications, the homogenous Bessel function is resulted as follows:

\[ r^2 \frac{d^2 R(r)}{dr^2} + r \frac{dR(r)}{dr} + R(r)(\nu^2 \pi^2 - n^2) = 0 \]

(25)

Wherein, \(k_r\) is the number of radial half waves, i.e.:

\[ k_r^2 = \left( \frac{m \pi}{l} \right)^2 = \left( \frac{\omega}{c_f} \right)^2 \]

(26)

For a shell exposed to an internal fluid, the factor of \(R(r)\) is always negative in relation 25; therefore, the general answer of equation 25 is expressed as:

\[ R(r) = A J_n(ik_r r) + B Y_n(ik_r r) \]

(27)

Where, \(J_n\) is the type one Bessel function, and \(Y_n\) is Bessel function of \(n\)-order, and \(r\) is the radial component in cylindrical coordinate system. For a cylinder filled up with fluid, the constant \(B\) must be set to zero, because the function is singular at the center of the cylinder \((r=0)\).

Kinetic energy of the fluid which is resulted from the movement of the internal fluid due to the shell displacement is derived as follows:
\[ T_{fl} = \frac{1}{2} \rho_{fl} \int_0^R \int_0^{2\pi} \int_0^1 \nu^2 r^2 \, dx \, d\theta \, dr \]  

(28)

Where, \( \nu \) is velocity, and \( \rho_{fl} \) is the fluid density.

The square of fluid velocity is equal to the sum of the squares of the velocity components in the axial, circumferential, and radial directions, as follows:

\[ \nu^2 = \nu_x^2 + \nu_\theta^2 + \nu_r^2 \]  

(29)

By considering the fluid as incompressible, and neglecting the effects of surface waves, the potential energy of the fluid is equal to zero.

The following functions are adopted to separate the spatial variable \( x, \theta \) and the time variable \( t \)

\[
\begin{align*}
    u_0(x, \theta, t) &= A \cos\left(\frac{m\pi x}{l}\right) \cos(n\theta) e^{i\omega t} \\
    v_0(x, \theta, t) &= B \sin\left(\frac{m\pi x}{l}\right) \sin(n\theta) e^{i\omega t} \\
    w_0(x, \theta, t) &= C \sin\left(\frac{m\pi x}{l}\right) \cos(n\theta) e^{i\omega t} \\
    \psi_x(x, \theta, t) &= D \cos\left(\frac{m\pi x}{l}\right) \cos(n\theta) e^{i\omega t} \\
    \psi_\theta(x, \theta, t) &= E \sin\left(\frac{m\pi x}{l}\right) \sin(n\theta) e^{i\omega t}
\end{align*}
\]

(30)

Formation of the potential energy function:

Now, having the potential and kinetic energies of the shell, fluid, and stiffeners, one can form the potential energy function as follows, where \( o \) is the number of rings and \( p \) is the number of stringers:

\[
F = T_{fl} - U_{shell} + T_{shell} + \sum_{i=1}^{o} \left( T_{ri} - U_{ri} \right) + \sum_{j=1}^{p} \left( T_{sj} - U_{sj} \right)
\]

(31)

In the above relation, \( T_{fl} \) is the kinetic energy of the fluid, \( U_{shell} \) is the potential strain energy of the shell, \( T_{shell} \) is the kinetic energy of the shell, \( T_{ri} \) is the kinetic energy of one ring, \( U_{ri} \) is the potential strain energy of one ring, \( T_{sj} \) is the kinetic energy of one stringer, and \( U_{sj} \) is the potential strain energy of one stringer.

Problem solving:

Reily-Ritz method has been used to solve the problem. This method is based on the minimum potential energy method. According to Reily-Ritz method, in order for the potential energy which is a function of \( A, B, C, D, \) and \( E \) to be a minimum, the differentiation of the total energy with respect to the factors applied in the displacement field, must go zero. Therefore, a differentiation is taken from the total energy of the system with respect to the stated factors, and is set to zero. Then, a 5 equation 5 unknown differential set of equations is arrived at, wherein the \( A, B, C, D, \) and \( E \) are its unknowns.

By reordering the terms, the following matrix relation is achieved:

\[
\begin{bmatrix}
    A \\
    B \\
    C \\
    D \\
    E
\end{bmatrix} - \omega^2 \begin{bmatrix} M \\ C \end{bmatrix} = 0
\]

(32)
Wherein, $K$ and $M$ are the stiffness and mass matrices of the structure, respectively. The components of the matrix $K$ involve the geometrical dimensions, and the physical specifications of the structure. In order to determine the non-evident answers of the relation (32), a generalized eigenvalue problem has to be solved. In order for the equation set of (32) to have a non-evident answer, the determinant of the factors must be set to zero:

$$|K - \omega^2 M| = 0 \quad (33)$$

From the above, the natural frequencies for each of the $(m, n)$ modes are achieved.

**Results and Discussion**

*Comparison of the results for the isotropic shell reinforced with ring and stringer:*

<table>
<thead>
<tr>
<th>Table 1: Specifications of the shell and stiffeners.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic</td>
</tr>
<tr>
<td>Number of stringer/ ring</td>
</tr>
<tr>
<td>Radius</td>
</tr>
<tr>
<td>Thickness</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Height of stringer/ ring</td>
</tr>
<tr>
<td>Width of stringer/ ring</td>
</tr>
<tr>
<td>$E$</td>
</tr>
<tr>
<td>$\nu$</td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
</tbody>
</table>

In Table 2, the natural frequencies of the present research are compared with those of an experimental research, and also those of an analytical research.

<table>
<thead>
<tr>
<th>Table 2: Comparison of the results of an isotropic shell reinforced with ring, and stringer without internal fluid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

As it is seen, there is a good convergence between the reference frequencies and the frequency achieved from the present research.

*Comparison of the results for the isotropic shell filled up with fluid:*

<table>
<thead>
<tr>
<th>Table 3: Specifications of the shell and fluid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic</td>
</tr>
<tr>
<td>Radius</td>
</tr>
<tr>
<td>Radius to thickness ratio</td>
</tr>
<tr>
<td>Length to radius ratio</td>
</tr>
<tr>
<td>Density of the fluid</td>
</tr>
<tr>
<td>Density of the shell</td>
</tr>
<tr>
<td>$E$</td>
</tr>
<tr>
<td>$\nu$</td>
</tr>
</tbody>
</table>

In the following table, natural frequencies of the present research are compared with a number of references. The unit of the frequencies is Hertz.

<table>
<thead>
<tr>
<th>Table 4: Comparison of the natural frequencies of the present research (isotropic shell with internal fluid) with a number of other references.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

As it is realized, there is a good convergence between the reference frequencies and the frequency achieved from the present research.
Investigation of the results:

Here, natural frequencies of the system in different states are studied, and the effective factors on natural frequencies have been investigated. The factors investigated and known to have effect on the natural frequencies are: effect of the fluid, dimensions of the stiffeners, and the distribution pattern of stiffeners on the surface of the shell.

**Table 5:** Geometrical specifications of the shell.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1m</td>
</tr>
<tr>
<td>R</td>
<td>0.2m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.002m</td>
</tr>
</tbody>
</table>

Characteristics of some materials used in different sections of the investigation of the results, are presented in the following table:

**Table 6:** Properties of the materials used in the current section.

<table>
<thead>
<tr>
<th>Number of material</th>
<th>Name of material</th>
<th>E11 (GPa)</th>
<th>E22 (GPa)</th>
<th>G12 (GPa)</th>
<th>ν</th>
<th>ρ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Steel</td>
<td>206</td>
<td>206</td>
<td></td>
<td>0.3</td>
<td>7800</td>
</tr>
<tr>
<td>2</td>
<td>Glass/ Epoxy</td>
<td>19</td>
<td>7.6</td>
<td>4.1</td>
<td>0.26</td>
<td>1643</td>
</tr>
<tr>
<td>3</td>
<td>Graphite/ Epoxy</td>
<td>110.31</td>
<td>15.17</td>
<td>4.96</td>
<td>0.25</td>
<td>1577</td>
</tr>
<tr>
<td>4</td>
<td>Carbon/ Epoxy</td>
<td>139.4</td>
<td>8.35</td>
<td>3.1</td>
<td>0.268</td>
<td>1542</td>
</tr>
</tbody>
</table>

It is noteworthy that in all the investigated sections of the present project, the material of the stiffeners is steel, and in the case of existence of fluid, water is the case. The boundary conditions of the shell are simply supported in two ends at all sections.

Investigation of the effect of radius to thickness ratio on natural frequencies of the shell without stiffener and fluid:

For a composite shell composed of material 4 with the stacking sequence of 90/0/0/90, natural frequency for different values of \( r/h \) is presented in Figure 5.

![Fig. 5: Effect of radius to thickness ratio on natural frequency of the shell without internal fluid.](image-url)
Regarding the above diagram, by decreasing the $r/h$ ratio, natural frequencies at different modes increase. But, for $n=1$ mode, there is an exception, wherein by changing the $r/h$ ratio; there is no change in natural frequency.

**Investigation of the effect of radius to thickness ratio on natural frequencies of the shell without stiffener and with internal fluid:**

For a composite shell composed of material 4 with the stacking sequence of 90/0/0/90, natural frequency for different values of $r/h$ is presented in Figure 6.

![Figure 6: Effect of radius to thickness ratio on natural frequency of the shell with internal fluid.](image)

Regarding the above diagram, by decreasing the $r/h$ ratio, natural frequencies at different modes increase. Contrary to the state of without fluid, wherein for $n=1$ by changing the $r/h$ ratio there is no change in natural frequency, for the state of internal fluid, the decrease of $r/h$ ratio will also increase the natural frequency of this mode.

**Investigation of effect of the amount of fluid on natural frequencies:**

If the axis of the shell is supposed to be vertical, the fluid inside it can have different depths. In this section, natural frequency of the shell, including different amounts of internal fluid are compared with each other. As it seen from diagram 3, the depth of the fluid changes from a full state of L to L/6, and the values of the natural frequency for $m, n=1$ are drawn from 1 to 5. It is seen that as the depth of the fluid increases, natural frequencies decrease. The effect of this little amount of fluid on natural frequency is hugely considerable. It is observed that the frequency difference between L/6, and 2L/6 depths is a considerable amount, while the frequency difference between L, and 5L/6 depths is very small.

**Finding the optimum states of free vibration, using genetic algorithm:**

**Finding the best fiber angles for the laminated composite shell:**

The composite laminated shell with 4 layers is considered. The material of the layers is chosen as that of the material number 4. The objective is finding the best fiber angle for these 4 layers, i.e. which stacking sequence for the fiber angles causes the maximum natural frequencies.

In order to do this, the procedure is started with an initial multitude of 10 members, i.e. a 10*4 matrix, where in each row of the 10 rows, 4 values exist for the angles of the 4 layers. These values for the initial multitude are randomly chosen from values between 0 and 180.
Fig. 7: Effect of height of the fluid on natural frequencies of the shell with internal fluid.

Table 7: Optimization specifications of the composite laminated shell (4 layers).

<table>
<thead>
<tr>
<th>Optimization parameter</th>
<th>Range of angle variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber angle in first layer</td>
<td>0 – 180</td>
</tr>
<tr>
<td>Fiber angle in second layer</td>
<td>0 – 180</td>
</tr>
<tr>
<td>Fiber angle in third layer</td>
<td>0 – 180</td>
</tr>
<tr>
<td>Fiber angle in fourth layer</td>
<td>0 – 180</td>
</tr>
</tbody>
</table>

The fitness function is the base natural frequency for a shell with each of the given angels. For each of the 10 members, natural frequency should be found for the modes, which have higher possibility for the emergence of base frequency. Now, by comparison of the frequencies of the stated modes, the minimum frequency would be the base frequency. Comparing the 10 achieved results for the base frequencies, the maximum values known as the chosen generation, will be transferred to the next generation.

The rest of the members of the next generation are produced from the present generation, by application of the crossover operator (with higher possibility) and the mutation operator (with low possibility), and the next generation is formed so.

In every stage of the application of the genetic operation, the above stated issues must be performed, and repeated in order. The best attained values from different stages are saved in an array, then by running the algorithm for several times, its diagram is drawn so as to observe the progress trend of the algorithm.

The value achieved from the genetic algorithm for the highest base frequency, and also the angles of the layers related to it, are as follows:

Table 8: Optimized angles and base frequency.

<table>
<thead>
<tr>
<th>characteristic</th>
<th>n=3, m=1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max base frequency</td>
<td>1637 rad/sec</td>
</tr>
<tr>
<td>Mode</td>
<td>n=3, m=1</td>
</tr>
<tr>
<td>Fiber angles</td>
<td>[104.3, 0.7, 159, 42.5]</td>
</tr>
</tbody>
</table>

It is necessary to mention that the base for the measurement of the angles is a line perpendicular to the axis of the cylinder, i.e. fibers parallel with the axis of the cylinder have a 90 degree angle.

The best frequency achieved for each generation is drawn in the following diagram. The achieved values for the optimized fiber angles, after implementing the genetic algorithm have been found for 60 generations.

Finding the optimized rectangular section of stringer for the state without internal fluid:

A composite shell composed of material number 2 with the stacking sequence of 90/0/0/90 has been reinforced with 8 stringers. The specifications of the shell are as follows:
Area of the stringer cross-section is considered as 0.0004 square meters and is always taken a constant. The objective is finding the best value for height (d) and width (b) of the stringer, in a way to maximize the base frequency of it.

The stringer must not be excessively thin; therefore, a limitation for the dimensions of the stringer has been determined as follows:

\[ 0.1 < \frac{d}{b} < 10 \]

Therefore, for the area of the stated cross-section, the maximum and minimum values for dimensions of the stringer are 0.06325 and 0.006325, respectively. According to the stated conditions, using the genetic algorithm the optimum value for dimensions of the stringer can be determined.

Using the genetic algorithm, for the initial multitude of 10 members, and with the production of 20 generations, the following values are achieved:

\[
\begin{array}{|c|c|}
\hline
\text{characteristic} & \text{size} \\
\hline
\text{Max base frequency} & 2208 \text{ rad/sec} \\
\text{Mode} & n=3, m=1 \\
\hline
\text{d & b} & d = 0.06325 \text{ m}, b = 0.006325 \text{ m} \\
\hline
\end{array}
\]

The maximum values of the base frequency in each of the generations are drawn in Figure 10.
Finding the optimized rectangular section of stringer for the state with internal fluid:

A composite shell composed of material number 2 with the stacking sequence of 90/0/0/90 has been reinforced with 8 stringers. The specifications of the shell are as follows:

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stringers</td>
<td>8</td>
</tr>
<tr>
<td>Radius of shell</td>
<td>0.2m</td>
</tr>
<tr>
<td>Thickness of shell</td>
<td>0.01m</td>
</tr>
<tr>
<td>Length</td>
<td>1m</td>
</tr>
<tr>
<td>Area of cross-section</td>
<td>0.0004m$^2$</td>
</tr>
<tr>
<td>Stacking sequence</td>
<td>90/0/0/90</td>
</tr>
<tr>
<td>Number of shell material</td>
<td>2</td>
</tr>
<tr>
<td>Density of the fluid</td>
<td>1000 kg/m$^3$</td>
</tr>
</tbody>
</table>

Area of the stringer cross-section is considered as 0.0004 square meters, and is always taken a constant. The objective is finding the best value for height (d) and width (b) of the stringer, in a way to maximize the base frequency of it.

The stringer must not be excessively thin; therefore, a limitation for the dimensions of the stringer has been determined as follows:

$$0.1 < (d/b) < 10$$

Therefore, for the area of the stated cross-section, the maximum and minimum values for dimensions of the stringer are 0.06325 and 0.006325, respectively. According to the stated conditions, using the genetic algorithm the optimum value for dimensions of the stringer can be determined.

Using the genetic algorithm, for the initial multitude of 10 members, and with the production of 18 generations, the following results are achieved:

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max base frequency</td>
<td>1214 rad/sec</td>
</tr>
<tr>
<td>Mode</td>
<td>$n=2$, $m=1$</td>
</tr>
<tr>
<td>d &amp; b</td>
<td>$d = 0.06325$ m, $b = 0.006325$ m</td>
</tr>
</tbody>
</table>

The maximum values of the base frequency have been drawn in the following diagram for each generation:

![Diagram showing the highest base frequencies of the shell with internal fluid in each generation for different ratios of d/b.](image-url)
Optimization of cylindrical shell reinforced with ring and stringer with internal fluid:

A cylindrical shell composed of material number 2 with water as internal fluid, with the stated specifications of the table below, has been considered.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>0.2m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.005m</td>
</tr>
<tr>
<td>Length</td>
<td>1m</td>
</tr>
<tr>
<td>Stacking sequence</td>
<td>90/0/0/90</td>
</tr>
<tr>
<td>Number of shell material</td>
<td>2</td>
</tr>
<tr>
<td>Area of rings and stringers</td>
<td>0.0001 m²</td>
</tr>
<tr>
<td>Material used in stiffeners</td>
<td>Steel</td>
</tr>
</tbody>
</table>

The objective is optimization of the frequency and weight of the shell with internal fluid. The shell possesses rings and stringers, whose numbers will be determined by a computer developed genetic algorithm. Also, the area of the cross-section of rings and stringers is considered the constant of 0.0001 square meters, but their optimum height to width ratio (d/b) is also determined through the computer program. Symbols b, and d have been shown below.

First the fitness function must be found. The value of the natural frequency must be maximized, while the weight must be minimized. Hence, the base frequency is the nominator, and weight is the denominator of the fitness function. As it is known, frequency and weight are not of one kind; therefore, they should be non-dimensioned for homogenization purposes. Therefore, the base frequency of the reinforced shell with internal fluid is divided on the frequency of the non-reinforced shell with internal fluid, and the result is the nominator of the fitness function. Also, the weight of the reinforced shell is divided on the weight of the non-reinforced shell, and the result is the denominator of the fitness function.
Table 14: Geometrical specifications of the shell.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base frequency of the reinforced shell with internal fluid</td>
<td>( \omega_s )</td>
</tr>
<tr>
<td>Base frequency of the non-reinforced shell with internal fluid</td>
<td>( \omega )</td>
</tr>
<tr>
<td>Weight of the reinforced shell</td>
<td>( w_s )</td>
</tr>
<tr>
<td>Weight of the non-reinforced shell</td>
<td>( w )</td>
</tr>
</tbody>
</table>

If the above nomenclature is used for the stated issues, then the fitness function will be found as:

\[
\text{fitness} = \left( \frac{\omega_s}{\omega} \right) \left( \frac{w_s}{w} \right)
\]  

(34)

The optimization parameters and the range of changes are as follows:

Table 15: Optimization parameters and the range of changes.

<table>
<thead>
<tr>
<th>Optimization parameter</th>
<th>Range of changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height to width ratio of ring (d/b)</td>
<td>0.1 – 10</td>
</tr>
<tr>
<td>Height to width ratio of stringer (d/b)</td>
<td>0.1 – 10</td>
</tr>
<tr>
<td>Number of rings</td>
<td>0 – 20</td>
</tr>
<tr>
<td>Number of stringers</td>
<td>0 – 20</td>
</tr>
</tbody>
</table>

Table 16: Optimization parameters for the first generation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shell with stiffener</th>
<th>Best result in first generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness function</td>
<td>1</td>
<td>0.976</td>
</tr>
<tr>
<td>Height to width ratio of ring (d/b)</td>
<td>-</td>
<td>4.76</td>
</tr>
<tr>
<td>Height to width ratio of stringer (d/b)</td>
<td>-</td>
<td>1.07</td>
</tr>
<tr>
<td>Number of rings</td>
<td>-</td>
<td>17</td>
</tr>
<tr>
<td>Number of stringers</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td>Base natural frequency</td>
<td>392 rad/sec</td>
<td>909 rad/sec</td>
</tr>
<tr>
<td>Weight of reinforced shell</td>
<td>10.323 kg</td>
<td>25.02 kg</td>
</tr>
<tr>
<td>Mode</td>
<td>m = 1, n = 2</td>
<td>m = 1, n = 2</td>
</tr>
</tbody>
</table>

With an initial multitude of 10 members, and production of 80 generations, the following optimum values are achieved:

Table 17: Optimization parameters for the first generation with the initial multitude of 10 members.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness function</td>
<td>1.54</td>
</tr>
<tr>
<td>Height to width ratio of ring (d/b)</td>
<td>9.73</td>
</tr>
<tr>
<td>Height to width ratio of stringer (d/b)</td>
<td>9.68</td>
</tr>
<tr>
<td>Number of rings</td>
<td>5</td>
</tr>
<tr>
<td>Number of stringers</td>
<td>4</td>
</tr>
<tr>
<td>Base natural frequency</td>
<td>912 rad/sec</td>
</tr>
<tr>
<td>Weight of reinforced shell</td>
<td>15.63</td>
</tr>
<tr>
<td>Mode</td>
<td>m = 1, n = 2</td>
</tr>
</tbody>
</table>

Best values in each generation are drawn in Figure 12.

Results:

- In the cases studied in the current research, if fluid is added to the shell (either reinforced or simple shell), the circumferential mode \( n \) in which the base natural frequency happens, remains unchanged or becomes smaller, but the value of \( m \) related to the base natural frequency remains unchanged.
- In the shell without internal fluid, the more the \( r/h \) ratio is decreased, the more the natural frequencies at different modes are increased. Of course, there is an exception for \( n=1 \), regarding the fact that by changing the \( r/h \) ratio, the natural frequency does not change.
- In the shell with internal fluid, the more the \( r/h \) ratio is decreased, the more the natural frequencies at different modes are increased. In this state, \( n=1 \) also shows the increase of the natural frequency.
- In the shell with internal fluid, as the depth of the fluid increases, natural frequencies decrease.
Fig. 12: Trend of variation of the fitness function in different generations.

References