ORIGINAL ARTICLES

Solar Tracker Modelling

E.T. El Shenawy, M. Kamal and M.A. Mohamad

Solar Energy Dept., National Research Centre, Elbehooth st., Dokki, Giza, Egypt

ABSTRACT

System modelling is a key element in process control. Applying the many methods of control to improve command response, stability and disturbance rejection requires an understanding of the system under control. Also, for the solar tracking systems, the choice of driving motors, gears, amplifiers, as well as the position detectors is very important step in an active system design. In the present paper the torque of the servomotor used for a single axis solar tracker under Egypt climate, as well as the selection criteria of a servomotor used for the tracker are calculated. The effect of the wind on the solar tracker in Egypt has been considered. Finally, the solar tracker and disturbance transfer functions have been developed.

Key words: Solar tracker, System modelling, Servo motor torque, Solar collectors, Azimuth axis trackers.

Introduction

The solar tracker is a system which follows the path of the sun and keeps the sun’s rays normal to the solar collector surface all time. The sun’s location in the sky relative to a point on the surface can be defined with two angles, the solar altitude angle and the solar azimuth angle. These angles are calculated through sun-earth relations [William et. al., 1980].

Sun tracking is essential for many solar energy based power systems to improve the system performance. It is essential for concentrator systems, moreover in flat-plate photovoltaic systems, an increase in power output by 30-60% (depending on the location) has been observed by the tracking as compared to the fixed tilt [Saxena and Dutta, 1990].

The solar collectors (flat, parabolic or dish) are rigidly mounted on the tracker surface. The solar trackers may be single or double axis trackers. The dual axes tracker must have the ability to track both the sun’s azimuth and altitude angles. Two servomotors will be used to apply a torque to the azimuth and elevation axes to seek the sun. There are several common implementations of single axis trackers. These include horizontal single axis trackers, perpendicular azimuth single axis trackers, tilted single axis trackers and polar aligned single axis trackers. In the present study, only the perpendicular azimuth axis tracker is considered.

For the proposed azimuth axis tracker, the total torque on the tracker is the sum of the motor torque and load torque. The load torque of moving system has the following components, which are reflected to the motor shaft is:

\[ T_i = T_j + T_f + T_d \] (1)

The inertial torque \( T_i \) is the torque to accelerate and decelerate the load, while the friction torque \( T_f \) due to static frictions of the mechanical system. \( T_d \) is the viscous damping torque which is proportional to the speed and it is small because the required speed of system is small. At normal operating conditions, both \( T_f \) and \( T_d \) are small and constant. In most cases, the inertial torque demands is the peak torque, during acceleration. The accelerating torque is proportional to the inertia [Benjamin, 2010],

\[ T_i = J * \alpha \] (2)

2. Wind Forces on the Solar Collector:

In Egypt, at the Hurghada City, that is located at the Red Sea coast and represents one of the most locations which is rich with wind, has annual average speeds between 5.6 m/s and 8.25 m/s [Mohamed et. al., 2001]. Then the system torque calculations at wind velocity 10 m/s is suitable to work at the Egyptian environment. The following section describes the pressure, lift, drag and moment effects on two dimensional structure forms.

Corresponding Author: E.T. El Shenawy, Solar Energy Dept., National Research Centre, Elbehooth st., Dokki, Giza, Egypt.
Tel: (002) 0122 5955303, E-mail: essamahame@hotmail.com
The air flow will develop local pressure \( P \) over the body [Simiu and Scanlan, 1996, Dune, 1998],

\[
P = 0.5 \rho v^2
\]  

(3)

The integration of the pressures over the body surface results in a net force and a moment. The net wind-pressure forces \( F_{\text{lift}} \) and \( F_{\text{drag}} \) in the lift and drag direction, respectively are,

\[
F_{\text{lift}} = 0.5 C_{\text{lift}} \rho v^2 A
\]  

(4)

\[
F_{\text{drag}} = 0.5 C_{\text{drag}} \rho v^2 A
\]  

(5)

The drag force is the component of the force along flow, and the across flow component direction is the lift force.

The net flow-induced moment \( J \) is,

\[
J = 0.5 C_{\text{moment}} \rho v^2 A_{\text{ref}} l_{\text{ref}}
\]  

(6)

\( A_{\text{ref}} \) is a reference area, generally selected to be representative of a plane area subjected to the flow by the body, here chosen as the aperture area of a collector module aperture (width*length). \( L_{\text{ref}} \) is a reference length selected to be representative of a characteristics body dimension (here is chosen as the collector aperture width).

3. The Perpendicular Azimuth Axis Solar Tracker Components:

The main components of the tracker, shown in Fig. 1, are:
1. D.C. motor with Pulleys and belts or Roller-chain.
2. The east-west motor aluminum stand.
4. The north-south motor aluminum stand.
5. Box contains the bearings of rotations and the fixing of the horizontal pipe with the stand.
6. The perpendicular aluminum tube of the collector stand.
7. The parallel aluminum tube of the collector stand.
8. The elevation tracking axle.
9. The azimuth tracking vertical axle.
10. The pylon.

4. Calculations of the Moment of Inertia:

For the perpendicular azimuth axis solar tracker, shown in Fig. 1, it is obvious that there are three components contribute significantly for the moment of inertia as follows;
- the elevation axis,
- the load stand,
- the load (solar collector)

4.1. Mass Moment of Inertia of the Horizontal Pipe:

The geometry of the horizontal pipe is a hollow cylinder with the inner radius of 10.16 cm, the thickness is 0.45 cm and the length is 200 cm, while the weight of the aluminum pipe is 2.769 kg/m. The pipe mass’s moment of inertia is [Craft and Davison, 1999],

\[
J_{\text{pipe}} = \frac{W \ast (r_o^2 - r_i^2)}{2}
\]  

(7)

\[
= 0.0011 \text{ kg} \text{.m}^2
\]

4.2. Mass Moment of Inertia of the Collector Stand:

The designed collector stand consists of two interconnected aluminum rectangular tubes, that are perpendicular to the horizontal pipe with dimensions of (2.54 cm X 5cm) as width and (0.318cm X 200cm) as
length. And the two tubes that parallel to the horizontal pipe with the same dimensions but spaced from the axis of rotation by a distance (d = 97 cm). The weight of the tube is 0.861 kg/m.

The moment of inertia of the two interconnected aluminum rectangular tubes that are perpendicular to the horizontal pipe ($J_{\text{tube-perpendicular}}$) can be calculated by [Craft and Davison, 1999],

$$J_{\text{tube-perpendicular}} = \frac{W \cdot (l^2 + D^2)}{12}$$

$$= 1.147 \text{ kg} \cdot m^2$$

Fig. 1: The suggested perpendicular azimuth axis solar tracker.

By parallel axis theorem the moment of inertia of any object about an axis parallel to its axis but a distance, d, away ($J_{\text{tube-parallel}}$) is [Craft and Davison, 1999],

$$J_{\text{tube-parallel}} = \frac{W \cdot D^2}{3} + W \cdot d^2$$

$$= 3.2435 \text{ kg} \cdot m^2$$
4.3 Mass Moment of Inertia of the Collector:

We will consider here a parabolic trough collector (PTC) with 0.73 m radius, 2.4 m length and 30 kg weight [Thomas, 1996]. The geometry of the PTC is approximated by half thin walled hollow cylinder and the axis of rotation through end (in the case of elevation tracking). The collector inertia is [Craft and Davison, 1999],

\[ J_{load\_collector} = \frac{W \cdot r^2}{4} = 3.675 \text{ kg } \text{m}^2 \]  

(10)

4.4. The Moment Effects of the Wind Disturbance on the Tracker:

The torque loads that are acting on PTC are wind load and gravity load [8]. For a parabolic trough collector in an array protected with a wind screen fence, drag and lift force, and moment coefficients equal respectively to 0.5, +0.5, +0.35 Air density is 1.229 kg/m$^3$ [Kalogirou, 1996]. Our suggested tracker is ground mounted, and the reference area \(A_{ref}\) and \(l_{ref}\) of the parabolic trough solar collector are 3.5 m$^2$ and 1.46 m respectively [Dune, 1998]. Using Eq.(6) at wind velocity 10 m/s then the additional wind load Inertia is:

\[ J_{Load\_wind} = 525.82 \text{ Kg } \text{m}^2 \]  

(11)

4.5. The Load Torque Calculations:

The total load inertia is,

\[ J = J_{pipe} + J_{tube\_perpend} + J_{tube\_parallel} + J_{load\_collector} + J_{Load\_wind} \]

\[ = 0.0011 + 1.147 + 3.2435 + 3.675 + 525.82 \]

\[ J = 533.892 \text{ Kg } \text{m}^2 \]  

(13)

The load angular velocity and displacement respectively \(\omega_l\) and \(\theta_l\) can be described by means of move profile graphs, as shown in Fig. 2, which shows the trapezoidal move profile [Richard, 1995]. In Fig. 2, we can define \(t_a\) as the acceleration time, \(t_r\) as the maximum speed time (constant speed), \(t_d\) as the deceleration time, and \(t_s\) as the off time. With the assumption that \(t_a = t_r = t_d\), the peak angular velocity \(\omega_l\) of the tracking system to move an angular displacement \(\theta_l = 0.0174\) rad, in a time equals 1 sec is [Richard, 1995],

\[ \omega_l = 1.5 \frac{\theta_l}{t} = 0.0261 \text{ rad } / \text{sec} \]

\[ = 0.25 \text{ RPM} \]  

(14)

The peak acceleration \(\alpha\) [11],

\[ \alpha = 4.5 \frac{\omega_l}{t} = 0.1175 \text{ rad } / \text{sec}^2 \]  

(15)

From Eq. (2), the acceleration torque is,

\[ T_l = J\alpha = 62.732 \text{ N } \text{m} \]  

(16)

Selecting a gear-reducer ratio \(n = \frac{1}{1000}\) then the reflected acceleration torque is (assuming that the gear-reducer efficiency=75%),
The reflected deceleration torque equals to the reflected acceleration torque\cite{11}. The minimal required motor angular speed is,

\[ \omega_m = 250 \quad \text{RPM} \quad (18) \]

Then the motor HP can be calculated as follow\cite{Richard, 1995},

\[ \frac{T_i \omega_m}{716.2} \quad \text{HP} \]

\[ = 0.035 \quad \text{HP} \quad (19) \]

![Fig. 2: The trapezoidal move profile.](image)

5. Motor Selection:

The peak torque required by the application must fall within the peak torque rating of the motor at the peak speed. Figure 3 shows the obtained system speed graph based on the system design, while Fig. 4 shows the speed torque curve of the selected motor and the required system torque. The maximum required torque by the application is 0.1 N.m at \( \omega_m = 250 \quad \text{RPM} \), this falls within the speed/torque of the selected motor as shown in Fig. 4. The safety margin is more than 74% that is enough for the additional constant low friction, viscous and other disturbance torques, where 20% is minimum enough safety margin \cite{Richard, 1995}. The main selected motor parameters are listed in table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low speed torque</td>
<td>0.13 N.m</td>
</tr>
<tr>
<td>Back e.m.f constant, ( K_b )</td>
<td>( 50.6\times10^{-3} \text{ V/rad/sec.} )</td>
</tr>
<tr>
<td>Viscous friction coefficient, ( f_m )</td>
<td>( 3.2\times10^{-6} \text{ N.m/rad/sec.} )</td>
</tr>
<tr>
<td>Coulomb friction torque</td>
<td>0.007 N.m</td>
</tr>
<tr>
<td>Armature Inductance, ( L_a )</td>
<td>0.001 H</td>
</tr>
<tr>
<td>Inertia of the motor, ( J_m )</td>
<td>( 5.8\times10^{-7} \text{ Kg.m}^2 )</td>
</tr>
<tr>
<td>Armature resistance, ( R_a )</td>
<td>1.93 ( \Omega )</td>
</tr>
<tr>
<td>Motor Mass</td>
<td>0.49 Kg</td>
</tr>
</tbody>
</table>
6. Model for the proposed solar tracker:

The solar collectors (flat, parabolic or dish) are rigidly mounted on a pedestal, and the pedestal is rotated by the application of a torque by a dc motor to seek the sun. The dc motor model used in this study is shown in Fig. 5. Since this type of motor uses a permanent magnet to generate the magnetic field in which the armature rotates, the electrical circuit in the armature alone can model the motor.

In this simple model $R_a$ and $L_a$ indicate the equivalent armature coil resistance and inductance respectively and the voltage supplied by the power source $V_a$ (control signal). The Inertia $J$ and friction $f$ contains the motor, the solar collector and pedestal components.

The equivalent moment of inertia $J$ and equivalent viscous friction $f$ referred to the motor shaft respectively are [Benjamin, 2010],

$J = J_m + n^2 J_l$  \hspace{1cm} (20)

$f = f_m + n^2 f_l$  \hspace{1cm} (21)

where $J_l$ and $f_l$ are respectively, the moment of inertia and friction of the load on the output shaft.

The block diagram of single axis solar tracker is shown in Fig. 6. The total torque on the pedestal is the sum of the motor torque, the wind-torque input and load friction. Both of the wind disturbance load torque, $T_{dist}(t)$,
and the coulomb friction torque input, $T_c$, have been added to the system [Fu et. al., 1987, Knudsen M. and J. Grue, 1995]. Kirchoff’s voltage law leads to the following equation [Fu et. al., 1987],

$$ v_a = e_m + R_a i_a + L_a \frac{di_a}{dt} $$ \hspace{1cm} (22)

By examining the effect of the magnetic field in the motor, and realizing that magnetic flux is constant, we can arrive at the following two equations relating the torque $T_m$ and the motor speed output $\omega$ to the supplied current and voltage:

$$ e_m = K_b \omega $$ \hspace{1cm} (23)

$$ T_m = Ki_a $$ \hspace{1cm} (24)

Where $K_b$ is back e.m.f constant, volt/rad/sec, and $K$ is motor torque constant, N.m/A. The armature current produces the torque which is applied to the inertia and friction and the disturbance torque. Then,

$$ K i_a = j \frac{d^2 \theta_m}{dt^2} + f \frac{d \theta_m}{dt} + \tau_c + \tau_{\text{dist}} (t) $$ \hspace{1cm} (25)

$$ T_{\text{dist}} (s) $$

$$ \theta_i (s) $$

$$ K_b $$

$$ 1 \over (s^2 + s \omega_0) $$

$$ \theta_m^* (s) $$

$$ 1 \over (s^2 + f) $$

$$ K $$

$$ 1 \over s $$

$$ \theta_i (s) $$

By examining the effect of the magnetic field in the motor, and realizing that magnetic flux is constant, we can arrive at the following two equations relating the torque $T_m$ and the motor speed output $\omega$ to the supplied current and voltage:

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$$ T_{\text{dist}} (s) $$

$$ \theta_i (s) $$

$$ K_b $$

$$ 1 \over (s^2 + s \omega_0) $$

$$ \theta_m^* (s) $$

$$ 1 \over (s^2 + f) $$

$$ K $$

$$ 1 \over s $$

$$ \theta_i (s) $$

By examining the effect of the magnetic field in the motor, and realizing that magnetic flux is constant, we can arrive at the following two equations relating the torque $T_m$ and the motor speed output $\omega$ to the supplied current and voltage:

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$$ T_m = Ki_a $$ \hspace{1cm} (24)

Where $K_b$ is back e.m.f constant, volt/rad/sec, and $K$ is motor torque constant, N.m/A. The armature current produces the torque which is applied to the inertia and friction and the disturbance torque. Then,
Assuming that all initial conditions are zero, and taking the laplace transform of equations (22 to 25). By superposition, the response of the open system including the gear ratio \( n \) is,

\[
\theta_l = G_p(s) \ V_a(s) \pm G_d(s) \ T_{\text{dist}}(s)
\]  

where \( G_p(s) \) is the open loop transfer function from the control input, \( V_a(s) \), to the output, \( G_d(s) \) is the open loop transfer function from the wind disturbance input, \( T_{\text{dist}}(s) \), to the output. While \( \theta_l \) is load Angular position and \( \theta_r \) is angular position reference input. Also, \( \Theta^*_m(s) \) is the motor angular velocity.

Using eqs(21 to 24), \( G_p(s) \) and \( G_d(s) \) are given by [Benjamin, 2010],

\[
G_p(s) = \frac{\theta_l(s)}{V_a(s)} \bigg|_{T_{\text{dist}}=0} = \frac{nK_m}{s[JL_a s^2 + (L_a f + R_a J)s + R_a f + KK_b]},
\]

and

\[
G_d(s) = \frac{\theta_l(s)}{T_{\text{dist}}(s)} \bigg|_{V_a=0} = \frac{-n(R_a + L_a s)}{s[JL_a s^2 + (L_a f + R_a J)s + R_a f + KK_b]}
\]

The inductance \( L_a \) in armature circuit is usually small and may be neglected [3], then transfer functions have been given by Eqs. (27, 28) are reduced to,

\[
G_p(s) = \frac{\theta_l(s)}{V_a(s)} \bigg|_{T_{\text{dist}}=0} = \frac{nK}{s[R_a Js + R_a f + KK_b]} = \frac{nK_m}{s(\tau_m s + 1)},
\]

And

\[
G_d(s) = \frac{\theta_l(s)}{T_{\text{dist}}(s)} \bigg|_{V_a=0} = \frac{-nR_a}{s[R_a Js + R_a f + KK_b]} = \frac{-nK_d}{s(\tau_m s + 1)}
\]

where,

\[
K_m = \frac{K}{(R_a f + KK_b)} = \text{Motor gain constant}
\]

\[
\tau_m = \frac{R_a J}{(R_a f + KK_b)} = \text{Motor time constant}
\]

\[
K_d = \frac{R_a}{(R_a f + KK_b)} = \text{Disturbance gain constant.}
\]

The reduced open loop response is,

\[
\theta_l = \frac{nK_m}{s(\tau_m s + 1)} V_a(s) - \frac{nK_d}{s(\tau_m s + 1)} T_{\text{dist}}(s)
\]

Based on the open loop reduced transfer function, the reduced open loop block diagram is shown in Fig. 7.

Using the selected motor parameter in table 1, load inertia in equation (12 and 13) considering the gear ratio \( n \) and substitute in Equations (20, 21, 29, and 30) neglecting the load viscous friction, then,

\[
J = J_m + n^2 J_L = 5.378 \times 10^{-4} \text{Kg.m}^2
\]

\[
f = f_m + n^2 f_L \approx f_m \approx 3.2 \times 10^4 \text{N.m/\text{rad/sec.}}
\]
The plant and disturbance transfer functions will be as follows,

\[ G_p(s) = \frac{0.0197}{s(s + 0.401)} \] (37)

\[ G_d(s) = \frac{-0.746}{s(s + 0.401)} \] (38)

![Diagram](image_url)

**Fig. 7:** The reduced block diagram of the open loop one axis tracking system.

7. **Conclusion:**

For the perpendicular azimuth axis solar tracker, torque of the servomotor used for a solar tracker in Egypt climate is studied, as well as the selection criteria of a servomotor that is suitable to the solar tracker is performed. Also, the effect of the wind on the solar tracker in Egypt has been considered. The analysis showed that the wind loads is the main factor in the servomotor torque calculations. A special design has been developed based on Egyptian environment and it can be modified to work at any others countries that have another environment conditions. Finally, the solar tracker and disturbance transfer functions have been developed.

**Nomenclature**

- A: area, m\(^2\).
- D: width, m.
- e: motor back electromotive force, V.
- f: friction coefficient of the motor and load, N.m/rad/sec.
- F: force, N.
- J: moment of inertia, kg.m\(^2\).
- l: length, m.
- L: inductance, H.
- n: gear ratio.
- r: radius, m.
- R: resistance, \(\Omega\).
- T: torque, N.m.
- P: pressure, N/m\(^2\).
- V: voltage, V.
- W: weight, Kg.

**Greek letters:**

- \(\alpha\): angular acceleration, rad/sec\(^2\).
- \(\rho\): air mass, Kg.
- \(\omega\): angular velocity, rpm.
- \(\theta\): angular displacement, rad.
- \(\tau\): torque
- \(\nu\): air velocity, m/s.
Subscripts

a armature
d damping
f friction
i inner
j inertia
l load
m motor
o outer

References