Original Articles

Fuzzy-based Multi-Objective Optimal Placement of Unified Power Flow Controller

Meisam Shabani, Mohammad Reza Alizadeh Pahlavani

Department of Electrical Engineering, Islamic Azad University, Borujerd branch, Borujerd, Iran.

Malek-Ashtar University of Technology (MUT), Shabanlo St., Lavizan, Tehran, Iran.

ABSTRACT

This paper presents a fuzzy optimization method to enhance the Optimal Power Flow (OPF) considering Unified Power Flow Controller (UPFC). The OPF problem is formulated as a nonlinear programming optimization problem using multiobjective framework wherein the total generation fuel cost and active power losses are considered as objective functions of the optimization problem. In the proposed approach, firstly, the total fuel cost generation and active power losses are optimized individually in order to obtain the fuzzy membership functions of the objectives, then, the multi-objective problem is reformulated as a new standard nonlinear problem using the fuzzy sets theory and max-min operator. Finally, it is solved by nonlinear programming by means of discontinuous derivatives method. The simulation results of the IEEE 30-bus test system show the performance of the proposed method.

Key words: Fuzzy optimization, Optimal Power Flow, Decision Making, UPFC

Introduction

Previously, Carpentier (J. Carpentier, 1962) introduced a generalized, nonlinear programming formulation of the economic dispatch problem including voltage and other operating constraints. This formulation was afterward termed the Optimal Power Flow problem (OPF) (H.W. Dommel and W.F. Tinney, 1968). The most important purpose of the OPF is to reach the optimal steady state operation of a power system, which concurrently minimizes the value of a selected objective function and meets certain physical and operating constraints. Thus the power flow (PF) and economic dispatch (ED) calculations have been ideally merged into OPF problem. Nowadays any problem that includes the determination of the immediate “optimal” steady state of an electric power system is an OPF problem. Different classes of the OPF problems, tailored towards special-purpose applications, are defined by selecting different functions to be optimized, different sets of controls and different sets of constraints (H.H. Happ, 1977; B. Stott, et al., 1987; B.H. Chowdury, 1992).

In multi-objective OPF due to being poor collaboration among design objectives and poor resolution of design conflicts, relations are complex and system operator (decision maker) encounter more uncertainty in planning to satisfy system preferences. To get rid of these problems a fuzzy multi-objective optimization model is utilize. Fuzzy set theory represents an interesting tool to assist research in optimization techniques when vague relationships or incompatible measurements among model parameters limit the specification of model objective functions and constraints. Recently, fuzzy set theory has been successfully applied in solving power system optimization problems, because it provides a new approach to coordinating multiple conflicting objectives of the problem. In this paper, constraints are classified into two parts: soft constraints and hard constraints. The OPF problem is formulated with fuzzy objective and fuzzy soft constraints. In the other words, security considerations of the network are considered as fuzzy soft constraint. An efficient nonlinear programming with discontinuous derivatives (DNLP) method is then modified to solve this new formulation. The numerical results demonstrate that the fuzzy OPF can be equivalent to the crisp OPF where a feasible solution exists. When there is no feasible solution for the crisp OPF, the fuzzy OPF can obtain a more realistic solution that “evenly” distributes violations of the limits, rather than violate a single normal limit excessively (X. Wang, 1999).

Also, security and reliability are the major concerns in the deregulated and unbundled electricity supply industry due to the increased number of market participants and the changing demand patterns. Congestion management has been debated much for increasing competition electricity power generation in both pool and bilateral dispatch models (X. Wang, 1999). In this paper, congestion is corrected by corrective actions using FACTS devices. Indeed, while system security constraints are softened using fuzzy approach, in return, FACTS devices are considered in the network to facilitate power system security control by the system operator.

Many conventional optimization techniques were developed to solve the OPF problem; the most popular

Corresponding Author: Meisam Shabani, Department of Electrical Engineering, Islamic Azad University, Borujerd branch, Borujerd, Iran.
E-mail: meysam_shabani206@yahoo.com
approaches are linear programming, sequential quadratic programming, generalized reduced gradient method, and the Newton method. References (J.A. Momoh, et al., 1999; J.A. Momoh, et al., 1999; M. Huneault, F.D. Galiana, 1991) suggest a complete list of the most commonly used conventional optimization algorithms with consider to the OPF. Despite the fact that some of these techniques have excellent convergence characteristics and various among them are widely used in the industry, some of their drawbacks are (M.R. AlRashidia, M.E. El-Hawary, 2009):

1. Convergence to the global or local solution is highly dependant on the selected initial guess, i.e. they might converge to local solutions instead of global ones if the initial guess happens to be in the vicinity of a local solution.

2. Each technique is tailored to suit a specific OPF optimization problem based on the mathematical nature of the objectives and/or constraints.

3. They are developed with some theoretical assumptions, such as convexity, differentiability, and continuity, among other things, which may not be suitable for the actual OPF conditions.

A new category of computational intelligence tools has emerged to cope with some of the traditional optimization algorithms’ shortcomings. The main modern techniques include evolutionary programming (EP) (Y.R. Sood, 2007; W. Ongsakul, T. Tantimaporn, 2007), genetic algorithm (GA) (M. Todorovski, D. Rajicic, 2006; D. Devaraj, B. Yegnanarayana, 2005), evolutionary strategies (ES) (J. Kennedy, R.C. Eberhart, 2001), artificial neural network (NN) (M.C. Dondo, M.E. El-Hawary, 2003; R.S. Hartati, M.E. El-Hawary, 2001), simulated annealing (SA) (C.A. Roa-Sepulveda, B.J. Pavez-Lazo, 2003), ant colony optimization (ACO) (B. Gasbaoui, B. Allaoua, 2009) and particle swarm optimization (PSO) (M.R. AlRashidi, M.E. El-Hawary, 2009; Y. Valle, et al., 2008). Majority of these partly new developed tools mimic a certain natural phenomenon in its search for an optimal solution like species evolution (EP, GA, and ES), human neural systems (NN), thermal dynamics of a metal cooling process (SA), data processing and interpretation in human brain (FST), or social behavior (ACO and PSO). They have been successfully applied to a wide range of optimization problems in which global solutions are more preferred than local ones or when the problem has non-differentiable regions. But these methods have some drawbacks too, such as:

1. These methods require significantly large computations and are not efficient enough for real-time use energy management system. Hence, there is a need an alternative approach, which can quickly respond to changes of power system conditions in possible shortest time.

2. Implementation of these methods is difficult.

3. Intelligence methods generate a Pareto solution set and decision maker must selects best compromise solution through Pareto solutions by a decision making approach.

4. Intelligence methods are stochastic and can’t strictly figure on solutions optimality.

To handle mentioned problems fuzzy optimization approach as a mathematical method which don’t have problems of traditional mathematical optimization and yet it covers intelligent methods problems can be effective key in solving multi-objective optimization problems.

Some of its advantages expressed as follows:

1. All objectives and constraints are considered as fuzzy form, simultaneously.


3. Fuzzy optimization generate an optimal solution in the end of algorithm and so don’t need to a decision making approach.

The remainder of this paper is organized as follows: UPFC model is presented in section II. In section III, the proposed mathematical formulations of the multi-objective OPF are expressed in the form of a nonlinear programming problem concerning system’s physical and technical constraints. In section IV, solution approach of the fuzzy optimization framework is mentioned. In the next section, the IEEE 30-bus test system is studied to demonstrate effectiveness of the proposed scheme. Some relevant conclusions are drawn in the section VI.

**UPFC Model:**

The basic schematic of the UPFC is presented in Fig. 1. The power injection model of the UPFC is shown in Fig.2.

\[
P_{ss} = -b_s r V_i V_j \sin(\theta_i - \theta_j + \gamma) 
\]

(1)

\[
Q_{ss} = -b_s r V_i^2 (r + 2 \cos(\gamma)) + b_s r V_i V_j \cos(\theta_i - \theta_j + \gamma) 
\]

(2)

\[
P_{sr} = -P_{ss} 
\]

(3)

\[
Q_{sr} = b_s r V_i V_j \cos(\theta_i - \theta_j + \gamma) 
\]

(4)
here \( r \) is the radius of the UPFC operating region; \( \gamma \) is the UPFC phase angle; \( b_1 \) is \( I/(X_S+X_B) \) where \( X_S \) is the transmission line reactance and \( X_B \) is the series transformer leakage reactance (M. Noroozian, et al., 1997).

\[
\begin{align*}
\text{Fig. 1:} & \quad \text{Basic schematic diagram of UPFC.} \\
\text{Fig. 2:} & \quad \text{The power injection model of UPFC.}
\end{align*}
\]

**Problem Formulation:**

The economic optimal operation of power systems, considering transmission constraints and supplying load demand, requires to minimize two objective functions (total generation fuel cost and active power losses) while satisfying several equality and inequality constraints. Generally the optimal operation problem which named OPF can be formulated as follows.

**Objective Functions:**

- **Minimization of total generation fuel cost:** The generation cost function is represented by a quadratic polynomial function as follows (R. Palma-Behnke, et al., 2004):

  \[
  F_1 = \sum_{i=1}^{g} C_i(P_{Gi}) = \sum_{i=1}^{g} \alpha_{0i} + \alpha_{1i} P_{Gi} + \alpha_{2i} P_{Gi}^2 \quad (\$/h) \tag{5}
  \]

  where \( P_{Gi} \) is the real power generation of unit \( i \). Also, \( \alpha_{0i}, \alpha_{1i}, \alpha_{2i} \) are cost coefficients of unit \( i; \) \( g \) is the number of generators.

- **Active power losses:** The total power loss to be minimized is as follows (A. Navarro, et al., 2007):

  \[
  F_2 = F(V, \delta) = \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} V_{ij} \cos(\alpha_{ij} + \theta_i - \theta_j) \tag{6}
  \]

- **Maximization of the system loadability:** It is well known that voltage stability in power systems is closely related to maximum power transfer. Therefore, the calculation of loadability limits is a main point in voltage security assessment (A. Navarro, et al., 2007). Loadability limits can be explained as the operating points, where the load demand reaches a maximum value that can be served subject to system and operational constraints. Any attempt to increase load consumption beyond system constraints by means of either a gradual increase of demand, or a load restoration processes, will result in instability and collapse (R.V. Tappeta, et al., 2000). The objective function for calculation of the maximum loading of the system is of the form proposed in (H. Yano, 2009). The loadability index can be formulated as follows:

Maximize \( F_3 = \rho \)

Subject to

\[
\begin{align*}
& f_1(V, \theta) - P_{Gi} + \rho P_{Di} = 0 \\
& f_2(V, \theta) - Q_{Gi} + \rho Q_{Di} = 0
\end{align*}
\tag{7}
\]

It is important to mention that not only the power loads but also the power generations can increase. That is, the active power generations also are decision variables.
- Minimization of investment cost of UPFC: Optimal placement of FACTS devices considering the cost of FACTS installation has been mathematically formulated. The installation cost of UPFC is taken from (T. Shieh, et al., 2009) and shown as following:

\[ C_{\text{UPFC}} = 0.0003S^2 - 0.2691S + 188.22 \]  

(8)

The investment cost of UPFC device is in US$/kVAr. It must be converted into US$/Hour. Normally, the FACTS devices will be in-service for many years. However, only a part of its lifetime is employed to regulate the power flow. In this paper, five years is applied to evaluate the cost function. Therefore the average values of the investment costs are calculated using the following equation:

\[ F_4 = IC = C(f) \times S \times 1000 \text{ ($/h)} \]  

where

\[ C(f) = \frac{C_{\text{UPFC}}}{8760 \times 5} \]  

(10)

Constraints:

- Generation Real Power limits: The real power output limits of generator i are formulated as:

\[ P_{Gi}^{\text{min}} \leq P_i \leq P_{Gi}^{\text{max}} \quad \forall i \in NG \]  

(11)

- Voltage Control and Reactive Support: The voltage limits and reactive power output limits assuming constant power factor for loads can be expressed using following inequalities:

\[ Q_{Gi}^{\text{min}} \leq Q_i \leq Q_{Gi}^{\text{max}} \quad \forall i \in NG \]  

(12)

\[ |V_i|_{\text{min}} \leq |V_i| \leq |V_i|_{\text{max}} \quad \forall i \in n \]  

(13)

where \( Q_{Gi} \), \( Q_{Gi}^{\text{min}} \), \( Q_{Gi}^{\text{max}} \) are stand for reactive power output, maximum and minimum reactive limits of generating unit i, respectively. Also, \( |V_i| \), \( |V_i|_{\text{min}} \) and \( |V_i|_{\text{max}} \) are related to bus voltage, minimum and maximum limits of voltage of \( i \)th bus, respectively.

- Power balance equations: Real and reactive power balance equations of \( i \)th bus considering injected active and reactive power of FACTS devices can be expressed as (M. Noroozian, 1997):

\[ P_{Gi} + P_{FACTS} = P_{Di} + \sum_{j=1}^{n} Y_{ij} V_j Y_j \cos(\alpha_j + \theta_j - \theta_i) \quad \forall i, j \in n \]  

(14)

\[ Q_{Gi} + Q_{FACTS} = Q_{Di} + \sum_{j=1}^{n} Y_{ij} V_j Y_j \sin(\alpha_j + \theta_j - \theta_i) \quad \forall i, j \in n \]  

(15)

where, \( i = 1,2,\ldots,n \); and \( n \) is the number of buses, \( P_{Gi} \) and \( Q_{Gi} \) are the generated real and reactive power of unit located at bus \( i \), respectively; \( P_{Di} \) and \( Q_{Di} \) are the real and reactive power of load located at bus \( i \), respectively; \( P_{FACTS} \) and \( Q_{FACTS} \) are active and reactive power injected by FACTS devices to the specific bus, respectively.

Transmission constraints:

\[ |S_i| \leq |S_i|_{\text{max}} \quad \forall i \in NB \]  

(16)

where \( |S_i| \), \( |S_i|_{\text{max}} \) are stand for the apparent power flow and the capacity of \( l \)th transmission line.

UPFC constraints:

\[ 0 \leq r \leq r_{\text{max}} \]  

(17)

\[ -\pi \leq \gamma \leq \pi \]  

(18)

In (17) and (18) represent the limits of the parameters of UPFC, i.e. \( r \) and \( \gamma \), parameters, respectively.
4. FUZZY Multiobjective Algorithm:

4.1. Finding the Optimum Value of each Objective Function:

The optimization model to find the optimum value of each objective is given by (R.V. Tappeta, 2000):

\[
\text{Minimize } F_t(X), \quad t = 1, 2
\]

Subject to \( h_i(X) = 0 \quad i = 1, 2, \ldots, M \)
\( g_j(X) \leq 0 \quad j = 1, 2, \ldots, N \)
\( X_k^l \leq X_k \leq X_k^u \)

where, \( F_t(X) \) refers to the objective functions; Also, \( h_i(X) \) and \( g_j(X) \) are equality and inequality constraints. Finally, \( X_k \) is the \( k \)-th decision variable.

The solution of the above model is the optimum solution of each objective function, \( X_j^* \), and the optimal value of the objective function at the optimum solution, \( F_t^u \), can be written as:

\[
F_t^u = F_t(X_j^*) \quad (t = 1, 2)
\]

where, \( F_t^u \) is the optimum value of \( t \)-th objective function.

4.2. Fuzzy Multiobjective Optimization Model:

Due to the compromised nature of the solutions of multi-objective optimization problems, it is well-fitted to implement fuzzy decision making approach (H. Yano, 2009; T. Shieh, 2009). Because of the conflicting objective functions and the role of human decision on the final solution of the multiobjective optimization solutions, the fuzzy method can be implemented to solve the problem.

In the fuzzy set theory, membership functions are established to fuzzify the fuzzy sets. The membership function values vary between zero and one. The elements in a fuzzy set with membership value 1 reflect that they are in the core of the fuzzy set. The membership function value is zero for the element outside the fuzzy set. The elements with membership function value between zero and one construct the boundary of the fuzzy set. In order to use fuzzy set theory to solve the optimization problems, the fuzzy constraints have to be formed first. These constraints originated from the given crisp constraints by relaxing the bounds. A corresponding membership function is established to describe the fuzziness of each constraint. In addition to fuzzy constraints, fuzzy objective functions are also needed. Each objective function is converted into a pseudo-goal. A membership function is associated with the pseudo-goal. The pseudo-goal has membership function value one if the design is located at the optimum from the single-objective optimization problem with the same constraints for the multiobjective design. It is obvious that solving the multiobjective optimization problem is essential to simultaneously make all membership function values of the pseudo-goals as large as possible.

The above-mentioned procedure is summarized as follows:

(a) Finding the minimal and maximum feasible value of each objective function considering constraints:

\[
m_i = \min_{1 \leq l \leq n} F_l(X_i^*) = F_l(X_i^*)
\]
\[
M_i = \max_{1 \leq l \leq n} f_l(X_i^*)
\]

where, \( m_i \) and \( M_i \) are the minimum and maximum feasible value of \( i \)-th objective function, respectively. For three objective functions (\( F_1 \), \( F_2 \) and \( F_3 \), which refer to total fuel cost, active power losses and loadability index functions, respectively), payoff table should be performed as Table 1 to determine the range of each objective function, i.e. \( m_i \) and \( M_i \), as follows:

\[
m_i = F_i^*(X_i^*) \quad i = 1, 2, 3
\]
\[
M_i = \max_{j=1, 2, 3} \{ F_j(X_i^*) \} \quad i = 1, 2
\]
\[
M_3 = \min_{j=1, 2, 3} \{ F_3(X_i^*) \}
\]
Table 1: Payoff table

<table>
<thead>
<tr>
<th></th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(min $F_1, F_2, F_3$)</td>
<td>$F_1^<em>(X_1^</em>)$</td>
<td>$F_2^<em>(X_2^</em>)$</td>
<td>$F_3^<em>(X_3^</em>)$</td>
</tr>
<tr>
<td>($F_1, \min F_2, F_3$)</td>
<td>$F_1 (X_1^*)$</td>
<td>$F_2^<em>(X_2^</em>)$</td>
<td>$F_3^<em>(X_3^</em>)$</td>
</tr>
<tr>
<td>($F_1, F_2, \max F_3$)</td>
<td>$F_1 (X_1^*)$</td>
<td>$F_2 (X_2^*)$</td>
<td>$F_3^<em>(X_3^</em>)$</td>
</tr>
</tbody>
</table>

(b) Establishing the membership function of each fuzzy objective function: Most applications that involve fuzzy set theory have a tendency to be independent of the specific shape of the membership functions. For total generation fuel cost, active power losses and loadability functions, it is suitable to use a membership function with trapezoidal form, as shown in Fig. 3; the membership function is as follows:

$$
\mu_{F_i}(X) = \begin{cases} 
1, & F_i(X) \leq m_i, \\
\frac{M_i - F_i(X)}{M_i - m_i}, & m_i < F_i(X) < M_i, \quad (i = 1, 2) \\
0, & F_i(X) \geq M_i, \\
0, & F_i(X) \leq m_i, \\
\frac{F_i(X) - M_i}{m_i - M_i}, & m_i < F_i(X) < M_i, \quad i = 3 \\
1, & F_i(X) \geq M_i.
\end{cases}
$$

Fig. 3: (a) total fuel cost membership function, (b) active power losses membership function, (c) loadability membership function

(c) Establishing the membership function of each fuzzy constraint function: For simplicity, a linear membership function is used to reflect the smooth transition. Other types of the membership function can also be used depending on the problems under consideration. The linear membership function is given by:

$$
\mu_{g_i}(X) = \begin{cases} 
1, & g_j(X) \leq b_j, \\
\frac{[(b_j + d_j) - g_j]}{d_j}, & b_j < g_j(X) < b_j + d_j, \\
0, & g_j(X) \geq b_j + d_j.
\end{cases}
$$

where $b_j$ and $b_j + d_j$ form an allowable fuzzy transition interval for the $j^{th}$ inequality constraint. In this paper, constraints are classified into two parts: soft constraints (transmission power flow constraint, i.e. (16)) and hard constraints (such as active and reactive power balance, i.e equations (14) and (15), respectively). Therefore, the above membership function, (28), can be used to fuzzify power flow of lines, (16). In this regard, in the fuzzy optimization model, the power flow of a transmission line ($S$) can increase up to $1.1S^\text{max}_l$. That is,
\[ b_j = S_j^{\text{max}} \] and \[ b_j + d_j = 1.1S_j^{\text{max}}. \]

(d) Establishing fuzzy multiobjective optimization model:

Maximize \[ \lambda. \]

Subject to

\[ \lambda \leq \mu_{f_i}(X^i), \quad i = 1, 2 \]
\[ \lambda \leq \mu_{f_i}(X), \quad i = 1, 2, \ldots, I \]
\[ \lambda \leq \mu_{f_j}(X^J), \quad j = 1, 2, \ldots, J \]
\[ 1 \geq \lambda \geq 0 \]
\[ X^+_k \geq X_k \geq X^-_k, \quad k = 1, 2, \ldots, k \]

The flowchart of finding the best location for the UPFC in the network is illustrated in Fig. 4. In this flowchart UPFC is located in different lines and the mentioned model, (29), is executed. After searching all lines, the best line selected based on the fuzzy decision making approach (X. Liu, et al., 2010).

Case Study:

MATLAB that is linked with GAMS (T. Niknam, 2010) which handles nonlinear programming with discontinuous derivatives (DNLP) method to solve it. Three different objective functions are considered: total fuel cost and active power loss to be minimized and loadability of system to be maximized. Different cases are considered for these objective functions with and without UPFC device. In each case optimal settings of the UPFC and its best location is determined.

In this section total fuel cost, active power losses, loadability of system and investment cost of UPFC functions are shown with \( F_1, F_2, F_3 \) and \( F_4 \), respectively. Firstly, the single objective optimization problem for each objective function is simulated; then double combination of objective functions are studied \( (F_1 & F_2, F_1 & F_3, F_2 & F_3) \); finally, three objective functions \( (F_1 & F_2 & F_3) \) are modeled, simultaneously in the multi-objective optimization problem. It is noted that, in the case of using UPFC, its investment cost will be added to the fuel cost function. For the better illustration of results, three states are considered for each case as follows:

State 1: OPF without UPFC

State 2: OPF considering UPFC and without including investment cost of UPFC \( (F_4) \) in the fuel cost function.

State 3: OPF considering UPFC and including investment cost of UPFC \( (F_4) \) in the fuel cost function.

IEEE 30-bus test system [29] is used to show the capability of the UPFC to control power flow and improve operating condition of the power system while power flow constraints are satisfied. Table 2 shows the OPF results for single objective optimization of total fuel cost before and after installing an UPFC. Also single objective optimization results for active power losses and loadability functions are shown in tables 3 and 4, respectively.

Tables 5, 6, 7, 8 show the OPF results before and after installing an UPFC for the following four cases:

Case 1: Multi-objective optimization of total fuel cost and active loss \((F_1 & F_2)\); the results of this case are shown in Table 5.

Case 2: Multi-objective optimization of total fuel cost and loadability \((F_1 & F_3)\); the results of this case are shown in Table 6.

Case 3: Multi-objective optimization of active power losses and loadability \((F_2 & F_3)\); the results of this case are shown in Table 7.

Case 4: The last case is to optimize all three objective functions, simultaneously; the results of this case are shown in Table 8.

Conclusion:

This paper presents a new multiobjective optimal power flow method while UPFC is considered. The equations for the inclusion of the UPFC devices are presented with an appropriate circuit model. A mathematical model is presented with a fuzzy optimization formulation. The solution of the fuzzy problem through a nonlinear programming using DNLP method is presented. The method is tested on the IEEE 30-bus system and its results are presented. The results of implementing fuzzy approach shows that using multiobjective optimization problem leads to enhance flexible framework for solving optimal power flow problems while meeting the uncertainty of some parameters of the model. In other words, the system operator can relax some of its constraints and limitations using fuzzy approach. Therefore the optimization problem of
network operation can be solved with more feasible region of candidate solutions. Consequently, the proposed method with respect to the crisp optimization makes more flexible optimization solution for the operator in the system.

**Fig. 4:** Flowchart of fuzzy multiobjective optimization (FMO) of fuzzy multiobjective optimization

**Table 2:** Results of IEEE 30-bus system for total fuel cost optimization

<table>
<thead>
<tr>
<th>Objective Functions (F_i)</th>
<th>Total Fuel Cost ($/h)</th>
<th>( \Sigma P_{loss} ) (MW)</th>
<th>( \Sigma Q_{loss} ) (MVAR)</th>
<th>Loadability Index</th>
<th>Investment cost ($/h)</th>
<th>FACTS Size (MVA)</th>
<th>FACTS Location</th>
<th>FACTS Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>802.25</td>
<td>9.447</td>
<td>37.789</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>State 2</td>
<td>790.83</td>
<td>6.362</td>
<td>25.715</td>
<td>1</td>
<td>383.19</td>
<td>102.47</td>
<td>Line 2-5</td>
<td>( r=20.44 )</td>
</tr>
<tr>
<td>State 3</td>
<td>800.03</td>
<td>8.837</td>
<td>35.675</td>
<td>1</td>
<td>42.36</td>
<td>10</td>
<td>Line 2-5</td>
<td>( \gamma = 85.17 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( r=0.21 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \gamma = 72.59 )</td>
</tr>
</tbody>
</table>
Table 3: Results of IEEE 30-bus system for active power losses optimization

<table>
<thead>
<tr>
<th>Objective Functions ((F_2))</th>
<th>(\lambda)</th>
<th>Total Fuel Cost (($/h))</th>
<th>(\sum P_{loss}) ((MW))</th>
<th>(\sum Q_{loss}) ((MVAR))</th>
<th>Loadability Index</th>
<th>Investment cost (($/h))</th>
<th>FACTS Size ((MVA))</th>
<th>FACTS Location</th>
<th>FACTS Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>-</td>
<td>968.11</td>
<td>3.29</td>
<td>16.245</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>State 2</td>
<td>-</td>
<td>965.12</td>
<td>2.031</td>
<td>11.668</td>
<td>1</td>
<td>270.44</td>
<td>69.27</td>
<td>2-5</td>
<td>(r=0.137) (\gamma=66.12)</td>
</tr>
<tr>
<td>State 3</td>
<td>0.724</td>
<td>966.364</td>
<td>2.555</td>
<td>13.457</td>
<td>1</td>
<td>105.531</td>
<td>25.459</td>
<td>2-5</td>
<td>(r=0.05) (\gamma=85.959)</td>
</tr>
</tbody>
</table>

Table 4: Results of IEEE 30-bus system for loadability index optimization

<table>
<thead>
<tr>
<th>Objective Functions ((Case1))</th>
<th>(\lambda)</th>
<th>Total Fuel Cost (($/h))</th>
<th>(\sum P_{loss}) ((MW))</th>
<th>(\sum Q_{loss}) ((MVAR))</th>
<th>Loadability Index</th>
<th>Investment cost (($/h))</th>
<th>FACTS Size ((MVA))</th>
<th>FACTS Location</th>
<th>FACTS Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>-</td>
<td>1319.402</td>
<td>12.532</td>
<td>51.846</td>
<td>1.402</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>State 2</td>
<td>-</td>
<td>1354.43</td>
<td>13.306</td>
<td>55.979</td>
<td>1.44</td>
<td>42.36</td>
<td>10</td>
<td>27-30</td>
<td>(r=0.083) (\gamma=105.184)</td>
</tr>
<tr>
<td>State 3</td>
<td>0.968</td>
<td>1327.352</td>
<td>14.073</td>
<td>59.283</td>
<td>1.423</td>
<td>21.391</td>
<td>5.014</td>
<td>27-30</td>
<td>(r=0.042) (\gamma=90.396)</td>
</tr>
</tbody>
</table>

Table 5: Results of IEEE 30-bus system for total fuel cost and active power losses optimization

<table>
<thead>
<tr>
<th>Objective Functions ((F_3))</th>
<th>(\lambda)</th>
<th>Total Fuel Cost (($/h))</th>
<th>(\sum P_{loss}) ((MW))</th>
<th>(\sum Q_{loss}) ((MVAR))</th>
<th>Loadability Index</th>
<th>Investment cost (($/h))</th>
<th>FACTS Size ((MVA))</th>
<th>FACTS Location</th>
<th>FACTS Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>0.048</td>
<td>813.169</td>
<td>6.938</td>
<td>28.725</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>State 2</td>
<td>0.457</td>
<td>801.340</td>
<td>4.382</td>
<td>18.868</td>
<td>1</td>
<td>363.380</td>
<td>96.430</td>
<td>2-5</td>
<td>(r=0.191) (\gamma=81.805)</td>
</tr>
<tr>
<td>State 3</td>
<td>0.436</td>
<td>815.481</td>
<td>6.660</td>
<td>27.828</td>
<td>1</td>
<td>5.558</td>
<td>1.296</td>
<td>27-30</td>
<td>(r=0.01) (\gamma=90.396)</td>
</tr>
</tbody>
</table>

Table 6: Results of IEEE 30-bus system for total fuel cost and loadability index optimization

<table>
<thead>
<tr>
<th>Objective Functions ((Case2))</th>
<th>(\lambda)</th>
<th>Total Fuel Cost (($/h))</th>
<th>(\sum P_{loss}) ((MW))</th>
<th>(\sum Q_{loss}) ((MVAR))</th>
<th>Loadability Index</th>
<th>Investment cost (($/h))</th>
<th>FACTS Size ((MVA))</th>
<th>FACTS Location</th>
<th>FACTS Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>0.120</td>
<td>852.813</td>
<td>10.228</td>
<td>40.969</td>
<td>1.048</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>State 2</td>
<td>0.145</td>
<td>852.645</td>
<td>7.378</td>
<td>30.163</td>
<td>1.060</td>
<td>439.440</td>
<td>120.136</td>
<td>1-3</td>
<td>(r=0.197) (\gamma=84.021)</td>
</tr>
<tr>
<td>State 3</td>
<td>0.124</td>
<td>858.624</td>
<td>10.272</td>
<td>41.268</td>
<td>1.054</td>
<td>5.375</td>
<td>1.253</td>
<td>27-30</td>
<td>(r=0.01) (\gamma=50.655)</td>
</tr>
</tbody>
</table>

Table 7: Results of IEEE 30-bus system for active power losses and loadability index optimization

<table>
<thead>
<tr>
<th>Objective Function ((Case3))</th>
<th>(\lambda)</th>
<th>Total Fuel Cost (($/h))</th>
<th>(\sum P_{loss}) ((MW))</th>
<th>(\sum Q_{loss}) ((MVAR))</th>
<th>Loadability Index</th>
<th>Investment cost (($/h))</th>
<th>FACTS Size ((MVA))</th>
<th>FACTS Location</th>
<th>FACTS Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>0.150</td>
<td>1012.147</td>
<td>4.164</td>
<td>19.571</td>
<td>1.060</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>State 2</td>
<td>0.549</td>
<td>1002.464</td>
<td>2.491</td>
<td>13.274</td>
<td>1.052</td>
<td>274.114</td>
<td>70.304</td>
<td>2-5</td>
<td>(r=0.138) (\gamma=93.148)</td>
</tr>
<tr>
<td>State 3</td>
<td>0.166</td>
<td>1017.115</td>
<td>4.221</td>
<td>19.898</td>
<td>1.067</td>
<td>15.960</td>
<td>3.734</td>
<td>25-26</td>
<td>(r=0.020) (\gamma=57.449)</td>
</tr>
</tbody>
</table>

Table 8: Results of IEEE 30-bus system for total fuel cost, active power losses and loadability index optimization

<table>
<thead>
<tr>
<th>Objective Functions ((Case4))</th>
<th>(\lambda)</th>
<th>Total Fuel Cost (($/h))</th>
<th>(\sum P_{loss}) ((MW))</th>
<th>(\sum Q_{loss}) ((MVAR))</th>
<th>Loadability Index</th>
<th>Investment cost (($/h))</th>
<th>FACTS Size ((MVA))</th>
<th>FACTS Location</th>
<th>FACTS Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>0.131</td>
<td>858.454</td>
<td>11.325</td>
<td>44.921</td>
<td>1.053</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>State 2</td>
<td>0.117</td>
<td>850.017</td>
<td>10.139</td>
<td>40.651</td>
<td>1.049</td>
<td>109.175</td>
<td>26.372</td>
<td>2-5</td>
<td>(r=0.061) (\gamma=50.352)</td>
</tr>
<tr>
<td>State 3</td>
<td>0.133</td>
<td>856.853</td>
<td>10.892</td>
<td>43.269</td>
<td>1.052</td>
<td>9.873</td>
<td>2.305</td>
<td>24-5</td>
<td>(r=0.01) (\gamma=107.4)</td>
</tr>
</tbody>
</table>

References


