MODELING AND CALCULATION ERROR CAUSED BY FREE ELECTRONS IN THE IONOSPHERE OF THE DELAY EFFECTS ON GPS SIGNALS

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ABSTRACT
In some situations, the ionospheric effect has the main role in positions determination. This paper therefore reviewed the theory of propagation delay caused by some ionospheric constituents to derive maximum order of ionosphere delay. In this article we calculate the error caused by the charged particle in the ionosphere on the signals carrying the GPS (L1 and L2) and explain the errors in terms of plasma refractive index and electron density in ionosphere. Also we use the ionosphere plasma media index dependence of phase and magnetic and plasma frequency to explain the errors in this region and the phase shift of radio waves in passing through this environments. We will see that first order of this equation has a main rule in signal propagation.

Key words: GPS signals, ionosphere plasma Delay, Atmospheric Delay, charged particle.

Material and Method
We use the ionosphere plasma media index dependence of phase and magnetic and plasma frequency to explain the errors in the this region and the phase shift of radio waves in passing through this environments. We can see that first order of this equation has a main rule in signal propagation.

Introduction
The history of the Global Positioning System (GPS) can be said to begin from the year 1973. Until then, the U.S. Navy and the U.S. Air Forces had been studying navigation separately. But in 1973 a Joint Program Office was formed by the U.S. Department of Defence (DOD). The new system called GPS or NAVSTAR, was planned. The first prototype satellites were launched in 1978 (Pisacane, V.L. and M.M. Feen, 1974; Dodson, A.H., 1986; Hopfield, H.S., 2006; Wells, D.E., 2007; Black, H.D., 2001). The full operational capability was reached by the end of year 1994 (Abdalla, K.A., 1992; Hartmann, G.K. and R. Leitinger, 1984; Chamberlain, J.W., 1978; Dodo, J.D., 2008). Each GPS satellite transmits low power radio signals at two carrier frequencies, called L1 (1575.42 MHz) and L2 (1227.60 MHz). The presence of free electrons in the geomagnetic field causes a nonlinear dispersion of electromagnetic waves traveling through the ionized medium (Kamarudin, M.N. and M.Z. Mohd, 2005; Qinshu, H.E., 2006; Ross, W.T., 1982; Ahn, Y.W., 2006; Beutler, G. and M. Rothacher, 2002; Richardous, P., 1985).

4. Ionospheric Effects on GPS signals:

The ionosphere is a region containing ions and free electrons extending from approximately 60 Km to 2000 Km. The ions and electrons are present in roughly equal quantities, making the ionosphere charge neutral. Despite the name ionosphere, however, ions and electrons only constitute approximately 0.4% of the particles at a given altitude. The low atmospheric density in this region allows them to persist with out recombining for extended periods of time, creating sufficient quantities to delay GPS signals (Pisacane, V.L. and M.M. Feen, 1974; Dodson, A.H., 1986).

The ionosphere is composed of plasma that alters transiting radio waves in three ways. First, the wave group velocity decreases as (Hopfield, H.S., 2006):

\[ V_g = c \sqrt{1 - \frac{\alpha e^2}{\omega^2}} \]  

(1)

Where:

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is called the plasma frequency with a value of typically 1-10 MHz. A decreased group velocity yields a code delay. Second, the phase velocity increases as:
\[ V_\phi = \frac{c}{\sqrt{1 - \omega_p^2 / \omega^2}} \]  
which yields a phase advance. Third, plasma density irregularities with scale lengths of the Fresnel length scatter radio signals, which then add constructively and destructively at the receiver. In the ionosphere the ionization gives rise to a refractive index less than unity (Burnside 1971, Chamberlain 1978, Hartmann and Leitinger 1984). The refractive index of the ionospheric perturbation can be expressed as a power series in the inverse of the frequency, \( f \) (L1 and L2), and has the form (Dodo, J.D., 2008):
\[ n = \sum_{i=1}^{\infty} a_i / f^{(i+1)} \]  
where \( a_i \) the constants which depend on the state of the ionosphere, and \( f \) is the carrier frequency.

5. Refractivity in the Ionosphere:

The ionosphere has, by definition, a large number of charged particles, and is therefore a dispersive medium. This means that while the group index of refraction is greater than unity, the phase index of refraction is less than unity, causing the signal to bend counter-intuitively. Further, the bending and delay experienced by the signal in this region will be a function of frequency.

The full expression of index of refraction in the ionosphere, the Appleton formula, is derived by Davies and described more succinctly by Kloucher (Prilepin, M.T., 1986; Adegoke, A.S. and M.A. Onasanya, 2008):
\[ n^2 = 1 - \frac{X}{1 - iZ} - \frac{Y_L^2 + Y_T^2}{2(1 - X - iZ)} \pm \left[ \frac{Y_L^4 + Y_T^4}{4(1 - X - iZ)^2} \right]^{1/2} \]  
That defined:
\[ Z = \frac{f_i}{f} \]  
\[ X = \frac{N_e^2}{\varepsilon_0 m_0 \omega^2} = \frac{f_i^2}{f^2} \]  
\[ Y_L = \frac{eB}{m_0 \omega} = \frac{f_i}{f} \cos \theta \]  
\[ Y_T = \frac{eB}{m_0 \omega} = \frac{f_i}{f} \sin \theta \]  
\[ Y = \frac{f_i}{f} \]  
\[ B_i = B \cos \theta \]  
\[ B_T = B \sin \theta \]  

That \( f \) is the frequency of GPS signals, \( f_i \) is the electron-neutral collision frequency, \( f_p \) is the plasma frequency of the ionosphere, \( f_H \) is the electron gyro frequency and \( \Theta \) is the angle of the signal with respect to the Earths magnetic field \( N_e \) is the electron density of the ionosphere, \( \varepsilon_0 \) is the permittivity of free space, \( m_0 \) is the rest mass of an electron and \( B \) is the strength of the Earths magnetic field. Also the refractive index, \( n \), can be expressed (Satirapod, C. and P. Chalermwattanachai, 2005):
The propagation delays in signals transmitted by GPS satellites are estimated using range difference or Doppler counts. Each GPS satellite transmits two stable carrier frequencies at approximately 1575.42MHz and 1226.7MHz. Thus, by measuring Doppler counts at these two frequencies, and then mixing them, we can apply a first order ionospheric correction. Since, the transmission of the navigation signal, at the two L-band frequencies is provided, the magnitude of the delay can be calculated by comparing the two-frequency observation. In the pseudo-range method, the total ionospheric group delay (TGD) is given by (Skone, S. and V. Hoyle, 2005; Adogoke, A.S. and M.A. Onasanya, 2008):

\[ TGD = R / c + a_1 / f^2 + a_2 / f^3 + a_3 / f^4 \]  

(17)

Where \( R \) is the satellite receiver range, \( C \) is the velocity of light, and \( f \) is the carrier frequency. Neglecting the higher order terms in equation (23), TGD can be approximated by:

\[ TGD = R / C + a_1 / f^2 \]  

(18)

For a device receiving both L1 and L2 signals the only difference between the total group delays is caused by the frequency dependent ionospheric group delay i.e. TGD_{L1} and TGD_{L2}. The first order refraction effect for GPS signals can be estimated by combining the two frequencies (L1 and L2) and using the approximate square law of equation (24):

\[ D_t = TGD_{L2} - TGD_{L1} = a_1 / f^2_{L2} - a_1 / f^2_{L1} \]  

(19)

We can use equation (16) and table (1) to derive the entity of constant \( a_1 \):

\[ a_1 = 40.3 \ N_e \]  

(20)

that \( N_e \) is the number of electrons found in a one meter square column from the receiver to the satellite:

\[ N_e = \int n_e d\rho \]  

(21)

Therefore with using equations (14),(16),(19) we can write:

\[ D_t = \frac{40.3}{c} \left[ \frac{f^2_{L1} - f^2_{L2}}{f^2_{L1} f^2_{L2}} \right] \times \int n_e d\rho \]  

(22)

<table>
<thead>
<tr>
<th>constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( L_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( \lambda_1 )</td>
</tr>
<tr>
<td>3</td>
<td>( L_2 )</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>24 cm</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( m_e )</td>
<td>( 9.107 \times 10^{-31} \text{ Kg} )</td>
</tr>
<tr>
<td>( \varepsilon_0 )</td>
<td>( 8.86 \times 10^{-12} \text{ Nm}^2 / \text{c}^2 )</td>
</tr>
<tr>
<td>( e )</td>
<td>( 1.602 \times 10^{-19} \text{ c} )</td>
</tr>
</tbody>
</table>

Fig. 1: Electron density versus height of ionosphere in day and night (Cerruti, A.P.,).

Fig. 2: Various layers of the ionosphere and their predominant ion populations. The density in the ionosphere varies considerably, as shown[27].
From Figure 1 it is seen that the greatest electron density is in about the height 300-400 Km from Earth's. Also Figure 2 shows the ionospheric layers and the principle ions that compose each region. One important layer from the standpoint of Navigation and Communication customers is the F2 layer, where electron concentrations reach their highest values. At high latitudes there is another source of ionization called the aurora. The aurora is a display of lights caused by electrons and protons striking the atmosphere at high speed. The particles come from the magnetosphere and spiral down the magnetic field lines of the Earth. These particles produce a spectacular array of light, and when they strike the atmosphere they also produce ionization. Therefore in this region the greatest free electron density is about $2.4 \times 10^{12} \text{m}^{-3}$ causing the greatest error in ionosphere that equal to:

$$D_i = 25 \times 10^{-9} \text{sec}$$

(23)

6. Conclusion:

In this paper we present ionosphere delay experienced by two band frequencies GPS signals traveling during D,E,F and topside of ionosphere region. Ionosphere signal delays vary as a function of electron density. Maximum value of electron density is $2.4 \times 10^{12} \text{m}^{-3}$ in $F_2$ layer and causes highest order of delay, that's equal to $D_i = 25 \times 10^{-9} \text{sec}$.

7. Discussion:

As will be shown, the delay emissions in the atmosphere will be due to changes in speed of the reference signal and the traveled path by waves. Here we have also with formulas show that for calculated higher-order terms of delay emissions we can use the three frequencies of GPS systems and compare them with each other.

References