Journal of Applied Sciences Research, 8(2): 1103-1113, 2012
ISSN 1819-544X
This is a refereed journal and all articles are professionally screened and reviewed

ORIGINAL ARTICLES

On Structural, Number Theoretic and Fuzzy Relational Properties of Large Multi-Connected Systems of Feedback Fuzzy State Space Models (FFSSM's).

[1]Izaz Ullah Khan, [2]Tahir Ahamd, [3]Normah Maan

[123]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Skudai, Johar Darul Ta'zim, Malaysia.

ABSTRACT

Large systems consist of many functional components. The interconnectivity of the components determines the output performance of large complex systems. In the current study, a detailed treatise on a multi-connected feedback fuzzy state space model (FFSSM) of a dynamical system is given. The concept of a single FFSSM is generalized to a large connected system of FFSSM. The convexity and normality of the induced solution of a single FFSSM helps generalize these properties to large connected systems of FFSSM. Likewise, the application of the Modified Optimized Defuzzified Value Theorem and Extended Modified Optimized Defuzzified Value Theorem help in deriving two important theorems for determining the optimal inputs in multi-connected systems of FFSSM. The BIBO stability of the multi-connected system of FFSSM's is also studied. Another two important parts of the study are the application of the number theoretic concepts and fuzzy relational calculus in the study of the physical properties of the FFSSM's.

Key words: Feedback Fuzzy State Space Modeling (FFSSM), Multi-connected Systems, Number Theory, Fuzzy Relational Calculus.

Introduction

The visual display of large systems with the help of blocked diagram helps in understanding the operational process (Ogata, 1997). The dynamic behavior of a system can be assessed by a blocked diagram, but it lacks the physical applicability of the system behavior. In order to reduce the complexity, a large multi-connected system is decomposed into subsystems. The system behavior is studied on the basis of subsystems and a conclusion is drawn about the overall behavior of the system. In composition, the properties of an FFSSM can be used to examine the properties of large multi-connected systems of FFSSM's.

Ahamd et al., (1997, 1998, 2004) used a fuzzy optimized algorithm for minimizing the cross talk in multi-connected mirco-strip lines. The idea was enhanced by Razidah et al., (2002a, 2002b, 2004, 2005a, 2005b, 2009) to propose a Fuzzy State Space Model (FSSM) for a dynamical system. A preliminary idea of the graph theoretic and fuzzy relational approach of the feed forward fuzzy state space model was given in Razidah (2005a, 2005b) and Khairi (2007). Harish (2010) presented a graph theoretic approach to the FSSM. In this paper the concept of a multi-connected system of fuzzy state space model is enhanced to a multi-connected system of feedback fuzzy state space modeling (FFSSM).

A single FFSSM can be represented as a blocked diagram having multiple inputs and outputs. Large complicated systems of FFSSM can be formed by interconnecting individual subsystems of FFSSMs. The characteristics and properties of the single FFSSM are preserved in a multi-connected system of FFSSM. The definition of a single system of FFSSM is given below.

(D1) Definition: (Razidah (2005a, 2005b)) A Feedback Fuzzy State Space Model (FFSSM) of a dynamical system is defined as

$$S_{gF}: \quad \begin{aligned} \dot{x}(t) &= Ax(t) + B\tilde{u}(t) \\ \tilde{y}(t) &= Cx(t) + D\tilde{u}(t) \end{aligned}$$

Here $\tilde{u}(t) = [u_1, u_2, ..., u_n]^T$ and $\tilde{y}(t) = [y_1, y_2, ..., y_m]^T$ represent the fuzzified input and fuzzified output vectors with initial conditions $t_0 = 0$ and $x_0 = x(t_0) = 0$. T is the matrix transposition. The matrixes $A_{p\times p}$, $B_{p\times m}$, $C_{m\times p}$, $D_{p\times n}$ are the state, input, output and feedback transmission matrixes respectively.

Corresponding Author: Izaz Ullah Khan, Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Skudai, Johar Darul Ta'zim, Malaysia.
E-mail: tahir@utm.my, izaz1982@yahoo.co.uk

Jamshidi (1997) described a multi-connected, time invariant system with N subsystems as given below.

$$\begin{aligned}
\dot{x}_1 &= A_1x_1 + B_1u_1 + g_1(x)\\
\dot{x}_2 &= A_2x_2 + B_2u_2 + g_2(x)\\
\dot{x}_3 &= A_3x_3 + B_3u_3 + g_3(x)\\
&\;\;.\quad\;\; .\quad\;\; .\quad\;\; .\\
&\;\;.\quad\;\; .\quad\;\; .\quad\;\; .\\
&\;\;.\quad\;\; .\quad\;\; .\quad\;\; .\\
\dot{x}_N &= A_Nx_N + B_Nu_N + g_N(x)
\end{aligned}$$

with $g_i(x) = \sum_{j=1, j\neq 1}^{N} A_{ij}x_j$

where $x_i \in \Re^{n_i}, x_i \in \Re^{n_i}$ and $g_i(x) \in \Re^{n_i}$ are the state, control/input and interaction matrices for the subsystem "i" respectively. A_i, B_i and A_{ij} are the constant matrices to be determined. The pair (A_i, B_i) is controllable. The system is further reduced to, $\dot{x}_1 = Ax + Bu + Hx$

where $\dot{x} = [x_1^T, x_2^T, x_3^T, ..., x_N^T], u = [u_1^T, u_2^T, u_3^T, ..., u_N^T]$ and $H = \begin{bmatrix} 0 & A_{12} & & ... & A_{1N} \\ A_{21} & 0 & & ... & A_{2N} \\ . & & & & . \\ . & & 0 & & . \\ . & & & & . \\ A_{N1} & ... & & ... & 0 \end{bmatrix}$ are the overall state, inputs and interconnection matrix.

In this study, multi-connected systems are classified on the basis of the input sources, that is either the inputs are from one source directly (Feed Forward) or they are from multi-connected subsystems.

(D 2) Definition: A feed forward multi-connected fuzzy state space model is a system SgF_1 consisting of the subsystems $SgF_{11}, SgF_{21}, SgF_{31}, ..., SgF_{n1}, i = 1, 2, 3, ..., n$ such that inputs of $SgF_{(i+1)1}$ are outputs of SgF_{i1}.

(D 3) Definition: A feedback multi-connected fuzzy state space model is a system SgF_2 consisting of the subsystems $SgF_{12}, SgF_{22}, SgF_{32}, ..., SgF_{n2}, i = 1, 2, 3, ..., n$ such that inputs of SgF_{c2} are outputs of SgF_{i2}, $c = 1, 2, 3, ..., n$.

Figure 1 and 2 shows two large multi-connected systems on the basis of definitions 1 and 2 respectively.

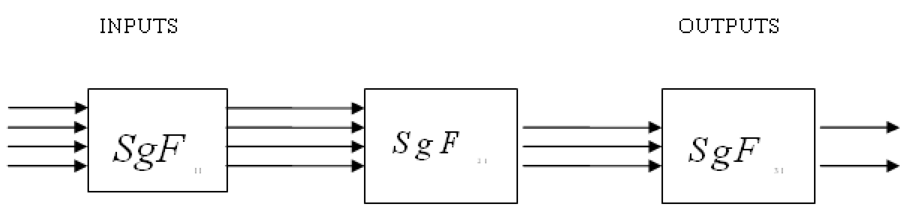

Fig. 1: A Feed Forward Multi-Connected System of Fuzzy State Space Models.

The paper is organized as follows. After the introduction, we discuss the structural properties of the FFSSM such as complexity, normality, and stability. The study of these properties results in some theorems and their proofs. This is followed by a number theoretic approach of the FFSSM. Subsequently we present the fuzzy relational approach for the FFSSM. Finally a summary of the findings and contributions is given in conclusions.

2 Properties of FFSSM in Large Multi-Connected Systems:

Each subsystem in a large multi-connected system exhibits characteristics of a single FFSSM. Thus all the theorems related to an individual FFSSM can be enhanced and generalized to a large multi-connected system of FFSSM based on definitions (D 2) and (D 3). These theorems are concerned with the normality, convexity, extended modified optimized defuzzified value theorem and BIBO stability of a large multi-connected system of FFSSM's.

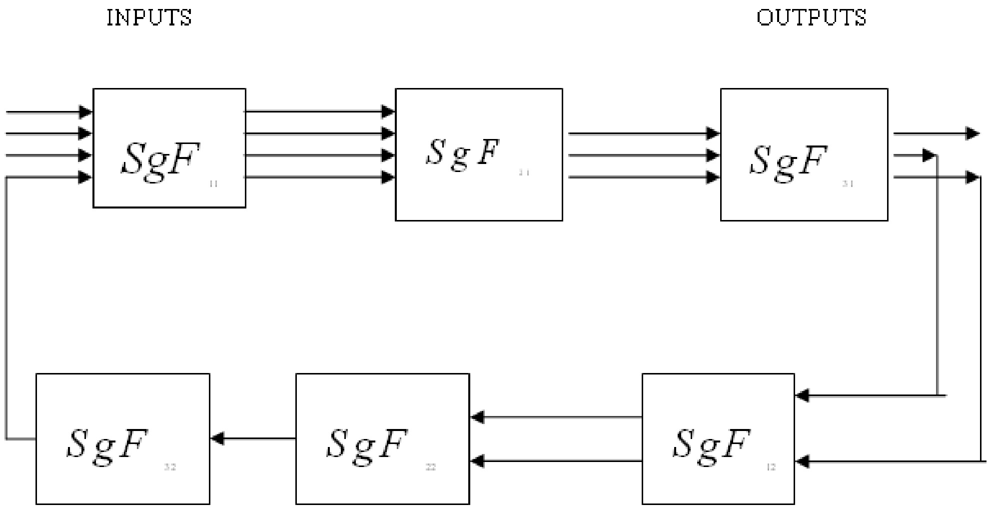

Fig. 2: A Feedback Multi-Connected System of Fuzzy State Space Models.

2.1 Convexity and Normality of Induced Solution of FFSSM's in Large Multi-Connected Systems:

The induced solution of each FFSSM in a large multi-connected system is normal and convex provided that the inputs are normal and convex Razidah (2005a). The related theorems are given below.

Theorem 1:

For a Feedback Fuzzy State Space Model $SgF : \Re^n \to \Re^m$ composing of the feed forward response $SgF_1 : \Re^n \to \Re^m$ and the feedback response $SgF_2 : \Re^n \to \Re^m$ with $SgF = SgF_1 + SgF_2$ and

$$\begin{matrix} (r) = SgF_1(u_{11}, u_{21}, ..., u_{n1}) \\ (d) = SgF_2(u_{12}, u_{22}, ..., u_{n2}) \end{matrix} \quad \text{and} \quad (r+d) = SgF(u_1, u_2, ..., u_n)$$

with $r = (r_1, r_2, ..., r_m)$ and $d = (d_1, d_2, ..., d_m)$

Given that $u_{i1}, u_{i2} \in I_{i1}, I_{i2} \subset \Re\,(i_1, i_2 \in \aleph)$ are convex then the induced solutions of SgF_1 and SgF_2 are convex which implies that SgF is convex.

Proof:

Suppose the performance parameter consists of a sequence of fuzzy state space models $SgF_{11}, SgF_{21}, SgF_{31}, \cdots, SgF_{n1}$ and $SgF_{12}, SgF_{22}, SgF_{32}, \cdots, SgF_{n2}$, $i = 1, 2, 3, ..., n$ as the feed forward and feedback responses based on definition (D 2 and D 3).

Since all the inputs $u_{i1}, u_{i2} \in I_{i1}, I_{i2} \subset \Re\,(i_1, i_2 \in \aleph)$ for the feed forward and feedback responses respectively are convex.

Now by Theorem 4.1, Razidah (2005a), the induced solution of SgF_{11} is convex. This induced solution becomes an input to SgF_{21} resulting in convex induced solution. The convex solution of SgF_{21} becomes input to SgF_{31} and so on. So the induced solution of SgF_1 is convex.

The same procedure applies to the feedback response. Given that the initial inputs to SgF_{12} is convex. By Theorem 4.1, Razidah (2005a), and definition (D 3) the outputs of SgF_{i2} becomes input of SgF_{c2}, $c = 1, 2, 3, ..., n$, $i = 1, 2, 3, ..., n$ which are convex, so the induced solution of SgF_2 is convex.
Since the induced solution of SgF_1 and SgF_2 are convex. So the system of SgF is convex.

Theorem 2:

For a multi connected system of Feedback Fuzzy State Space Models $SgF : \Re^n \to \Re^m$ composing of the feed forward response $SgF_1 : \Re^n \to \Re^m$ and the feedback response $SgF_2 : \Re^n \to \Re^m$ with $SgF = SgF_1 + SgF_2$ and

$(r) = SgF_1(u_{11}, u_{21}, ..., u_{n1})$ and $(r + d) = SgF(u_1, u_2, ..., u_n)$
$(d) = SgF_2(u_{12}, u_{22}, ..., u_{n2})$

with $r = (r_1, r_2, ..., r_m)$ and $d = (d_1, d_2, ..., d_m)$

Given that $u_{i1}, u_{i2} \in I_{i1}, I_{i2} \subset \Re\,(i_1, i_2 \in \aleph)$ are normal then the induced solutions of SgF_1 and SgF_2 are normal and so the induced solution of SgF is normal.

Proof:

Given that the inputs $u_{i1}, u_{i2} \in I_{i1}, I_{i2} \subset \Re\,(i_1, i_2 \in \aleph)$ for the feed forward and feedback responses in a multi-connected system are normal. To prove that the induced solution of SgF_1 and SgF_2 are normal.
For the feed forward response there exist $u_{i1}^* \in u_{i1}$ such that $\mu(u_{i1}^*) = 1$ for all $i = 1, 2, 3, ..., n$.
Let $y_1 = SgF_1(u_{11}^*, u_{21}^*, ..., u_{n1}^*)_{\alpha=1}$, so $\mu(y_1) = \mu(SgF_1(u_{11}^*, u_{21}^*, ..., u_{n1}^*)_{\alpha=1}) = 1$.
Showing that the induced solution of SgF_1 is normal.
For feedback response there exist $u_{i2}^* \in u_{i2}$ such that $\mu(u_{i2}^*) = 1$ for all $i = 1, 2, 3, ..., n$.
Let $y_2 = SgF_2(u_{12}^*, u_{22}^*, ..., u_{n2}^*)_{\alpha=1}$, so $\mu(y_2) = \mu(SgF_2(u_{12}^*, u_{22}^*, ..., u_{n2}^*)_{\alpha=1}) = 1$.

Showing that the induced solution of SgF_2 is normal. The normality of SgF_1 and SgF_2 implies the normality of SgF.

2.2 Optimization of the Inputs of FFSSM's in Large Multi-Connected Systems:

In order to get the optimal values of the inputs in a large multi-connected system of FFSSM, the Modified Optimized Defuzzified Values Theorem Razidah (2005a) is enhanced. In case of a MISO system Modified Optimized Defuzzified Values Theorem is used to determine the optimal inputs whereas in the case of a MIMO system Extended Modified Optimized Defuzzified Values Theorem is employed for getting the optimal values of inputs.

Theorem 3:

Given that a large multi-connected system of FFSSM's $SgF : \Re^n \to \Re$ consisting of the feed forward response $SgF_1 : \Re^n \to \Re$ and the feedback response $SgF_2 : \Re^n \to \Re$ with $SgF = SgF_1 + SgF_2$ so that

$$(r) = SgF_1(u_{11}, u_{21}, ..., u_{n1}) \\ (d) = SgF_2(u_{12}, u_{22}, ..., u_{n2}) \quad \text{and} \quad (r+d) = SgF(u_1, u_2, ..., u_n)$$

Provided that the preference feed forward and feedback responses are given then Theorem 4.3, Razidah (2005a) holds.

Proof:

Suppose a large multi-connected system SgF is composed of feed forward response SgF_1 and feedback response SgF_2 where these systems are connected by definitions (D 2) and (D 3). Given that the preferences for the feed forward and feed back responses are given then the application of Theorem 4.3, Razidah (2005a) helps the process of defuzzification and selection of optimal values of the input parameters.

Theorem 4:

For a Feedback Fuzzy State Space Model $SgF : \Re^n \to \Re^m$ composing of the feed forward response $SgF_1 : \Re^n \to \Re^m$ and the feedback response $SgF_2 : \Re^n \to \Re^m$ with $SgF = SgF_1 + SgF_2$ and

$$(r) = SgF_1(u_{11}, u_{21}, ..., u_{n1}) \\ (d) = SgF_2(u_{12}, u_{22}, ..., u_{n2}) \quad \text{and} \quad (r+d) = SgF(u_1, u_2, ..., u_n)$$

with $r = (r_1, r_2, ..., r_m)$ and $d = (d_1, d_2, ..., d_m)$

Provided that the preference feed forward and feedback responses are given then Theorem 4.4, Razidah (2005a) holds.

Proof:

Consider a large multi-connected system SgF composed of feed forward response SgF_1 and feedback response SgF_2 where these systems are connected by definitions (D 2) and (D 3). Given that the preferences for the feed forward and feed back responses are given then the application of Theorem 4.4, Razidah (2005a) helps the process of defuzzification and selection of optimal values of the input parameters.

2.3 BIBO Stability of FFSSM's in Large Multi-Connected Systems:

If every bounded input has a bounded output, the system is BIBO stable. Theorem 4.5, Razidah (2005a) deals with the BIBO stability of FFSSM. This theorem is revisited in the following to accommodate a multi-connected system of FFSSM.

Theorem 5:

For a Feedback Fuzzy State Space Model $SgF : \Re^n \to \Re^m$ composing of the feed forward response $SgF_1 : \Re^n \to \Re^m$ and the feedback response $SgF_2 : \Re^n \to \Re^m$ with $SgF = SgF_1 + SgF_2$ and

$$(r) = SgF_1(u_{11}, u_{21}, ..., u_{n1})$$

$$(d) = SgF_2(u_{12}, u_{22}, ..., u_{n2}) \quad \text{and} \quad (r+d) = SgF(u_1, u_2, ..., u_n)$$

with $r = (r_1, r_2, ..., r_m)$ and $d = (d_1, d_2, ..., d_m)$

Given that fuzzy inputs of the multi-connected system are bounded, then Theorem 4.5 Razidah Razidah (2005a) holds.

Proof:

Consider a large multi-connected system SgF composed of feed forward response SgF_1 and feedback response SgF_2 where these systems are connected by definitions (D 2) and (D 3). Provided that fuzzy inputs of the multi-connected system are bounded, then a repeated application of the Theorem 4.5, Razidah (2005a) proves the BIBO stability of the system.

3 A Number Theoretic Approach to the Multi-Connected System of FFSSM's:

In this section the fundamental concepts of number theory are applied to a large multi-connected system of FFSSM's. Razidah (2005a, 2005b) and Khairi (2007) presented a preliminary overview of the application of number theory to a feed forward multi-connected system of fuzzy state space model. In this study, the concepts of FSSM are enhanced to accommodate a feedback multi-connected system of fuzzy state space model. The basic definitions of the multi-connected feed forward and feedback system (D 2) and (D 3) remains the same.

The basic concepts of the divisors, common divisors and greatest common divisors play an important role in this study (Hillman and Alexanderson, 1994; Gilbert and Gilbert, 2000). An integer "c" is a common divisor of the integers "a" and "b' if it divides both "a" and "b" written as "c/a" and "c/b". If another integer "d" is a greatest divisor dividing "a" and "b", then "d" is the greatest common divisor denoted as d= gcd(a, b). These common definitions from the number theory are used to give some new definitions applied to physical system modeling.

(D 4) Definition: Let SgF_{11} and $SgF_{21} \in SgF_1$ be the performance parameters concerning with the feed forward response then SgF_{11} is said to be a feeder of SgF_{21} denoted as $SgF_{11}\big/SgF_{21}$ if and only if the outputs of SgF_{11} are the inputs of SgF_{21}. In the same way, if SgF_{12} and $SgF_{22} \in SgF_2$ be the performance parameters concerning with the feedback response then SgF_{12} is said to be a feeder of SgF_{22} denoted as $SgF_{12}\big/SgF_{22}$ if and only if the outputs of SgF_{12} are the inputs of SgF_{22}.

(D 5) Definition: Let SgF_{11} and $SgF_{21} \in SgF_1$ be the performance parameters concerning with the feed forward response then S_{gFF} is said to be a common feeder of SgF_{11} and SgF_{21} if and only if $S_{gFF}\big/SgF_{11}$ and $S_{gFF}\big/SgF_{21}$.In the same way, if SgF_{12} and $SgF_{22} \in SgF_2$ be the performance parameters concerning with the feedback response then S_{gFB} is said to be a common feeder of SgF_{12} and SgF_{22} if and only if $S_{gFB}\big/SgF_{12}$ and $S_{gFB}\big/SgF_{22}$.

(D 6) Definition: Let SgF_{11} and $SgF_{21} \in SgF_1$ be the performance parameters concerning with the feed forward response with S_{gFF} as the common feeder then S_{GFF} is said to be a greatest common feeder of SgF_{11} and SgF_{21} denoted as $S_{GFF} = \gcd(SgF_{11}, SgF_{21})$ if

1. $S_{gFF}\big/SgF_{11}$ and $S_{gFF}\big/SgF_{21}$.
2. $S_{GFF}\big/SgF_{11}$ and $S_{GFF}\big/SgF_{21} \Rightarrow S_{GFF}\big/S_{gFF}$.

In the same way, if SgF_{12} and $SgF_{22} \in SgF_2$ be the performance parameters concerning with the feedback response with S_{gFB} as a common feeder then S_{GFB} is said to be a greatest common feeder of SgF_{12} and SgF_{22} denoted as $S_{GFB} = \gcd(SgF_{12}, SgF_{22})$ if

1. $S_{gFB}\big/SgF_{12}$ and $S_{gFB}\big/SgF_{22}$.
2. $S_{GFB}\big/SgF_{12}$ and $S_{GFB}\big/SgF_{22} \Rightarrow S_{GFB}\big/S_{gFB}$.

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Feeder in this approach is denoting an input source for the system. The Figure 6.3 show that SgF_{11} is a feeder of SgF_{21} denoted as SgF_{11}/SgF_{21} for the feed forward response whereas SgF_{12} is a feeder of SgF_{22} denoted as SgF_{12}/SgF_{22} for the feedback response.

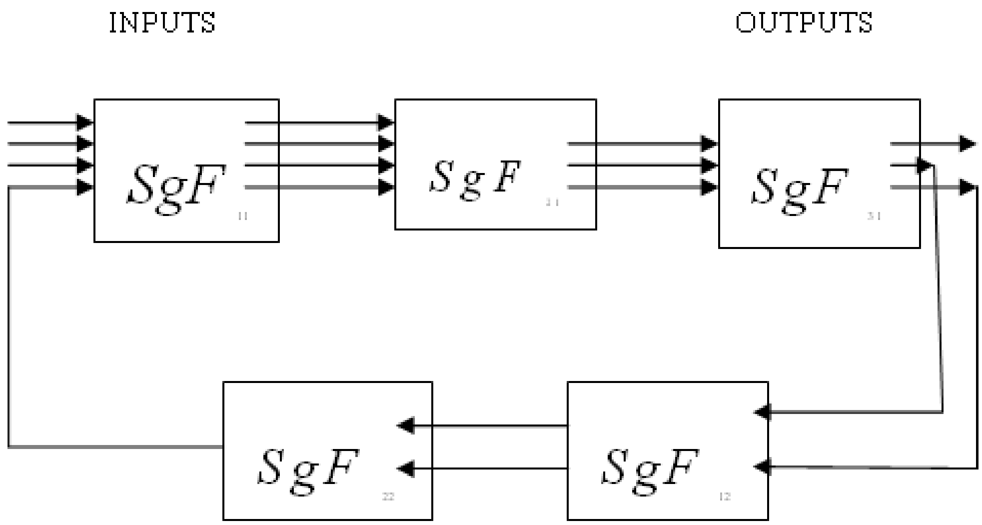

Fig. 3: The Concept of Feeder for the Feed Forward and Feedback Responses.

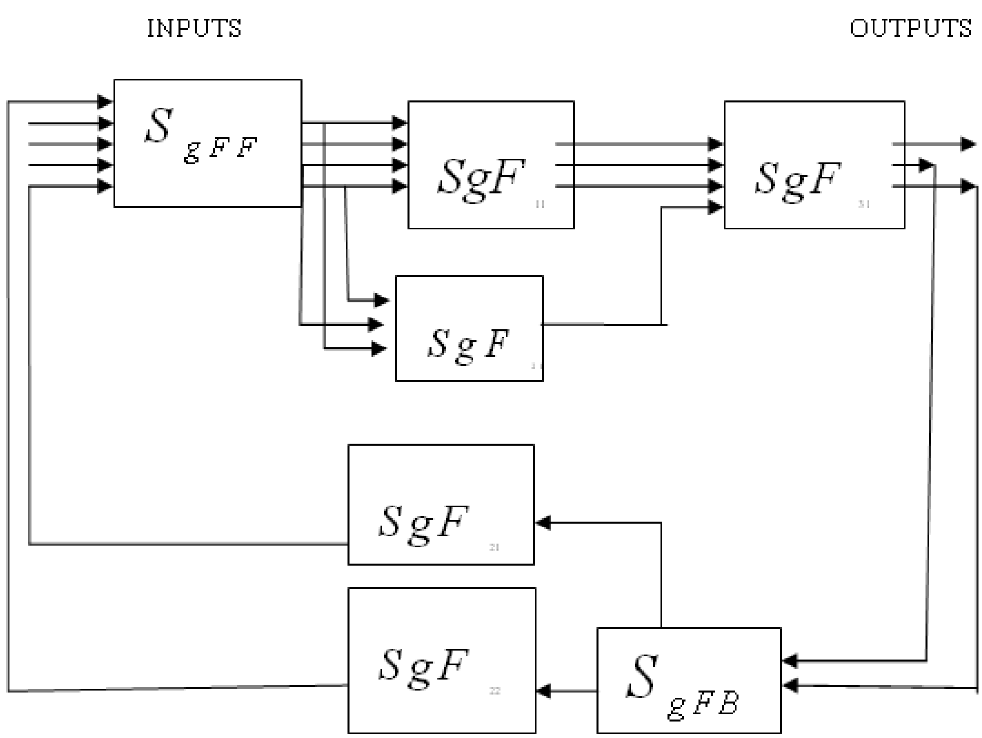

Fig. 4: Greatest Common Feeder for the Feed Forward and Feedback Responses.

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Figure 4 shows that S_{gFF} is a common feeder of SgF_{11} and SgF_{21} for the feed forward response whereas S_{gFB} is a common feeder of SgF_{12} and SgF_{22} for the feedback response.

Figure 5 shows that S_{GFF} is a greatest common feeder of SgF_{11} and SgF_{21} for the feed forward response and S_{GFB} is a greatest common feeder of SgF_{12} and SgF_{22} for the feedback response.

4 A Fuzzy Relational Approach to the Multi-Connected System of FFSSM's:

In this section, some structural properties of a multi-connected system of FFSSMs are studied using the basic idea of fuzzy relations. A fuzzy relation " $\tilde{R}$ " is a subset of $X \times Y$ mapping members of universe "X" on a universe "Y" by the Cartesian product of two universes. The degree of membership expresses the strength of association of the relation and is defined and denoted as below.

$$\tilde{R} = \left\{((x, y), \mu_{\tilde{R}}(x, y)) \text{ where } (x, y) \in X \times Y\right\}$$

Let $\tilde{A} \subseteq X$ and $\tilde{B} \subseteq Y$, the Cartesian product of $\tilde{A} \times \tilde{B}$ generate a fuzzy relation " $\tilde{R}$ " defined and denoted as $\mu_{\tilde{R}}(x, y) = \mu_{\tilde{A} \times \tilde{B}}(x, y) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))$.

Assume that SgF_{1} consisting of the subsystems $SgF_{11}, SgF_{21}, SgF_{31}, \cdots, SgF_{n1}$ whereas SgF_{2} is consisting of all the subsystems $SgF_{12}, SgF_{22}, SgF_{32}, \cdots, SgF_{n2}$ concerning feedback responses. In this case " $\tilde{R}_1$" represents the degree of association of two subsystems denoted as $\left(SgF_{11} \tilde{R}_1 SgF_{21}\right)$ or $SgF_{11} \tilde{R}_1 SgF_{21}$ and is an ordered pair on $\left(SgF_{1} \times SgF_{1}\right)$. Similarly $\left(SgF_{11} \tilde{R}_1' SgF_{21}\right)$ denotes the degree of non association of the two subsystems on the domain $\left(SgF_{1} \times SgF_{1}\right)$. It must be noted that " $\tilde{R}_1$" is not symmetric and not an equivalence relation. Then the association can be represented as follows.

$$\begin{array}{c|ccccc}
\tilde{R}_1 & SgF_{11} & SgF_{21} & SgF_{31} & \cdots & SgF_{n1} \\
\hline
SgF_{11} & 1 & \min(\mu_{(SgF_{11})}, \mu_{(SgF_{21})}) & \min(\mu_{(SgF_{11})}, \mu_{(SgF_{31})}) & \cdots & \min(\mu_{(SgF_{11})}, \mu_{(SgF_{n1})} \\
SgF_{21} & \min(\mu_{(SgF_{21})}, \mu_{(SgF_{11})}) & 1 & \min(\mu_{(SgF_{21})}, \mu_{(SgF_{31})}) & \cdots & \min(\mu_{(SgF_{21})}, \mu_{(SgF_{n1})} \\
SgF_{31} & \min(\mu_{(SgF_{31})}, \mu_{(SgF_{11})}) & \min(\mu_{(SgF_{31})}, \mu_{(SgF_{21})}) & 1 & \cdots & \min(\mu_{(SgF_{31})}, \mu_{(SgF_{n1})}) \\
. & . & & & & \\
. & . & & & & \\
. & . & & & & \\
SgF_{n1} & \min(\mu_{(SgF_{n1})}, \mu_{(SgF_{11})}) & \min(\mu_{(SgF_{n1})}, \mu_{(SgF_{21})}) & \min(\mu_{(SgF_{n1})}, \mu_{(SgF_{31})}) & \cdots & 1
\end{array}$$

Similarly for the feedback response, " $\tilde{R}_2$ " represents the degree of association of two subsystems denoted as $\left(SgF_{i2} \tilde{R}_2 SgF_{j2}\right)$ or $SgF_{i2} \tilde{R}_2 SgF_{j2}$ and is an ordered pair on $\left(SgF_{2} \times SgF_{2}\right)$. Similarly $\left(SgF_{i2} \tilde{R}_2' SgF_{j2}\right)$ denotes the degree of non association of the two subsystems on the domain

$\left(SgF_{2} \times SgF_{2}\right)$. Again " $\tilde{R}_2$ " is not symmetric and not an equivalence relation. Then the association can be represented as below.

$$
\begin{array}{c|ccccc}
\tilde{R}_2 & SgF_{12} & SgF_{22} & SgF_{32} & \cdots & SgF_{n2} \\
\hline
SgF_{12} & 1 & \min(\mu_{(SgF_{12})}, \mu_{(SgF_{22})}) & \min(\mu_{(SgF_{12})}, \mu_{(SgF_{32})}) & \cdots & \min(\mu_{(SgF_{12})}, \mu_{(SgF_{n2})} \\
SgF_{22} & \min(\mu_{(SgF_{22})}, \mu_{(SgF_{12})}) & 1 & \min(\mu_{(SgF_{22})}, \mu_{(SgF_{32})}) & \cdots & \min(\mu_{(SgF_{22})}, \mu_{(SgF_{n2})} \\
SgF_{32} & \min(\mu_{(SgF_{32})}, \mu_{(SgF_{12})}) & \min(\mu_{(SgF_{32})}, \mu_{(SgF_{22})}) & 1 & \cdots & \min(\mu_{(SgF_{32})}, \mu_{(SgF_{n2})}) \\
. & . & & & & \\
. & . & & & & \\
. & . & & & & \\
SgF_{n2} & \min(\mu_{(SgF_{n2})}, \mu_{(SgF_{12})}) & \min(\mu_{(SgF_{n2})}, \mu_{(SgF_{22})}) & \min(\mu_{(SgF_{n2})}, \mu_{(SgF_{32})}) & \cdots & 1
\end{array}
$$

The α -cuts of the relation give all combinations of the members of the fuzzy relation with the degree of association greater than or equal to α for the feed forward and feed back responses respectively.

$$\left(\tilde{R}_1\right)_\alpha = \left\{ SgF_{i1}, SgF_{j1} \middle/ \mu_{\tilde{R}_1}(SgF_{i1}, SgF_{j1}) \geq \alpha, i = 1, 2, 3, ..., n, \ j = 1, 2, 3, ..., n \right\}$$

$$\left(\tilde{R}_2\right)_\alpha = \left\{ SgF_{i2}, SgF_{j2} \middle/ \mu_{\tilde{R}_2}(SgF_{i2}, SgF_{j2}) \geq \alpha, i = 1, 2, 3, ..., n, \ j = 1, 2, 3, ..., n \right\}$$

The following program developed on matlab® computational software gives all subsystems within a large multi-connected system of FFSSM's connected with a degree of association greater than or equal to α at any time.

```
clear all
%Input the Fuzzy Relation.
R=input('Enter the Fuzzy Relation')
%Enter the degree of Association.
lamda=input('Enter the degree of Association=')
[m,n]=size(R);
for i=1:m
   for j=1:n
      if (R(i,j)<lamda)
         c(i,j)=0;
      else
         c(i,j)=1;
      end
   end
end
%output
display('The following systems has a degree of Associaton greater than or equal to lambda.')
display(c)
Output
Enter the Fuzzy Relation[0 0.6 0.7 0.9;0.9 0.0 0.6 0.4;0.2 0.4 0.0 0.5;0.7 0.9 1 0.0]
R =

   0.0000    0.6000    0.7000    0.9000
   0.9000    0.0000    0.6000    0.4000
   0.2000    0.4000    0.0000    0.5000
   0.7000    0.9000    1.0000    0.0000

Enter the degree of Association=0.6
```

lamda =

0.6000

The following systems have a degree of Associaton greater than or equal to lambda.

c =

0	1	1	1
1	0	1	0
0	0	0	0
1	1	1	0

The result shows that SgF_{31} is connected to none of the subsystems. Also there is no connectivity between SgF_{11} and SgF_{41} at the degree of association equal to 0.6000.

5 Conclusions:

This paper is devoted to the properties of a large multi-connected system of FFSSM's. These structural properties are concerning with convexity, normality, BIBO stability and optimal operational conditions for a multi-connected system of FFSSM's. A flow of some definitions resulted in the derivation and proof of some important theorems. A new application to the multi-connected system of feedback dynamical system is the number theoretic approach to the system of FFSSM's. In this connection, the basic concepts of the divisors, common divisors and greatest common divisors are applied to the system of FFSSM's. Finally the fuzzy relational approach to the system of FFSSM's presents the degree of association of various systems within a large multi-connected system of FFSSM's. The preliminary number theoretic and fuzzy relational approach presented in this study can be further investigated to generate new studies concerning the multi-connected system of FFSSM's. Also some new methodologies can be researched in the proposed area.

Acknowledgment

We are thankful to Univerisiti Teknologi Malaysia and COMSATS University of Science and Technology Abbottabad for providing necessary research environment and requirements for conducting this research.

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