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An Application of Interactive Fuzzy Multi-Objective Approach to Optimal Location of PST

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ABSTRACT

Optimal power flow (OPF) and optimal location of FACTS devices are very important tools in planning and operation of power system. In multi-objective OPF due to being poor collaboration among design objectives and poor resolution of design conflicts, relations are complex. To handle these problem, a fuzzy interactive multi-objective optimization model is developed based on Pareto solutions while the metric function and some additional constraints are added to ensure the collaboration among design objectives. A new multi-objective OPF algorithm introduced by this method. It is able to interact with decision maker (DM) against all OPF method, and DM can direct process to side that it meet his priority for scheduling power systems. In this paper OPF is formulated as a nonlinear optimization problem with both equality and inequality constraints. The proposed objective functions are: minimization of total fuel cost, minimization of active power losses. Numerical examples are presented in order to show efficiency of proposed technique through 118-bus IEEE test system using MATLAB and Generalized Algebraic Modeling of System (GAMS) softwares.

Key words: Fuzzy optimization, Optimal Power Flow, Decision Making, PST.

Introduction

The main objective of an OPF is to find out the optimal operating state of a power system by optimizing a particular objective while satisfying certain specified physical and operating constraints (Nualhong, S., 2004; Huneault, M. and F.D. Galiana, 1991). A number of conventional optimization techniques have been applied to solve the OPF problem. They consist of linear Programming (LP) (Stot, B. and J.L. Marinho, 1979; Alsac, O. and B. Scott, 1974), Non-Linear Programming (NLP) (Reid, G.F. and L. Hasdorf, 1973), Quadratic Programming (QP) (Reid, G.F. and L. Hasdorf, 1973; Momoh, J.A. and J.Z. Zhu, 1999), and Interior Point Methods (IPM) (Bakirtzis, A.G., 2002). All these techniques rely on convexity of the functions of the equations to get the global optimum. However, due to the non-differentiable feature, nonlinear and non-convex nature of the OPF problem, the methods based on these assumptions do not guarantee the global optimum solution. During the last two decades, the interest in applying Artificial Intelligence (AI) in non-convex and non-linear optimization problems has grown rapidly.

Fuzzy set methods have been applied to obtain more practical models. Fuzzy set methods have already been used in many applications such as control, scheduling, robotics, and soft computing issues. In the field of power engineering, they have been applied to some areas including OPF (Terasawa, Y. and S. Iwamoto, 1988; Tomsovic, K., 1992; Miranda, V. and J.T. Saraiva, 1992; Abdul-Rahman, K.H. and S.M. Shahedehpour, 1993). These developments have made it possible to use this technology to overcome some of the limitations of the conventional OPF.

This paper presents the application of a fuzzy set method to the OPF problem taking into account the uncertainty of the inequality constraints in a standard power system. The OPF with fuzzy constraints is formulated as a fuzzy optimization problem and converted into a crisp optimization problem. Nonlinear programming with discontinuous derivatives (DNLP) method is then modified to solve the new problem. Numerical test results on a standard power system show that the developed fuzzy OPF method could give a good trade-off between reducing total fuel cost and active power losses while satisfying constraints. Besides, the proposed approach produces more realistic generation schedules when a feasible solution cannot be obtained by using crisp OPF.

2. PST Injection Model:

Combinations of shunt- and series-connected FACTS technology provide additional functionality to the FACTS device. The most known device of this type is the Phase-Shifting Transformer (PST), which is widely used in power systems.
used throughout the world. The topology is based on one shunt transformer (the exciting unit) and one series transformer (the boost unit) that is. PST capable of changing the voltage phase angle difference across a line, leading to a change in the power flow on the line (Nicklas Johansson, M., 2008).

The basic schematic of the PST is shown in Fig. 1. The power injection model of the PST is shown in Fig. 2 where

\[ P_{ss} = -b_s k V_i V_j \sin(\theta_i - \theta_j + \sigma) \]  
(1)

\[ Q_{ss} = -b_s k V_i^2 (k + 2 \cos(\sigma)) + b_s k V_i V_j \cos(\theta_i - \theta_j + \sigma) \]  
(2)

\[ p_o = -p_i \]  
(3)

\[ Q_o = b_s k V_j V_j \cos(\theta_i - \theta_j + \sigma) \]  
(4)

Where

\[ k = \tan(\sigma) \]

\[ b_s = \frac{1}{(X_s + X_b)} \]

here \( k \) is the transfer ratio of PST; \( \sigma \) is the PST phase angle; where \( X_s \) is the transmission line reactance and \( X_b \) is the series transformer leakage reactance.

![Fig. 1: Per-phase schematic diagram of PST.](image)

![Fig. 2: The power injection model of PST.](image)

3. Problem Formulation:

The economic optimal operation of power systems, considering transmission constraints and supplying load demand, requires to minimize two objective functions (total generation fuel cost and active power losses) while satisfying several equality and inequality constraints. Generally the optimal operation problem which named OPF can be formulated as follows.
3.1. Objective Functions:

Minimization of total generation fuel cost: The generation cost function is represented by a quadratic polynomial function as follows (Palma-Behnke, R., 2004):

\[ F_1 = \sum_{i=1}^{g} C_i(P_{gi}) = \sum_{i=1}^{g} \alpha_{0i} + \alpha_{1i} P_{gi} + \alpha_{2i} P_{gi}^2 \quad (\$/h) \tag{5} \]

where \( P_{gi} \) is the real power generation of unit \( i \). Also, \( \alpha_{0i}, \alpha_{1i} \) and \( \alpha_{2i} \) are cost coefficients of unit \( i \); \( g \) is the number of generators.

- Active power losses: The total power loss to be minimized is as follows (Navarro, J.A., 2007):

\[ F_2 = F(V', \delta) = \sum_{i=1}^{n} \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\alpha_{ij} + \theta_j - \theta_i) \tag{6} \]

3.2. Constraints:

- Generation Real Power limits: The real power output limits of generator \( i \) are formulated as (Palma-Behnke, R., 2004):

\[ p_{min}^{Gi} \leq P_{gi} \leq p_{max}^{Gi} \quad \forall i \in NG \tag{7} \]

- Voltage Control and Reactive Support: The voltage limits and reactive power output limits assuming constant power factor for loads can be expressed using following inequalities:

\[ q_{min}^{Gi} \leq Q_{gi} \leq q_{max}^{Gi} \quad \forall i \in NG \tag{8} \]

\[ |V_i'_{\min}| \leq |V_i'_{\max}| \quad \forall i \in n \tag{9} \]

where \( Q_{gi}, q_{min}^{Gi}, q_{max}^{Gi} \) are stand for reactive power output, maximum and minimum reactive limits of generating unit \( i \), respectively. Also, \( |V_i'_{\min}| \) and \( |V_i'_{\max}| \) are related to bus voltage, minimum and maximum limits of voltage of \( i \)th bus, respectively.

- Power balance equations: Real and reactive power balance equations of \( i \)th bus considering injected active and reactive power of FACTS devices can be expressed as (Palma-Behnke, R., 2004):

\[ P_{gi} + P_{FACTS} = P_{Di} + \sum_{j=1}^{n} V_j V_j Y_{ij} \cos(\alpha_{ij} + \theta_j - \theta_i) \quad \forall i, j \in n \tag{10} \]

\[ Q_{gi} + Q_{FACTS} = Q_{Di} + \sum_{j=1}^{n} V_j V_j Y_{ij} \sin(\alpha_{ij} + \theta_j - \theta_i) \quad \forall i, j \in n \tag{11} \]

where, \( i=1,2,\ldots,n \) ; and \( n \) is the number of buses, \( P_{gi} \) and \( Q_{gi} \) are the generated real and reactive power of unit located at bus \( i \), respectively; \( P_{Di} \) and \( Q_{Di} \) are the real and reactive power of load located at bus \( i \), respectively; \( P_{FACTS} \) and \( Q_{FACTS} \) are active and reactive power injected by FACTS devices to the specific bus, respectively.

- Transmission constraints:

\[ |S'_i| \leq S'_{i_{\max}} \quad \forall i \in NB \tag{12} \]

where \( S'_{i_{\max}} \) are stand for the apparent power flow and the capacity of \( i \)th transmission line.

- PST constraints (Lashkar, A., 2011):

\[ -20^\circ \leq \sigma \leq 20^\circ \tag{13} \]
4. Interactive Fuzzy Multi-Objective Optimization Model:

4.1. Finding the Optimum Value of each Objective Function:

The optimization model to find the optimum value of each objective is given by (Tappeta, R.V., 2000):

Minimize $F_t(X), \quad t = 1, 2$

Subject to $h_i(X) = 0 \quad i = 1, 2, \ldots, M$
$g_j(X) \leq 0 \quad j = 1, 2, \ldots, N$
$X_k^l \leq X_k \leq X_k^u$

where, $F_t(X)$ refers to the objective functions; Also, $h_i(X)$ and $g_j(X)$ are equality and inequality constraints. Finally, $X_k$ is the $k$th decision variable.

The solution of the above model is the optimum solution of each objective function, $X_t^{*}$, and the optimal value of the objective function at the optimum solution, $F_t^{*}$, can be written as:

$$F_t^{*} = F_t(X_t^{*}) \quad (t = 1, 2)$$

where, $F_t^{*}$ is the optimum value of $t$th objective function.

4.2. Establishing Fuzzy Interactive Multi-Objective Optimization Model:

Let $X$ be the ideal solution, and $F_t^{*}$ be the ideal value of $t$th objective function, then the metric function which evaluates $X$ is determined as follows (Shih, C.J., C.J. Chang, 1995):

$$d(X) = \sqrt{\alpha \sum_{i=1}^{n} \frac{F_i(X) - F_i^{*}}{F_i^{*}} }$$

where, $\alpha$ can be chosen from the universe $[1, \infty]$. $\alpha = 2$ is usually used. Minimizing this metric function results in a commonly encountered min–max method (Shih, C.J., C.J. Chang, 1995), since for this metric the optimum $X$ can be defined as:

$$F(X) = \min_x \max_i \left| \frac{F_i(X) - F_i^{*}}{F_i^{*}} \right|$$

The degree of priority of each objective criterion can be incorporated in metric function with the following additional constraints

$$w_i \left| \frac{F_i(X) - F_i^{*}}{F_i^{*}} \right| \leq \varepsilon$$

And

$$\sum_{i=1}^{n} w_i = 1$$

where $w_i$ depict the degree of significance of the $i$th objective criterion, and $\varepsilon$ represents the allowable degree of deviation from the ideal solution for objectives (the ideal value is 0).
The process of a multi-objective optimization problem itself is subjective and can be modeled by fuzzy decision making due to the conflicting objectives and the nature of human decision on conflict resolution. In fuzzy set theory, membership functions are established to give the fuzziness of fuzzy sets. The membership function values vary between zero and one. The elements in a fuzzy set with membership value 1 means that they are in the core of the fuzzy set. The membership function value is zero for the element outside the fuzzy set. The elements with membership function value between zero and one construct the boundary of the fuzzy set. In order to use fuzzy set theory to solve the optimization problems, the fuzzy constraints must be formulated first. These constraints originated from the given crisp constraints by relaxing the bounds. A corresponding membership function is established to explain the fuzziness of each constraint. In addition to fuzzy constraints, fuzzy objective functions are also required. Each objective function is converted into a pseudo-goal. A membership function is associated with the pseudo-goal. The pseudo-goal has membership function value one if the design is situated at the optimum from the single-objective optimization problem with the same constraints for the multi-objective design. It is clear that solving the multi-objective optimization problem is necessary to simultaneously make all membership function values of the pseudo-goals as large as possible.

The proposed procedure is summarized as follows (Huang, H.Z., 2006):

1. Finding the minimum feasible value and maximal feasible value of each objective function with consideration constraints:

\[ m_i = \min \{ F_i(X_i) \} = F_i(X_i^*) \]  \hspace{1cm} (20)

\[ M_i = \max \{ F_i(X_i) \} \]  \hspace{1cm} (21)

where, \( m_i \) and \( M_i \) are the minimum feasible value and maximum feasible value of \( i^{th} \) objective function.

For two functions (\( F_1 \) and \( F_2 \), which refer to total fuel cost and active power losses, respectively) payoff table should be performed as table 1 to determine the range of each objective function, i.e. \( m_i \) and \( M_i \), as follows.

**Table 1:** Payoff table.

<table>
<thead>
<tr>
<th></th>
<th>( F_1 )</th>
<th>( F_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (\min F_1, F_2) )</td>
<td>( F_1^<em>(X_1^</em>) )</td>
<td>( F_2(X_1^*) )</td>
</tr>
<tr>
<td>( (F_1, \min F_2) )</td>
<td>( F_1(X_2^*) )</td>
<td>( F_2^<em>(X_2^</em>) )</td>
</tr>
<tr>
<td>( (F_1, F_2, \max F_3) )</td>
<td>( F_1(X_3^*) )</td>
<td>( F_2(X_3^*) )</td>
</tr>
</tbody>
</table>

\[ m_i = F_i^*(X_i^*) \quad i = 1, 2 \]  \hspace{1cm} (22)

\[ M_i = \max_{\substack{j=1,2}} \{ F_j(X_j^*) \} \quad i = 1, 2 \]  \hspace{1cm} (23)

2. Establishing the membership function of each fuzzy objective function: Most applications that involve fuzzy set theory inclined to be independent of the specific shape of the membership functions.

The fuzzy objective function used by a decision maker can be quantified by selecting a corresponding membership function using trapezoidal representation, if aim be objective minimization, the membership function should be performed as follows:
(3) Establishing the membership function of every fuzzy constraint function: In the deterministic optimization problem, the problem feasibility is considered as either feasible or infeasible. For many applied problems in the power systems, the transition from infeasibility to feasibility is not obvious, because of not only the uncertain nature of problem data which are exist in the problem constraints. Therefore, the constraints are modeled in such a way that the conversion from infeasible state to feasible state is smooth and gradual with subjectivity. For simplicity, a linear membership function is used to reflect the smooth conversion. The linear membership function is given by:

\[
\mu_{F_i}(X) = \begin{cases} 
1, & F_i(X) \leq m_i, \\
\frac{M_i - F_i(X)}{M_i - m_i}, & m_i < F_i(X) < M_i, \ (i = 1, 2) \\
0, & F_i(X) \geq M_i.
\end{cases}
\] (24)

(4) Additional constraints: The ideal value of the degree of objective deviation in additional constraints is 0. Considering the mutual relationship among objectives and their membership functions, the additional constraints are established by the following equality constraints (Shih, C.J., C.J. Chang, 1995):

\[
\left| \frac{F_i(X) - F_i^u}{F_i^u} \right| = \left| \frac{F_j(X) - F_j^u}{F_j^u} \right| \\
i, j = 1, 2, \ldots, n, \ i \neq j
\] (26)

And

\[
\sum_{i=1}^{n} w_i = 1
\] (27)

The aim of adding additional constraints into the multi-objective optimization model is to make trade-off among the objective functions. By doing so, not only the degree of significance of each objective is considered, but also the deviation between each objective and its ideal value can be minimized.

(5) Establishing fuzzy multi-objective optimization model: The fuzzy multi-objective optimization model can be improved

Maximize \( \lambda \)

Subject to

\( \lambda \leq \mu_{F_i}(X), \ i = 1, 2, \ldots, n, \)

\( \lambda \leq \mu_{g_j}(X), \ i = 1, 2, \ldots, I, \)

\( \lambda \leq \mu_{h_k}(X), \ j = 1, 2, \ldots, J, \)

\[
\left| \frac{F_i(X) - F_i^u}{F_i^u} \right| = \left| \frac{F_j(X) - F_j^u}{F_j^u} \right| \\
i, j = 1, 2, \ldots, n, \ i \neq j
\] (28)

\[
\sum_{i=1}^{n} w_i = 1
\]
1 ≥ λ ≥ 0

\[ X^*_i \geq X^*_k \geq X^*_k, \quad k = 1, 2, ..., k \]

4.3. Selecting the weighting coefficients \( w_i \):

Generally, the degrees of significance of individual objectives are various, which are affected by many factors which may be having impact on the problem conditions. Therefore, the significance of objective functions has a very important affect on modeling and solving multi-objective optimization problems. The weighting coefficient \( w_i \) can be used to represent the design degree of importance corresponding to the \( i \)th objective criterion. The selection of \( w_i \) is highly subjective and correlated with other \( w_j (j \neq i) \). In this regard, \( w_i \) coefficients can be determined by decision maker based on their preferences. The flowchart of the proposed method is given in Fig.3.

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**Fig. 3:** Flowchart of fuzzy interactive multi-objective optimization.
5. Case Study:

The optimal power flow problem is implemented in MATLAB and GAMS softwares and nonlinear programming with DNLP method is employed to solve it. Two different objective functions are considered: total fuel cost ($F_1$) and active power loss ($F_2$) to be minimized. Different cases are considered for these objective functions with and without PST device. In each case, the optimal settings of the PST and its best location are determined. The PST performance is tested on the IEEE 118-bus system. Data of the 118-bus system contains 186 lines, 54 generators are taken from (Power system test case, 2000). Firstly, the single objective optimization problem for each objective functions are simulated; then two objective functions are studied simultaneously.

In the proposed model, however, not only the degree of importance of the objective functions, but also the objectives deviations with respect to their ideal values, are considered. Therefore, the results of the optimization problem, solved by the fuzzy interactive optimization based on Pareto solutions, are more realistic. Also the final solution based on selected weights of decision maker's vision is signed by (*) in all of states while different weights is available.

Single objective optimization results of the total fuel cost and active power losses minimization are shown in the tables 2 and 3, respectively. In the state 2 of all tables, PST is located in the system and improves values of objective functions. Simultaneous optimization results of total fuel cost and active power losses are shown in Table 4 and the best compromised solutions for this case are obtained.

| Table 2: Results of IEEE 118-bus system for total fuel cost optimization. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Objective Function ($F_1$) | Total Fuel Cost ($$/h) | $\sum P_{loss}$ (MW) | $\sum Q_{loss}$ (MVAR) | FACTS Size (MVA) | FACTS Location | FACTS Setting |
| Without PST | 129660.997 | 77.408 | 507.250 | - | - | - |
| With PST | 129539.83 | 74.872 | 789.287 | 200 | Line 27-25 | $\sigma^* = 11.845$ |

| Table 3: Results of IEEE 118-bus system for active power losses optimization. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Objective Function ($F_2$) | $\sum P_{loss}$ (MW) | Total Fuel Cost ($$/h) | $\sum Q_{loss}$ (MVAR) | FACTS Size (MVA) | FACTS Location | FACTS Setting |
| Without PST | 9.248 | 166390.285 | 69.383 | - | - | - |
| With PST | 8.983 | 166178.350 | 68.409 | 200 | Line 90-89 | $\sigma^* = 13.987$ |

| Table 4: Results of IEEE 118-bus system for total fuel cost and active power losses optimization. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Objective Functions ($F_1$&$F_2$) | $\lambda$ | $w_1$($w_1$, $w_2$) | Total Fuel Cost ($$/h) | $\sum P_{loss}$ (MW) | $\sum Q_{loss}$ (MVAR) | FACTS Size (MVA) | FACTS Location | FACTS Setting |
| Without PST | 0.712 | (0.5, 0.5) | 137628.342 | 24.033 | 159.278 | - | - | - |
| With PST | 0.654 | (0.6, 0.4) | 136143.458 | 27.292 | 191.627 | - | - | - |
| | 0.592 | (0.7, 0.3) | 134562.135 | 30.470 | 202.036 | - | - | - |
| | 0.397 | (*0.9, 0.1) | 131002.504 | 45.015 | 303.872 | - | - | - |
| With PST | 0.711 | (0.5, 0.5) | 137325.714 | 23.184 | 164.746 | 198.895 | Line 5-1 | $\sigma^* = 2.741$ |
| | 0.654 | (0.6, 0.4) | 134944.900 | 23.895 | 176.548 | 200.00 | $\sigma^* = 2.773$ |
| | 0.590 | (0.7, 0.3) | 134460.174 | 29.548 | 198.358 | 192.952 | $\sigma^* = 2.811$ |
| | 0.395 | (*0.9, 0.1) | 131272.789 | 37.167 | 226.127 | 200.00 | $\sigma^* = 2.934$ |

6. Conclusion:

In this paper, the fuzzy interactive multi-objective optimization framework for allocation PST is introduced to involve the objective functions. In order to evaluate the effectiveness of the proposed approach, the performance of PST is investigated on the IEEE 118-bus test system. The optimal location of PST and its settings to optimize total fuel cost and active power losses as single and multi-objective functions, under equality and inequality constraints of the both system and PST, were obtained and discussed. Proposed method advantage is interactive property that help decision maker to get better judgment for selecting best compromised solution. In other words, in the proposed approach, the system operator can utilize PST in the system for power flow control in such manner that its objective functions based on their importance can be optimized, concurrently.

| Table 4: Results of IEEE 118-bus system for total fuel cost and active power losses optimization. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Objective Functions ($F_1$&$F_2$) | $\lambda$ | $w_1$($w_1$, $w_2$) | Total Fuel Cost ($$/h) | $\sum P_{loss}$ (MW) | $\sum Q_{loss}$ (MVAR) | FACTS Size (MVA) | FACTS Location | FACTS Setting |
| Without PST | 0.712 | (0.5, 0.5) | 137628.342 | 24.033 | 159.278 | - | - | - |
| With PST | 0.654 | (0.6, 0.4) | 136143.458 | 27.292 | 191.627 | - | - | - |
| | 0.592 | (0.7, 0.3) | 134562.135 | 30.470 | 202.036 | - | - | - |
| | 0.397 | (*0.9, 0.1) | 131002.504 | 45.015 | 303.872 | - | - | - |
| With PST | 0.711 | (0.5, 0.5) | 137325.714 | 23.184 | 164.746 | 198.895 | Line 5-1 | $\sigma^* = 2.741$ |
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