A Proposed Scheme of Multidimensional Time Domain Equalizer

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ABSTRACT

This paper proposes a scheme to achieve time-domain equalization for multidimensional communication systems. This scheme depends on decomposing the multidimensional received signal into multiple one-dimensional signals. Next, removing the effects of the multidimensional inter-symbol interference (ISI) from each of these signals in parallel. The resulted one-dimensional equalized signals are then concatenated to form the original undistorted multidimensional signal. It is shown that the mathematical derivation of the proposed scheme was given, as well as a numerical example that demonstrates its performance in multidimensional channel. Processing speed gain over a regular one-dimensional equalization is linearly proportional to the multidimensional communication system in use.

Key words: Multidimensional Communication, Multidimensional Convolution, Multidimensional Equalization.

Introduction

Equalization techniques which can combat the frequency selectivity of the wireless channel are of enormous importance in the design of high data rate wireless systems. Although such techniques have been studied for over 40 years, recent developments in signal processing, coding and wireless communications suggest the need for paradigm shifts in this area. On one hand, two-dimensional intersymbol interference (ISI) channels have received a lot of attention lately because of the emergence of two-dimensional paradigms for data storage (N. Singla and J.A. O’Sullivan, 2005; Hiroyuki Shinoda, 2004).

In general, changing to multidimensional communication systems increases the speed of communication (or the capacity of storage for recording systems), but this comes at a price. The multidimensional ISI distorts the multidimensional signal which arises the computational complexity of the equalization process, where the regular one-dimensional such as Maximum Likelihood equalization is not straightforward. Ordentlich and Roth have shown that the problem of maximum-likelihood sequence detection for two-dimensional ISI channels is NP complete (E. Ordentlich and R.M. Roth, 2006). Many schemes had been proposed with varying degrees of success in dealing with two-dimensional ISI (K.M. Chugg, 1999; K.M. Chugg, 1997; A.H.J. Immink et al., 2003; R. Krishnamoorthi, 1998; P.S. Kumar and S. Roy, 1994; M. Marrow and J.K. Wolf, 2003; A. Moinian, 2005; O. Shental, 2004; N. Singla, 2002; N. Singla and J.A.O’Sullivan, 2004; Y. Wu, 2003; W. Weeks IV, 2000; Z. Zhao and R. Blahut, 2005 Mahmoud, W.A., 2009). However, none of these have the form of generalization. Here, a general procedure for curing the ISI effect of any dimensional communication system by decimating the received message in dimension repeatedly until having multiple messages with a dimension where a relatively low complexity equalization algorithm could be applied. Next, the message is reconstructed from the result of the multiple equalized signals.

The rest of the paper is arranged as follows; section 2 depicts the realization of the multidimensional convolution. In section 3; a multidimensional time domain equalization scheme is proposed, the proposed scheme is demonstrated with a simple 2-D communication system in section 4, and then section 5 concludes the paper.

Realization of Multidimensional Convolution:

In (Mahmoud, W.A., 2001) a method for computing multidimensional convolution using polynomial method had been proposed. Here, an alternative approach will be given depending on the principle of decimation.

Consider the following multidimensional arrays:

\[ A^N = \{a_{g_1,g_2,\ldots,g_N}\}\]  

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$B^N = \{b_{i_1, i_2, \ldots, i_N}\}$  \hspace{1cm} (2)

Where:

$0 \leq g_n \leq X_n \quad \forall \ 0 \leq n \leq XN$

$0 \leq l_n \leq Y_n \quad \forall \ 0 \leq n \leq YN$

N: number of dimensions processed by the array ($0 \leq N \leq \infty$).

$X_n, Y_n$: number of elements in the $n^{th}$ dimension.

$g_{o, i}, l_{o, i}$: is the number of $g_{o, i}, l_{o, i}$ arrays in the $n^{th}$ dimension.

Now, assume that:

$C^N = A^N *^N B^N$

Where $*^N$ represents an N-dimensional convolution operator.

In general, multidimensional discrete time convolution is defined as:

$C_{i_1, i_2, \ldots, i_N} = \sum_{k_1 = -\infty}^{\infty} \sum_{k_2 = -\infty}^{\infty} \ldots \sum_{k_N = -\infty}^{\infty} a_{k_1, k_2, \ldots, k_N} \times b_{i_1 - k_1, i_2 - k_2, \ldots, i_N - k_N}$  \hspace{1cm} (3)

The goal of this section is to suggest a method to perform the multidimensional convolution operation. This is done by decimating the convolved arrays in dimension as follows; if an array $(A^N)$ of N dimensions is decimated into $X_1$ arrays, each of $(N-1)$ dimensions; then $A^N$ may be described as:

$A^N = \{A^{N, i}\}$  \hspace{1cm} (4)

Where $0 \leq b_1 \leq X_1$. This may be called the first decimation in dimension of the array $A^N$.

In this way, the general form of multidimensional discrete time convolution could be written as:

$C^{N, i} = \sum_{k_1 = -\infty}^{\infty} \sum_{k_2 = -\infty}^{\infty} \ldots \sum_{k_N = -\infty}^{\infty} A^{N, k} *^{N-1} B^{N, i-k}$  \hspace{1cm} (5)

Thus, an N dimensional convolution may be obtained first by decimating the original convolved arrays into $(N-1)$ dimensions. Then applying (3), and then to find the resulted $(N-1)$ dimension convolution, another decimation is made, and so on until a zero dimensional convolution is reached which is simply a multiplication.

**The Proposed Multidimensional Equalization Scheme:**

Let $S$ be a multidimensional signal of the dimension $N$ where $0 \leq N \leq \infty$. And let $H$ to be a channel's impulse response (CIR) which has the same dimensions as $S$ (equal to $N$).

When the signal $S$ is transmitted through the channel with the CIR of $H$, then the received signal (with the noise term neglected for now) will be:

$R^N = H^N *^N S^N$  \hspace{1cm} (6)

Now let $EQ_x(A^N)$ to be an $(X)$ dimensional equalization process for the $(X)$ dimensional signal $A$, this equalizer is supposed to be modeled according to the characteristics of the channel in order to give:

$EQ_N(H^N *^N S^N) = \tilde{S}^N$  \hspace{1cm} (7)

Where $\tilde{S}$ is the detected multidimensional signal at the receiver.

Convolution in (6) may be replaced by the multidimensional convolution formula in (3):

$R^{N, i} = \sum_{k_1 = -\infty}^{\infty} H^{N, k} *^{N-1} S^{N, i-k} = H^{N, 0} *^{N-1} S^{N, i} + \sum_{k_1 = -\infty}^{\infty} H^{N, k} *^{N-1} S^{N, i-k}$  \hspace{1cm} (8)

To reconstruct the $i^{th}$ decimation of the original signal $S^{N, i}$ from the $i^{th}$ decimation of the received signal $R^{N, i}$, the equalization process will be applied as follows:

$\tilde{S}^{N, i} = EQ_N(H^{N, 0} *^{N-1} S^{N, i}) = EQ_N(R^{N, i} - \sum_{k_1 = -\infty}^{\infty} H^{N, k} *^{N-1} S^{N, i-k})$  \hspace{1cm} (9)
So, when starting from $i=0$, the term $\hat{S}^{N,i-k}$ required in each step to be found from the results of previous steps (resembling decision feedback equalizer). In words, equalization of $N$ dimensional signal $R^N$ may be performed by equalizing multiple $(N-1)$ dimensional signals decimated from $R^N$ as shown in Fig. 1.

Fig. 1: A block diagram of the proposed scheme.

**Applying the Proposed Scheme on A 2-D Communication System:**

Assume a two dimensional Pulse Amplitude Modulation (PAM) communication system with a channel of impulse response of $H^2$, where:

$$H^2 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}$$

For instance, assume a PAM signal of four levels (-3, -1, 1, and 3) is transmitted through the channel, to clarify the process, let a message signal of 16 symbols is rearranged as two dimensional signal as follows:

$$S^2 = \begin{bmatrix} 3 & 1 & -1 & 1 & 3 & -3 & 1 & -3 \\ 1 & 3 & -1 & -3 & 1 & -3 & 3 & -1 \end{bmatrix}$$

As in (4), $S^2$ could be decimated in dimension to form:

$$S^{2,0} = [3 \ 1 \ -1 \ 1 \ 3 \ -3 \ 1 \ -3]$$
$$S^{2,1} = [1 \ 3 \ -1 \ -3 \ 1 \ -3 \ 3 \ -1]$$

And the same for $H^2$:

$$H^{2,0} = [1 \ 0.5]$$
$$H^{2,1} = [0.5 \ 0.2]$$

Let $R^2$ to be the received two dimensional signal resulted from convolving $S^2$ with $H^2$:

$$R^2 = S^2 \ast^N H^2 = \begin{bmatrix} 3.0 & 2.5 & -0.5 & 0.5 & 3.5 & -1.5 & -0.5 & -2.5 & -1.5 \\ 2.5 & 4.6 & 0.2 & -3.2 & 1.2 & -3.4 & 1.4 & -0.8 & -1.1 \\ 0.5 & 1.7 & 0.1 & -1.7 & 0.1 & -1.3 & 0.9 & 0.1 & -0.2 \end{bmatrix}$$

Now, decimating $R^2$ yields:

$$R^{2,0} = [3.0 \ 2.5 \ -0.5 \ 0.5 \ 3.5 \ -1.5 \ -0.5 \ -2.5 \ -1.5]$$
$$R^{2,1} = [2.5 \ 4.6 \ 0.2 \ -3.2 \ 1.2 \ -3.4 \ 1.4 \ -0.8 \ -1.1]$$
$$R^{2,2} = [0.5 \ 1.7 \ 0.1 \ -1.7 \ -0.1 \ -1.3 \ 0.9 \ 0.1 \ -0.2]$$

$S^2$ is recovered by applying equation (9) to find $S^{2,0}$ and then $S^{2,1}$.
\[ \hat{S}^{2.0} = EQ_2 \left( R^{2.0} - \sum_{k=-\infty}^{\infty} H^{2k} \cdot S^{2-k} \right) = EQ_2 \left( R^{2.0} - (0) \right) = EQ_2 (R^{2.0}) \]

Which is simply a one dimensional linear equalization for the one dimensional array \( R^{2.0} \). Here, a zero forcing equalizer with four taps (derived from \( H^{2.0} \)) is used to perform equalization, so the result of equalization is:

\[ \hat{S}^{2.0} = [3.0 \quad 1.0 \quad 1.0 \quad 3.0 \quad 3.0 \quad 1.0 \quad 3.0] \]

Continuing to \( \hat{S}^{21} \):

\[ \hat{S}^{21} = EQ_2 \left( R^{2.0} - \sum_{k=-\infty}^{\infty} H^{2k} \cdot S^{21-k} \right) = EQ_2 \left( R^{2.0} - (H^{21} \cdot \hat{S}^{2.0}) \right) \]

\[ = EQ_2 ([1.5 \quad 0.6 \quad 0.5 \quad 0.4 \quad 1.5 \quad 1.4 \quad 0.3 \quad 1.3 \quad 0.3 \quad 0.2 \quad 0.1 \quad 0.1]) \]

Hence the result of equalization will be equal to:

\[ \hat{S}^{21} = [1.0 \quad 3.0 \quad 1.0 \quad 3.0 \quad 3.0 \quad 1.0 \quad 1.0] \]

It is obvious that the original two dimensional message could be concatenated from the decimated results. Thus this numerical example demonstrated the idea of the proposed scheme. This procedure can be applied to any communication system with any dimension which results in the simplification of the equalization process.

Conclusion:

The proposed multidimensional scheme given in this paper offers a systematic way to decimate the received signal in which resulted in the simplification of the equalization process. A regular one dimensional linear equalizer (such as zero forcing, or minimum mean square error equalizer) can be used to recover the original multidimensional signal through continuous decimation of the received signal in lower order dimension till reaching the one dimensional level. Further decimation may be done to reach the zero dimensional signal, in this way, a zero dimensional signal equalizer may be defined, which is simply a division process (degenerates to a decision feedback equalizer).

References


