Boundary Layer Flow and Heat Transfer over a Permeable Shrinking Sheet with Partial Slip

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Abstract: The steady, laminar flow of an incompressible viscous fluid over a shrinking permeable sheet is investigated. The governing partial differential equations are transformed into ordinary differential equations using similarity transformation, before being solved numerically by the shooting method. The features of the flow and heat transfer characteristics for different values of the slip parameter and Prandtl number are analyzed and discussed. The results indicate that both the skin friction coefficient and the heat transfer rate at the surface increase as the slip parameter increases.

Key words: Dual solutions; heat transfer; partial slip; permeable sheet; shrinking sheet.

INTRODUCTION

Enormous works have been done in various aspects related to the steady flow over a stretching sheet as presented in the literature. Such investigations are motivated by their momentousness in the technology and engineering fields, for example in the manufacture of plastic film, in the extraction of a polymer sheet from a die and in fibre industries and glass fibre production. In contrast, less work has been done on the flow over a shrinking sheet. The non-uniqueness of steady viscous hydrodynamic flow due to a shrinking sheet for a specific value of the suction parameter were studied by Miklavčič and Wang and they have reported an exact solution of the Navier-Stokes equations. The flow due to a shrinking boundary with partial slip has yet become relevance in many situations. For example, there is a slip regime where Navier-Stokes equation is valid but slip occurs in the rarefied gases as mentioned by Sharipov and Seleznjev. As the solid surface may be rough and porous, an equivalent slip exists. The no slip condition is replaced by Navier’s partial slip condition, where the amount of relative slip is proportional to the local shear stress. The effect of stagnation slip flow on the heat transfer from a moving plate was recently considered by Wang. Very recently, Fang et al. have solved the problem of viscous fluid flow over a shrinking sheet with a second order slip flow model, without considering the heat transfer aspects, and they presented an exact solution of the governing Navier-Stokes equations.

Motivated by the above investigations, the present paper investigates the boundary layer flow and heat transfer over a permeable shrinking sheet with a first order slip flow model. The effects of the slip parameter on the skin friction coefficient and the heat transfer rate at the surface will be investigated and discussed.

Mathematical Formulation: Consider a two-dimensional laminar boundary layer flow over a shrinking boundary where the lateral surface velocity is proportional to the distance towards the origin, i.e.

\[ U = -cx, \text{ where } c > 0. \]

The boundary layer equations are

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \]

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \]

where \((u,v)\) are the fluid velocities in the \((x,y)\) directions, \(T\) is the temperature in the boundary layer, \(\nu\) is the kinematic viscosity and \(\alpha\) is the thermal diffusivity. The appropriate boundary conditions for the velocity components with partial slip condition at the surface and the temperature are given by
\[ u = U + \kappa v \frac{\partial u}{\partial y}, \quad v = V_w, \quad T = T_w \quad \text{at} \quad y = 0, \]
\[ u \to 0, \quad T \to T_n \quad \text{as} \quad y \to \infty, \quad (4) \]

where \( T_w \) is constant surface temperature, \( V_w \) is the mass transfer velocity at the surface of the sheet with \( V_w > 0 \) for injection (blowing), \( V_w < 0 \) for suction and \( V_w = 0 \) corresponds to an impermeable sheet. Further, \( \kappa \) is a proportional constant and \( v \) is the kinematic viscosity of the bulk fluid.

We introduce now the following similarity transformation:
\[ \eta = \left( \frac{v}{V_w} \right)^{1/2}, \quad \psi = (v c)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_w}{T_n - T_w}, \quad (5) \]

where \( \eta \) is the independent similarity variable, \( f(\eta) \) is the dimensionless stream function, \( \psi \) is the dimensionless temperature and \( \Psi \) is the stream function defined as \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial y \) which identically satisfies Eq. (1). Using (5) we obtain
\[ u = c x f'(\eta) \quad \text{and} \quad v = -(v c)^{1/2} f(\eta), \quad (6) \]

where primes denote differentiation with respect to \( \eta \). In order that similarity solutions of Eqs. (1)–(3) exist, we take
\[ V_w = -(v c)^{1/2} s, \quad (7) \]

where \( s = f(0) \) is a non-dimensional constant which determines the transpiration rate at the surface, with \( s > 0 \) for suction, \( s < 0 \) for injection and \( s = 0 \) corresponds to an impermeable sheet.

Substituting (5) into Eqs. (2) and (3) we obtain the following nonlinear ordinary differential equations:
\[ f'' + ff' - f'^2 = 0, \quad (8) \]
\[ \theta'' + Pr f \theta' = 0, \quad (9) \]

where \( Pr = v / \alpha \) is the Prandtl number. The boundary conditions (4) now become
\[ f(0) = s, \quad f'(0) = -1 + Kf''(0), \quad \theta(0) = 0, \]
\[ f'(\eta) = 0, \quad \theta(\eta) = 0 \quad \text{as} \quad \eta \to \infty, \quad (10) \]

where \( K = \kappa \sqrt{c v} \) is a non-dimensional parameter indicating the relative importance of partial slip. If \( K = 0 \) there is no slip, and if \( K \to \infty \) the surface is stress-free (see Wang [11]).

The physical quantities of interest are the skin friction coefficient \( C_f \) and the local Nusselt number \( N_u \), which are defined as
\[ C_f = \frac{\tau_w}{\rho U^2 / 2}, \quad N_u = \frac{x q_w}{k (T_w - T_n)}, \quad (11) \]

where the wall shear stress \( \tau_w \) and the heat flux \( q_w \) are given by
\[ \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad (12) \]

with \( \mu \) and \( k \) being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables (5), we obtain
\[ \frac{1}{2} C_f \text{Re}^{1/2}_x = f''(0), \quad \text{Nu}_x / \text{Re}^{1/2}_x = -\theta'(0), \quad (13) \]

where \( \text{Re}_x = U x / \nu \) is the local Reynolds number.

**RESULTS AND DISCUSSION**

The nonlinear ordinary differential equations (8) and (9) subjected to the boundary conditions (10) were solved numerically using shooting method for some values of slip parameter \( K \) and suction/injection parameter \( s \), with the Prandtl number \( Pr \) is fixed to unity. The effects of the slip parameter \( K \) on the skin friction coefficient \( f''(0) \) and the local Nusselt number (heat transfer rate at the surface) \( -\theta'(0) \) are shown in Figs. 1 and 2, respectively. It is noticeable that dual solutions exist for the selected values of \( K \) when the suction parameter is greater than 2. This result appears to be in accordance with the result reported by Fang [4]. For each selected values of \( K \), there is indeed a critical value \( s_c \) of \( s \) for which the solution exists. Based on our computations, we found that \( s_c = 2, 1.484436, 1.280629 \) and 1.159900 for \( K = 0, 1, 2 \) and 3, respectively.

In the following discussion, we indentify the upper branch solution as the solution with higher values of \( f''(0) \) and \( \text{Nu}_x / \text{Re}^{1/2}_x \) as \( s \) increases. The opposite
trend can be observed for the lower branch solution, where the skin friction coefficient for the no slip condition \((K = 0)\) is seen to increase or decrease quite dramatically compared to the other values of \(K\). It can also be observed that the heat transfer rate at the surface for both branches increase swiftly as the suction parameter \(s\) increases and the curves become similar (in shape) as \(K\) increases. It may be pointed out here that the effect of the partial slip is to widen the range of the values of \(s\) for which the solutions exist.

Figure 3 presents the velocity profiles for selected values of the slip parameter \(K\) when \(s = 3\) for both upper and lower branch solutions. It can be seen that for the upper branch, the velocity at the wall decreases (in absolute sense) and in consequence decreases the skin friction coefficient \(f''(0)\). This observation is in agreement with the results presented in Figs. 1 and 2. Further, the lower branch solution exhibits a larger boundary layer thickness compared to the upper branch solution. It can be seen that the boundary layer thickness becomes thinner for the upper branch solution with the increase of the slip parameter while the velocity profiles for the lower branch solution have crossover points due to the slip effect.

Figure 4 illustrates the samples of temperature profiles for some values of \(K\) with fixed values of Prandtl number \(Pr\) and suction parameter \(s\). It is seen that an increase in the wall temperature gradient produces an increase in the surface heat transfer rate. Thus, the effect of the slip parameter is to increase the heat transfer from the sheet to the fluid. Figs. 3 and 4 show that the boundary conditions (10) are satisfied asymptotically, hence support the validity of the numerical results obtained, besides supporting the existence of the dual solutions shown in Figs. 1 and 2.

**Conclusions:** The present paper investigated the flow and heat transfer characteristics due to a shrinking permeable sheet with partial slip. The boundary layer equations governing the flow are reduced to ordinary differential equations using a similarity transformation. Using a numerical technique, these equations are then solved to obtain the velocity and temperature distributions as well as the skin friction coefficient and the local Nusselt number for various values of the slip parameter, while the Prandtl number is fixed to unity. It was found that the skin friction coefficient and the

![Fig. 1: Variation of the skin friction coefficient \(f''(0)\) with mass suction parameter \(s\) for some values of \(K\)](image1)

![Fig. 2: Variation of local Nusselt number \(-\theta'(0)\) with mass suction parameter \(s\) for some values of \(K\) when \(Pr1\).](image2)
Fig. 3: Velocity profiles $f'($η$)$ for some values of $K$

Fig. 4: Temperature profiles $\theta($η$)$ for some values of $K$
local Nusselt number (heat transfer rate at the surface) increase with increasing values of the slip parameter. The effect of slip and mass suction parameter strongly influenced the flow velocity and the temperature distribution in the boundary layer.

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