Transportation Cost Using Deterministic Inventory Model with Deteriorating Items Receives Price Discount

K. Jeyaraman and C. Sugapriya

Dept of Mathematics, Alagappa Govt arts college, Karaikudi, Tamilnadu, India.
Department of Mathematics, P.S.N.A College of Engg. and Tech., Dindigul, Tamilnadu, India.

Abstract: In this paper the problem deciding the optimal ordering quantity and frequency for a supplier-retailer system is considered. Here the instantaneous deteriorating items receive price discount. The cost of transportation due to use of multiple vehicles are considered. An algorithm is presented for a modified EOQ model. The EOQ model which adheres to the traditional EOQ model has deviations. The numerical results of the modified model and its corresponding algorithm are provided.

Key words: Supplier-retailer system, Deterioration, Price discount, Transportation cost

INTRODUCTION

Contemporary research in logistics management relies on an increased recognition that an integrated plan requires coordinating different functional specialties within a system. In keeping with this trend, we focus on the integration of production, inventory and transportation arising in a supplier-retailer logistic system.

In the general inventory models, costs of such issues are usually accounted according to the following assumptions: The production cost is proportional to the quantity of products produced. The ordering cost, which refers to the charge for preparing of production, is independent of the quantity ordered. The inventory cost (shortage cost) is proportional to the quantity of products stored (out of order) as well as the duration for which these items are stored (stock out).

When products are delivered from the supplier to the consumer, transportation costs are incurred. In the traditional economic order quantity (EOQ) model, the transportation cost is calculated together with the production cost, or with the ordering cost. However, in a practical logistic system, the transportation cost of a vehicle includes both of the fixed cost and the variable cost. The fixed cost, which is considered to be a constant sum in each period, refers to some necessary expenses such as parking fare and rewards to the driver. As to the variable cost, it depends mainly on the oil consumed, which is related directly to the distance traveled. In short, considering the real condition, it is unreasonable to assume that the transportation cost is proportional to the quantity delivered or is a constant sum. For example, the optimal ordering quantity $y^*$ gained according to the general EOQ formula may be partly loaded by the vehicles and the cost of the logistic system may not be the lowest.

In this study, we address the problem of minimizing the production, inventory and transportation costs for a supplier-retailer logistic system. Unlike most of the prior inventory models, both of the fixed cost and the variable cost of the vehicles are accounted in the model. In addition, since the multiple use of the vehicle can share the fixed cost and may reduce the total cost arising in the logistic system, the permitted working duration of the vehicle as well as the travel time of such vehicle along the trip is also considered.

It is worth noting that inventory lot-sizing models where transportation costs are considered explicitly are rich. Burns, Hall, developed analytic methods for minimizing total inventory and distribution costs under know demand. presented a selected dispatching policy each time a demand arrives so as to specify a shipment release schedule. identified practical operating routines for temporal consolidation such that the service requirements are met and scale economics can be realized. An extensive review of the routing and inventory models for freight distribution problem was given by. presented an analytical model for coordinating inventory and transportation decisions in a vendor-managed inventory system. However, these earlier papers do not consider the effects of multiple uses of the vehicles in which both the fixed cost and the variable costs are considered. Thus, from a practical point of view, there is still a need for analytical models that take into account the transportation cost in the inventory model.

The supply chain literature, recently surveyed by, develops an economic rationale for why retailers and vendors may choose different levels of inventory investment.

There is a large set of studies on channel coordination, provide a comprehensive review of the topic. A few notable studies related to buyer-vendor coordination include, which analyze the separate
or joint optimal ordering policies with discount schedules. Through channel coordination, inventory and other production costs will likely be reduced while capacity utilization is increased, as demonstrated by [13,15].

This paper is an extension of Model and algorithm of an inventory problem with the consideration of transportation cost[9]. It is assumed that a modified economic ordering quantity model for a single supplier-retailer system permitting price discount in which the transportation costs is calculated based on the practical operation. And an algorithm for such a model is designed so as to find the optimal solution within limited steps. The remainder of this paper is organized as follows. The modified model and some lemmas are presented in Section 2. An algorithm used to find the optimal solution for the model is given in Section 3. A few examples are followed in Section 4, and conclusion is given in Section 5.

The model: In the supplier-retailer problem considered, it is assumed that the demand is static during the whole planning horizon and the products can be delivered after they have been ordered for L time, where L is called as the lead time and it is predetermined parameter. In addition, the replenishment should be completed without product shortage occurring. There exists a set of homogeneous vehicles with limited capacity for delivery. In this study, it is assumed that the vehicles are hired from the third logistic party whenever the delivery needs to be completed. The objective of the study is to minimize the whole average costs of the logistic system on the long planning horizon.

Denote the demand quantity per unit time (referring to a day in this study) by D, and y as the ordering quantity of products. Then the highest inventory occurs when y is received, and after y/D time periods the inventory quantity will be reduced to zero. Denote the capacity of the vehicle as p, the fixed cost of such a vehicle as f, in this study, f represents the lowest cost of hiring such a vehicle in a working day, no matter how long the vehicle will be traveled. And the vehicle transportation cost per trip is c. U is the permitted working duration per day, t is the traveling time along each trip, A is the cost of preparing an order, h is the unit inventory cost per unit time and s is the unit production cost. r is the price discount and θ is the deterioration cost per unit time. Then the problem can be formulated as the following model (P)

Minimize $TCLU_y = \frac{A}{y/D} + \frac{m_p}{y/D} + \frac{n_p}{y/D} + \frac{r_l y}{y/D} + \frac{s y}{y/D} + \frac{θ y}{2}$ \quad (1)

Subject to

$(n-1)p < y \leq np$ \quad (2)

d \leq U/t \quad (3)

md \geq n \quad (4)

m, n, d are integers \quad (5)

where $TCLU_y$ is the total cost per unit time associated with the logistic system, y is the ordering size, m is the number of vehicles for delivering y, n is the total trips of these vehicles.

Constraint (2) specifies the number of trips finished by the vehicles for delivering quantity y. Since d in constraint (3) and (4) represents the maximum trips each vehicle is able to complete in a working day, we can regard it as a predetermined parameter in the following paragraphs and sections. Let $m = g(n)$, model $P_1$ can further be expressed as the following $P_2$ model:

Minimize $TCLU_y = \frac{A}{y/D} + \frac{s y}{y/D} + \frac{r_l y}{y/D} + \frac{h y}{y/D}$ \quad (6)

Subject to Equations (2) and (5) and the following constraint (7):

\[
g(n) = m, \quad \left\lceil \frac{n}{d} \right\rceil \quad (7)
\]

In the following, we give an example to illustrate each item in function $TCLU_y$.

In this example, assume $d = 2$, and the vehicle’s number used for delivering the quantity $y \in (n' - p, (n' + 2)p)$ is same, where $n'$ is a positive integer. We can deduce from the assumption that $fg(n, +1) = fg(n, +2)$ and

$fg(n, +3) = fg(n, +4) = l + 2 + f$

Based on the above assumption, the relationship of each item in $TCLU_y$ with y is given in Fig.1.

It can be seen from Fig.1 that $TCLU_y$ is not a continuous function; it cannot be differentiated during the whole interval. However, observe that when n is fixed, the value of $g(n)$ is also a constant. Denote $TCLU_y$ with a given n as $TCLU_{n_j}(y)$, that is
Theorem 2: For any function $TCU_n(y)$, if $y^*_n$ satisfies constraint (2), then $f(n) = TCU_n(y^*_n)$, where the meaning of $f(n)$ is the same as that defined in conclusion 2; otherwise, $f(n) = \min\{TCU_y(y'), TCU_{\alpha}(y')\}$ where $y' = (n - 1)p + 1$.

Proof: Based on the conclusion 1, we know that $TCU_n(y)$ is convex.

The proof of Theorem 2 can be derived as follows:

1. As $g(n)$ is a non-decreasing function of $n$, it can be seen from formulation (9) and (10) that, with the increment of $n$, $y^*_n$ and $TCU(y^*_n)$ become larger. So, for all $n > n_i, TCU_n(y^*_n)$ then $f(n) = TCU_n(y^*_n) > TCU_{\alpha}(y^*_n) = f(n)$. Therefore, the optimal solution of model $P_\alpha$ can be obtained by the steps listed below to find $f(n)$.

2. It can be deduced from the given condition that $f(n) \leq TCU_{\alpha}(y^*_n) \leq f(n)$, since $TCU_{\alpha}(y^*_n)$ is a non-decreasing function of $n$. Therefore, it can be conclude that for all $n > n_i, f(n) = TCU_{\alpha}(y^*_n) < TCU_{\alpha}(y^*_n) \leq f(n)$.

Proof: It can be deduced from above theorems.

3. The algorithm

Based on the above analysis, if denote $TCU_{\alpha}$ as the optimal objective value of model $P_\alpha$ and $v$ is the ordering quantities associated with $TCU_{\alpha}$, we can follow the steps listed below to find $TCU_{\alpha}$ as well as $y^*_n$.

Step1. For a positive integer $n$ (the initial value of $n = \min\{\frac{D}{p}\} y_n$), calculate according to formulation (9). If $y^*_n$ satisfies constraint (2), record $f(n) = TCU_{\alpha}(y^*_n)$, where $TCU_{\alpha}(y^*_n)$ is calculated according to formulation (10), let $\min\{TCU_{\alpha}, f(n)\}$ the initial value of $TCU_{\alpha}$ is set to be infinite), then stop; else if $y^*_n$ does not satisfy constraint (2), go to step (2).

Step2. According to formulation (8), calculate $TCU_{\alpha}(y^*_n)$ and $TCU_{\alpha}(y^*_n)$ respectively, where

$$y^*_1 = \left(\frac{n_1 - 1}{\beta} + 1\right), y^*_2 = n_1p$$

Proof: Based on the conclusion 1, we know that $TCU_n(y)$ is convex.

The examples: Three examples are given in this section in order to verify the given model as well as the algorithm in this first example, it is assumed that the transportation cost is proportional to the quantities
delivered and no traveling duration is considered. In this second example, the transportation is calculated based on travel distance of the vehicles and no fixed cost is considered. Whereas in the third example, the transportation costs include not only the fixed cost which is a fixed sum whenever a vehicle is employed, but also the variable cost which is calculated based on the travel distance of the vehicle. In addition, in the last two examples, the permitted working duration as well as the travel time of any vehicle along the trip is taken into account.

4.1 Example1: It is assumed that in this problem \( D = 100, k = 100, h = 0.02, s = 0.3, L = 2, r = 0.1, = 0.06 \). In addition, we assume transportation cost is proportional to the quantity delivered and the unit transportation cost, defining by \( a \), equal to 0.1. The objective is to decide optimal value of \( y^*, x^*, z^* \) with respect of minimizing the total average cost of the logistic system, where \( y^* \) is the economic ordering quantity, \( x^* \) is the optimal ordering points and \( z^* \) is the ordering cycle.

According to the traditional EOQ formula, we find:

\[
y^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2\times100\times100}{0.02}} = 1000
\]

Table 1: Computational results for Example 2

<table>
<thead>
<tr>
<th>( n )</th>
<th>( y )</th>
<th>( TCU_y(y^*) )</th>
<th>( TCU_y(y) )</th>
<th>( TCU_y(y) )</th>
<th>( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>69.66</td>
<td>140.46</td>
<td>118.00</td>
<td>118.00</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>72.83</td>
<td>137.56</td>
<td>95.00</td>
<td>95.00</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>75.67</td>
<td>104.87</td>
<td>88.67</td>
<td>88.67</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
<td>78.25</td>
<td>95.27</td>
<td>86.50</td>
<td>86.50</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>80.64</td>
<td>91.46</td>
<td>86.00</td>
<td>86.00</td>
</tr>
<tr>
<td>6</td>
<td>1200</td>
<td>82.88</td>
<td>89.98</td>
<td>86.33</td>
<td>86.33</td>
</tr>
<tr>
<td>7</td>
<td>1400</td>
<td>84.99</td>
<td>89.65</td>
<td>87.14</td>
<td>87.14</td>
</tr>
<tr>
<td>8</td>
<td>1600</td>
<td>86.99</td>
<td>89.99</td>
<td>88.25</td>
<td>88.25</td>
</tr>
</tbody>
</table>

The reason that the results of the above two examples are equal is that we design example2 based on the results from example1, that is, the parameters given in example2 guarantee that vehicle used for delivery of \( y^* \) in example 1 is fully loaded. However, in example2, when \( n = 5 \), the value of calculated according to formulation (9) is \( 1732.05 \), whereas based on the algorithm, \( y^* \) equals to 1000. In other words, the solution method using the traditional economic ordering quantity formula is not suitable for the given problem in this example.

We can also see from the results that for all \( n \leq 8 \), \( f(n) < TCU_y(y^*) \), and the value of \( TCU_y(y^*) \) increase along with the increment of \( n \). Following computation indicates that when \( n = 13 \), \( f(n) = TCU_y(y^*) = 95.80 > TCU_{\min} \). Such results further verify the algorithm.

4.2 Example2: It is assumed that in this problem, \( D = 100, k = 100, h = 0.02, s = 0.3, p = 200, c = 40, f = 0, U = 8, t = 4, r = 1, \theta = 0.06 \) to decide the optimal solutions \( y^*, x^*, z^* \).

The algorithm described in Section 3, and the computational results are listed in Table 1. We can see from the results that, based on the stopping criterion in step 2, the algorithm stops when \( n = 8 \). The optimal solution occurs at \( n = 5, TCU_{\min} = 86.00, y^* = 1000, z^* = 10, x^* = 200 \). The number of vehicle used for delivery is 3.

4.3 Example3: It is assumed that in this problem, \( D = 100, A = 100, h = 0.02, s = 0.3, p = 200, c = 40, U = 8, t = 4, f = 30, r = 1, \theta = 0.06 \) to decide the optimal solutions \( y^*, x^*, z^* \).

The computational results are listed in Table 2. We can from the results that, based on the criterion in step 2, the algorithm stops when \( n = 9 \). The optimal solution occurs at \( n = 5, TCU_{\min} = 93.50, y^* = 1000, x^* = 200 \). The number of vehicle used for delivery is 3.
Fig. 1: The relationship of each item in TCU with $y$.

Table 2: Computational results for Example 3

<table>
<thead>
<tr>
<th>$n$</th>
<th>$y_*$</th>
<th>TCU($y_*$)</th>
<th>TCU($y$)</th>
<th>TCU($y$)</th>
<th>$f(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>70.90</td>
<td>15546.01</td>
<td>125.50</td>
<td>125.50</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>74.98</td>
<td>152.49</td>
<td>102.50</td>
<td>102.50</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>78.56</td>
<td>116.10</td>
<td>96.17</td>
<td>96.17</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
<td>81.78</td>
<td>105.26</td>
<td>94.00</td>
<td>94.00</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>84.73</td>
<td>100.83</td>
<td>93.50</td>
<td>93.50</td>
</tr>
<tr>
<td>6</td>
<td>1200</td>
<td>87.47</td>
<td>98.97</td>
<td>93.83</td>
<td>93.83</td>
</tr>
<tr>
<td>7</td>
<td>1400</td>
<td>90.05</td>
<td>98.39</td>
<td>94.64</td>
<td>94.64</td>
</tr>
<tr>
<td>8</td>
<td>1600</td>
<td>92.48</td>
<td>98.55</td>
<td>95.75</td>
<td>95.75</td>
</tr>
</tbody>
</table>

Following computation indicates that when $n=16$, $f(n) = \text{TCU}(y_*) = 108.61 > \text{TCU}_{\min}$ So the optimal solution of the problem is the results of the trade-off of the transportation cost and the inventory cost.

5.0 Conclusion: In this study, we address an inventory problem arising from supplier-retailer system on the integration of production, inventory and transportation. In this study, unlike most of the prior inventory models, both of the fixed transportation cost and the variable transportation cost are accounted. In addition, multiple use of the vehicle is also considered since such arrangement can share the fixed transportation cost. A model for such problem is set up for the purpose of trading-off all of the costs related to the system and an algorithm for such model is presented. Computational results verify the proposed model as well as the efficiency of the algorithm.

REFERENCES


