

Nonlinear Identification of Wavenet Based Hammerstein Model – Case Study on High Purity Distillation Column

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Abstract: The application of Hammerstein model consists of a nonlinear static element described by wavenet based nonlinear function, followed by a linear block described by the Output Error (OE) model was investigated to represent the nonlinearity of the chemical processes. The model parameters were identified using iterative prediction-error minimization method and the proposed model was applied to high purity distillation column. The results proved that the Hammerstein model was capable of capturing the nonlinear dynamics of such process.

Keywords: Nonlinear identification, Hammerstein model, Distillation column

INTRODUCTION

Development of nonlinear model is the critical step in the application of nonlinear model based control strategies. Nonlinear behavior is the rule, rather than the exception, in the dynamic behavior of physical systems. Most physical devices have nonlinear characteristics outside a limited linear range. In most chemical processes, understanding the nonlinear characteristics is important for designing controllers that regulate the process^[1]. Many authors^[2,6] have noted the difficulty of developing the models required for nonlinear model-based control strategies. With carefully designed data collection experiments, the dominant behavior of plant can be fitted into one of the several possible structures. The main challenge in this task is to select a reasonable structure for the nonlinear model to capture the process nonlinearities. The nonlinear model used in control purposes should be as simple as possible, warranting less computational load and at the same time retain most of the nonlinear dynamic characteristics of the system.

The practical difficulty of nonlinear dynamic model development arises from several sources, of which the following two are fundamental. First fact is that model utility can be measured in several conflicting ways and second, the fact that the class of nonlinear models does not exhibit the unity that the class of linear models does. The four extremely important measures of model utility are approximation accuracy, physical interpretation, suitability for control and ease of development.

With carefully designed data collection experiments, the dominant behavior of plant can be

fitted into one of the several possible structures. Real world processes are nonlinear as a rule, rather than the expectation. Capturing the process nonlinearities is a crucial yet challenging task in nonlinear system identification. Many model structures have been proposed for the identification of nonlinear systems. The nonlinear static block followed by dynamic block in the Hammerstein structure has been found to be a simple and effective representation for capturing the dynamics of typical chemical engineering processes such as distillation columns^[7], heat exchangers^[1] and CSTR^[8]. The identification algorithm proposed by Pearson and Pottmann^[7] is simple, but cannot be used if the steady-state behavior is unknown. Nonlinear system identification involves the following tasks.

- Structure selection – Selection of suitable nonlinear model structure and number of model parameters
- Input sequence design – Determination of the input sequence $u(t)$ which is injected into the plant to generate the output sequence $y(t)$.
- Noise modeling – Determination of the dynamic model which generates noise input $e(t)$.
- Parameter estimation – Estimation of the remaining model parameters from the dynamic plant data $u(t)$ and $y(t)$ and the noise input $e(t)$.
- Model validation – Comparison of plant data and model predictions for data not used in model development

Many different techniques have been proposed for the black-box estimation of Hammerstein systems from

input-output measurements. These techniques mainly differ in the way that static nonlinearity is represented and in the type of optimization problem that is finally obtained. In parametric approaches, the static nonlinearity is expressed in a finite number of parameters. Both iterative and non-iterative methods have been used for determination of the parameters of the static-nonlinear and linear-dynamic parts of the model.

The Narendra and Gallman^[9] proposed an iterative algorithm which is referred as Narendra Gallman Algorithm (NGA) provided the initial momentum to block oriented modeling. This algorithm updates the linear dynamic element and the nonlinear gain polynomial separately and sequentially. The NGA provides accurate parameter estimation and is actually faster for higher order systems. Eskinat *et al.*,^[11] established the robustness of NGA to various level of noise. Also they compared the NGA with other well known identification methods such as prediction error method (PEM) and the recursive prediction error method (RPEM). They concluded that NGA and PEM are giving almost similar results for all types of noise levels where as RPEM is not suitable for all types of conditions. Lakshminarayanan *et al.*,^[10] have presented the identification of Hammerstein models using multivariate statistical tools. They obtained the parameters of linear system in state space form using canonical correlation analysis and adjusted the coefficients of the polynomial type nonlinear elements until convergence occurs.

Duwaish and Karim^[11] have developed the recursive algorithm for the identification of Hammerstein model and in their model the static nonlinear part is modeled by a multilayer feed forward neural network (MFNN) and the linear part is modeled by an autoregressive moving average (ARMA) model. Bai reported a two-stage identification algorithm based on the recursive least-squares and on the singular value decomposition^[12] and a blind identification approach^[13] to the Hammerstein model. Two identification algorithms, an iterative least-squares and a recursive least-squares, are developed for Hammerstein nonlinear systems with memoryless nonlinear blocks and linear dynamical blocks described by ARMAX/CARMA models^[14].

In this work, Hammerstein model consisted of a nonlinear static element described by wavenet based nonlinear function, followed by a linear block described by the Output Error (OE) model was explained. Even model parameters of the complicated nonlinear static functions can be estimated using the proposed iterative prediction-error minimization algorithm. Pilot plant distillation column is used as a platform for the identification of Hammerstein model. The remainder of

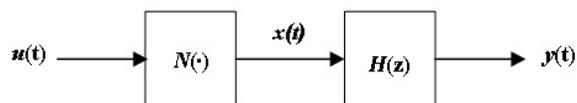


Fig. 1: Hammerstein model

this paper is organized as follows. In section 2, the wavenet based Hammerstein model structure is discussed and pilot plant distillation column details are given in section 3. Hammerstein model identification using iterative prediction-error minimization algorithm is explained in section 4 along with model validation results. Finally the concluding remarks are mentioned in section 5.

Hammerstein Model: The Hammerstein model consists of a nonlinear static element followed in series by a linear dynamic element. Hammerstein model has been considered as alternatives to linear models in a number of chemical process applications such as distillation column^[7], CSTR^[8], pH process^[15] etc.. The structure of the Hammerstein model is shown in Fig. 1, where $u(t)$ is an input and $x(t)$ is an output of the nonlinear static block. Simultaneously $x(t)$ is an input and $y(t)$ is an output of the linear dynamic block. In this work, a new wavenet based nonlinear function is used to describe the nonlinear static block and Output Error (OE) model is used to describe the linear dynamic block.

The linear block is the Output Error (OE) model, given by the following equation.

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})} x(t) \tag{1}$$

$$B(q^{-1}) = b_1 q^{-nk} + b_2 q^{-(nk+1)} + \dots + b_{nb} q^{-(nk+nb+1)} \tag{2}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \dots + a_{na} q^{-na} \tag{3}$$

where:

- nb is the number of coefficients in $B(q^{-1})$
- na is the number of coefficients in $A(q^{-1})$
- nk is the delay from input to output
- b_1, b_2, \dots, b_{nb} are the coefficients of polynomial B
- a_1, a_2, \dots, a_{na} are the coefficients of polynomial A

Wavenet structure based nonlinear function $x = F(u)$ is used to represent the static nonlinearity of the Hammerstein model.

$$\begin{aligned} F(u) = & (u - r)PL + as_1 f(bs_1((u - r)Qcs_1)) \\ & + \dots + as_k f(bs_k((u - r)Qcs_k)) \\ & + aw_1 g(bw_1((u - r)Q - cw_1)) \\ & + \dots + aw_k g(bw_k((u - r)Q - cw_k)) + d \end{aligned} \tag{4}$$

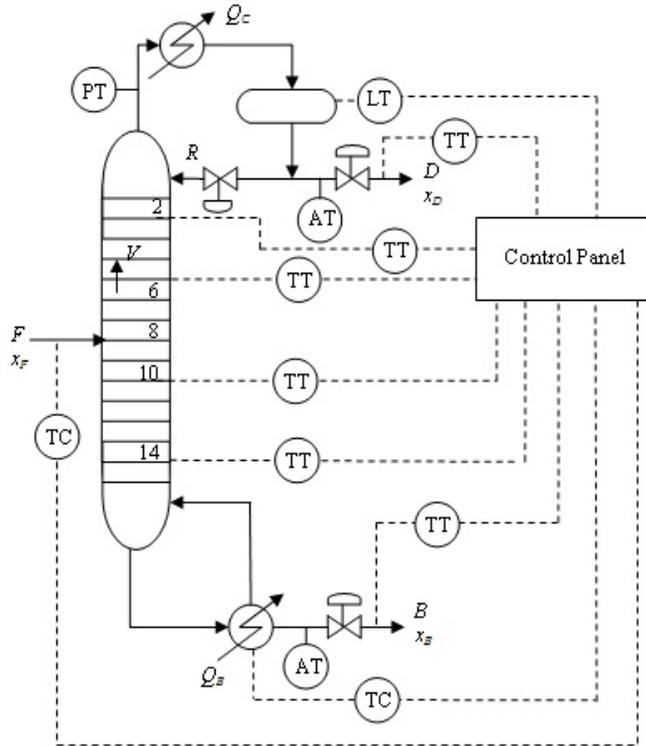


Fig. 2: Pilot plant distillation column

where:

$f(u)$ is the scaling function given by

$$f(u) = \exp(-0.5u'u) \tag{5}$$

$g(u)$ is the wavelet function given by

$$g(u) = (\dim(u) - u'u) \exp(-0.5u'u) \tag{6}$$

P is a $m \times p$ matrix

Q is a $m \times q$ matrix

r is a $1 \times m$ vector

L is a $p \times 1$ vector

cs_k is a $1 \times q$ vector

cw_k is a $1 \times q$ vector

d, as_k, bs_k, aw_k and bw_k are scalars

Parameters with the 's' are scaling parameters, and with the 'w' are wavelet parameters.

Distillation Column Case Study: The schematic of pilot plant distillation column utilized in this study is shown in Fig. 2. The methanol-water binary mixture was used as feed. The top and bottom product compositions are the controlled variables in distillation

Table 1: Nominal operating conditions and the column parameters

No. of trays	=	15
Feed tray number	=	8
Column diameter	=	10.8 cm
Weir height	=	2 cm
Weir length	=	8.3 cm
Feed composition	=	0.308
Feed flow rate	=	1 kmol/h
Distillate composition	=	0.98
Distillate flow rate	=	0.31 kmol/h
Bottoms flow rate	=	0.69 kmol/h
Reflux flow rate	=	1.09 kmol/h
Reflux drum hold up	=	0.015 m ³
Reboiler hold up	=	0.007 m ³
Mol. weight of methanol	=	32 kg/kmol
Mol. weight of water	=	18 kg/kmol

column. The reflux flow rate and reboiler vapor boil up rate are used as manipulated variables, whereas feed flow rate and feed composition are considered as disturbances. The nominal operating conditions and the column parameters are listed in Table 1. The reboiler and feed temperatures were controlled using separate PID controllers. In all the experiments, tray2, tray6, tray10, tray 14, distillate and bottom product temperatures are measured. The top and bottom product compositions are determined using refractive index analysis.

In this work, the data required for the nonlinear identification of Hammerstein model was generated

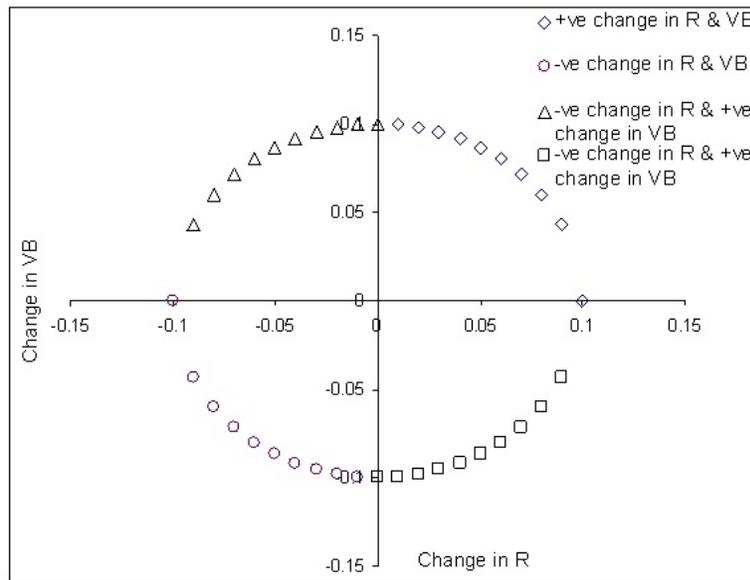


Fig. 3: Simultaneous changes in manipulated variables R and VB

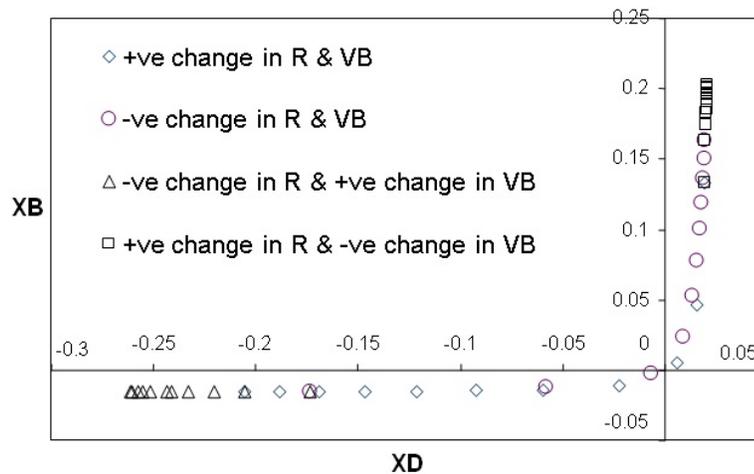


Fig. 4: Responses in top and bottom product compositions for simultaneous changes in R and VB

from validated first principle model by giving multilevel changes in the reflux flow rate. The calculation of activity and fugacity coefficients are included in order to account for the non-ideality. The activity coefficients are calculated using UNIFAC model and the fugacity coefficients are calculated using virial equation of state. The detailed model equations and the experimental validation of model were discussed in somewhere else^[16].

Nonlinearity Studies: The steady-state and dynamic simulation studies are carried out in MATLAB environment in order to verify the nonlinearity of the process. The steady-state behavior of the process is

studied by making simultaneous changes in the manipulated variables reflux flow rate and vapor boil up rate. If both the manipulated variables were changed simultaneously from +10% to -10% in a circular path as shown in Fig. 3, then the linear system will give the output in an elliptical form. When the system deviates from linearity then this ellipse will become non-elliptical. The responses in top and bottom product compositions for simultaneous changes in manipulated variables reflux flow rate and vapor boil up rate was shown in Fig. 4. The non-elliptical form of the response shows the nonlinearity of the system. The response surface has indicated that the values of process gains were different at different operating

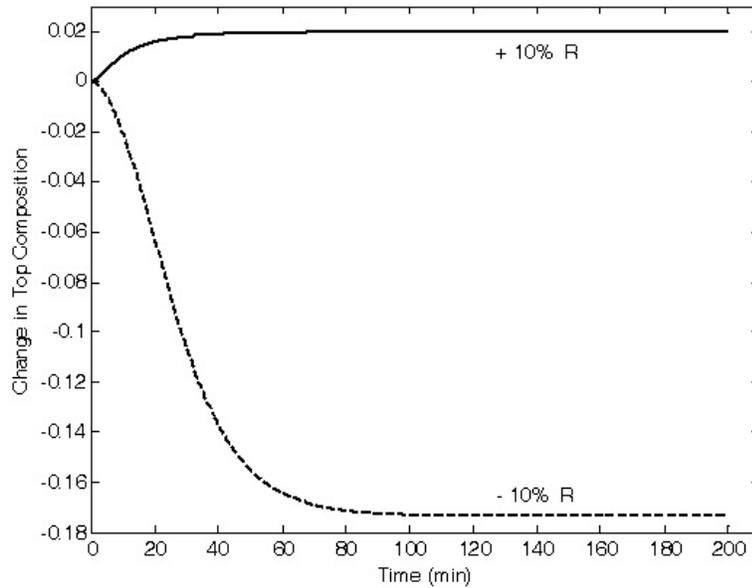


Fig. 5: Responses in top product composition for $\pm 10\%$ change in R

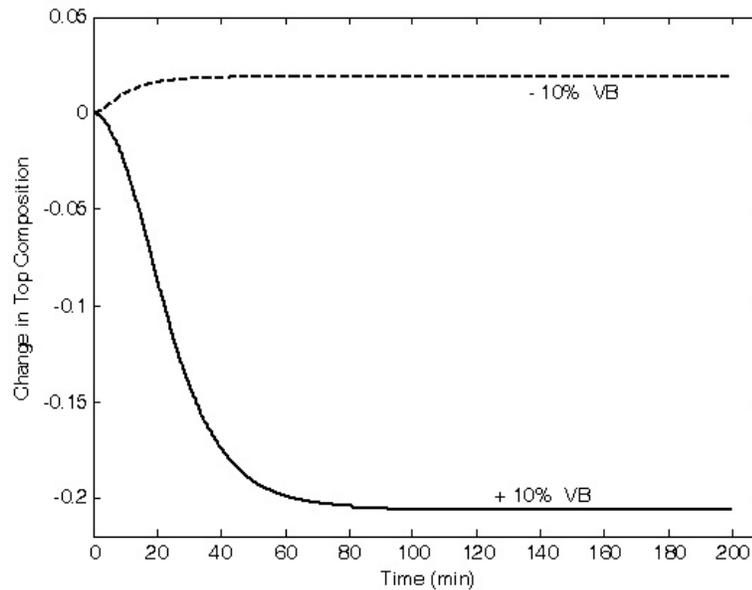


Fig. 6: Responses in top product composition for $\pm 10\%$ change in VB

conditions. Few points in Fig.4 have shown that there is a similar output for two distinctly different inputs, i.e. few points due to positive change in R and VB have produced similar outputs as that of few points due to negative change in R & VB, which clearly indicates the input multiplicity of the system.

The dynamic behavior of the process is studied by giving positive and negative step changes in reflux flow rate and vapor boil up rate. The response in top product composition change for + 10% change in reflux flow rate (R) is shown in Fig. 5. It

was observed that the top product composition change for -10% R is very high than that of +10% change in R, i.e. asymmetric responses to symmetric input changes and it indicates the violation of odd symmetry of the linear systems. The response in top product composition change for + 10% change in vapor boil up rate (VB) is shown in Fig. 6. It was noted that the top product composition was very much affected for +10% change in the vapor boil up rate compare to - 10% change.

Model Identification: The Hammerstein model considered here ultimately requires the estimation of unknown model parameters from input/output data, which uses iterative prediction-error minimization method for identification. Consequently, the iterative prediction-error minimization method to fit a Hammerstein model to data is discussed.

The output of the Hammerstein model is given by

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}((u-r)PL + as_1f(bs_1((u-r)Qcs_1)) + \dots + as_n f(bs_n((u-r)Qcs_n)) + aw_1g(bw_1((u-r)Q-cw_1)) + \dots + aw_k g(bw_k((u-r)Q-cw_k)) + d) \quad (7)$$

where the polynomials $A(q^{-1})$ & $B(q^{-1})$ are:

$$A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_nq^{-n} \quad (8a)$$

$$B(q^{-1}) = b_1q^{-1} + \dots + b_nq^{-n} \quad (8b)$$

The predictor of $y(t)$ is given by

$$\bar{y}(t, \theta) = \frac{B(q^{-1})}{A(q^{-1})}((u-r)PL + as_1f(bs_1((u-r)Qcs_1)) + \dots + as_n f(bs_n((u-r)Qcs_n)) + aw_1g(bw_1((u-r)Q-cw_1)) + \dots + aw_k g(bw_k((u-r)Q-cw_k)) + d) \quad (9)$$

To obtain the model parameters:

$$\theta = [a_1, \dots, a_n, b_1, \dots, b_n, as_1, \dots, as_k, bs_1, \dots, bs_k, aw_1, \dots, aw_k, bw_1, \dots, bw_k, d]^T \quad (10)$$

d, as_k, bs_k, aw_k and bw_k are nonlinear model parameters
We define the criterion

$$V(\theta) = \frac{1}{N} \sum_{t=1}^N \varepsilon^2(t, \theta) \quad (11)$$

N is the number of data points collected and $\varepsilon(t, \theta)$ is the prediction given by

$$\varepsilon(t, \theta) = y(t) - \bar{y}(t, \theta) \quad (12)$$

The model parameters θ can be obtained by minimizing the criterion given in Eq. (12). For the Hammerstein model, the derivative of the criterion is not a linear function of the parameters, so an analytical

solution cannot be found. Therefore iterative methods must be used to obtain a solution.

The iteration steps for the parameters can be expressed as:

$$\theta_{k+1} = \theta_k + \delta H(\theta_k)^{-1} g(\theta_k) \quad (13)$$

where:

δ - step length

$H(\theta)$ - Hessian of $V(\theta)$ or its approximation

$g(\theta)$ - Gradient of $V(\theta)$

For the Hammerstein model the gradient can be computed as:

$$g(\theta) = \frac{dV}{d\theta} = \frac{2}{N} \sum_{t=1}^m \varepsilon(t, \theta) \frac{\partial \varepsilon}{\partial \theta} \quad (14)$$

The derivatives $\frac{\partial \varepsilon}{\partial \theta}$ in the above equation can be

computer from eqs. (9) & (12) as

$$\frac{\partial \varepsilon(t)}{\partial a_j} = -\frac{1}{A(q^{-1})} \sum_{i=1}^m aw_i \frac{B(q^{-1})}{A(q^{-1})} u^i(t-j) \quad (15a)$$

$$\frac{\partial \varepsilon(t)}{\partial b_j} = -\frac{1}{A(q^{-1})} \sum_{i=1}^m bw_i u^i(t-j) \quad (15b)$$

$$\frac{\partial \varepsilon(t)}{\partial as_k} = \frac{B(q^{-1})}{A(q^{-1})} u^k(t) \quad (15c)$$

Since the Hessian is difficult to compute, it is usually approximated by another function. The most common method, Levenberg-Marquardt method, approximates the Hessian of the quadratic criterion to be minimized with:

$$H(\theta) \approx \frac{1}{N} \sum_{t=1}^N \frac{\partial \varepsilon(t, \theta)}{\partial \theta} \frac{\partial \varepsilon(t, \theta)^T}{\partial \theta} + \mu I \quad (16)$$

The identification algorithm for the Hammerstein model can be summarized as follows.

- Start iterations with initial estimate of parameters θ_0
- Compute the predictions of the model and $V(\theta_k)$
- Pick small value for μ (a typical choice is 0.0001)
- Compute the gradient and Hessian through Eqs. (15) and (16)
- Update parameter estimates through Eq.(13), and calculate the new $V(\theta_k)$

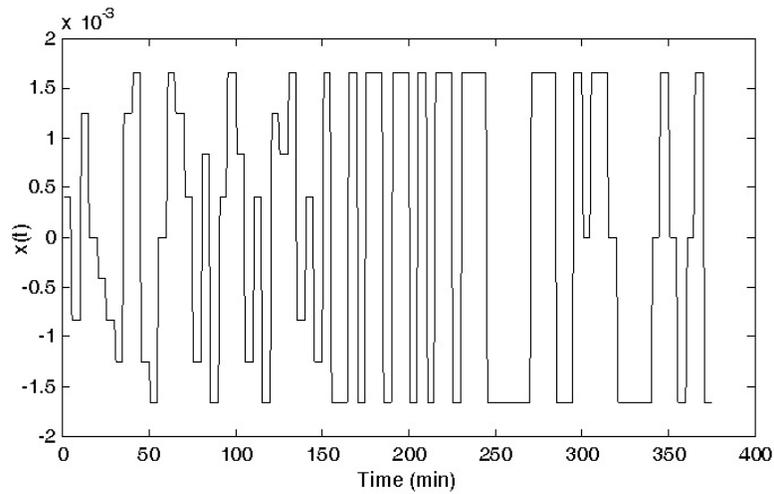


Fig. 7: Input $u(t)$ for multilevel changes in reflux flow rate

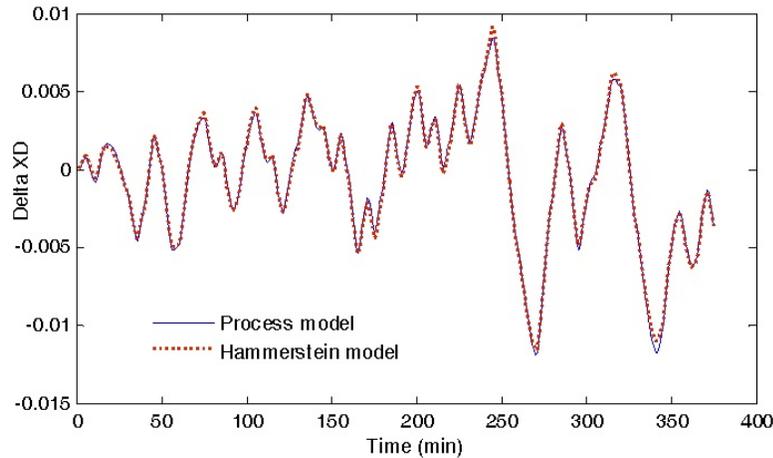


Fig. 8: Comparison of the output of the Hammerstein and fist principle model

If $V(\theta_k) > V(\theta_{k-1})$, update solution and decrease μ by a factor (say) 10 and go to step 2

If $V(\theta_k) > V(\theta_{k-1})$ increase μ by a factor (say) 10 and go to step 4.

Stop when the norm of gradient is below a criterion value or when the maximum number of iterations is reached.

Model Results and Validation: The linear Output Error (OE) model parameters are given by

- $a_1 = -0.523$
- $a_2 = 0.544$
- $b_1 = 0.872$
- $b_2 = 0.4479$

The values of other two terms P and Q associated with wavenet function model are same for this case.
 $P = Q = 709.64$

Linear term coefficient $L = 2.4045 \times 10^{-4}$

The values of other nonlinear parameters are given by

The output offset $d = 8.685 \times 10^{-6}$

Regressor mean $r = 2.77 \times 10^{-5}$

Wavelet coefficients $aw_k = 1 \times 10^{-3}$

$$\begin{bmatrix} -0.2075 \\ 0.1284 \\ -0.1584 \\ -0.0149 \end{bmatrix}$$

Wavelet dilation coefficients $bw_k =$

$$\begin{bmatrix} 8.0612 \\ 4.0357 \\ 4.3204 \\ 3.9636 \end{bmatrix}$$

$$\text{Wavelet translation coefficients } cw_k = \begin{bmatrix} 0.1802 \\ 0.1730 \\ 0.0551 \\ -1.1216 \end{bmatrix}$$

The different sets of data generated from validated first principle model were used for parameter estimation and model validation. The data used in the Hammerstein model validation is the one which is not used in the parameter estimation. The multilevel changes were made in reflux flow rate to generate $u(t)$ which is the input to the Hammerstein model. $u(t)$ is the difference between the steady-state value and the value of the reflux flow rate in the particular time instant.

$y(t)$ is the output of the Hammerstein model which is the difference between the steady-state value and the current value of the top product composition in the particular time instant. The comparison of the Hammerstein model output and the validated first principle model output was shown in Fig. 8. It has been found that the Hammerstein model results have shown 95.64% agreement with the experimentally validated first principle model results, which proved that the wavenet based nonlinear static element has captured the nonlinearity of the process.

Conclusion: Hammerstein model consisted of a nonlinear static element described by wavenet based nonlinear function, followed by a linear block described by the Output Error (OE) model have been presented for a nonlinear process and applied on high purity distillation column. The case study considered is an experimentally validated first principle distillation model. The nonlinear dynamic and steady-state characteristic of the distillation column has proved the nonlinearity of the process. The Hammerstein model parameters were identified using iterative prediction-error minimization method and the proposed model was applied to distillation column. The results proved that the wavenet based Hammerstein model was capable of capturing the nonlinear dynamics of distillation column.

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Nomenclature:

$A(q^{-1})$ - Polynomial function in linear model
 a_n - Coefficient of polynomial A

as_k - Scaling coefficient
 AT - Analysis transmitter
 aw_k - Wavelet coefficient
 $B(q^{-1})$ - Polynomial function in linear model
 b_n - Coefficient of polynomial B
 bs_k - Scaling dilation
 bw_k - Wavelet dilation
 cs_k - Scaling translation
 cw_k - Wavelet translation
 D - Distillate flow rate (kmol/h)
 d - Output offset
 $f(u)$ - Scaling function
 FL - Feed flow rate (kmol/h)
 $g(u)$ - Wavelet function
 $g(\theta)$ - Gradient of $V(\theta)$
 $H(z)$ - Linear dynamic model
 $H(\theta)$ - Hessian of $V(\theta)$ or its approximation
 LC - Level controller
 N - Number of data points
 $N(\cdot)$ - Static nonlinearity
 na - Number of coefficients in $A(q^{-1})$
 nb - Number of coefficients in $B(q^{-1})$
 nk - Delay from input to output
 P - Linear subspace matrix
 PT - Pressure transmitter
 Q - Nonlinear subspace matrix
 R - Reflux flow rate (kmol/h)
 R - Regressor mean vector
 TC - Temperature controller
 TT - Temperature transmitter
 $u(t)$ - Input to the Hammerstein model
 VB - Vapor boilup rate (kmol/h)
 $V(\theta)$ - Quadratic criterion
 XB - Bottom product composition
 XD - Top product composition
 XF - Feed composition
 $y(t)$ - Output of the Hammerstein model
 δ - Step length
 θ - Model parameter vector

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