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Interactive Three-Level Multiobjective Stochastic Linear Programming (TL-MSLP) Problem

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Abstract: In this study, consider three-level multiobjective stochastic linear programming (TL-MSLP) problem with chance constraints. Assume that there is randomness in the left-hand sides of the constraints and that the random variables are normally distributed. An interactive algorithm for solving such problem is presented. By using the chance-constrained technique, the problem converted from probabilistic into deterministic three-level multiobjective deterministic linear programming (TL-MOSLP) problem. This problem can be transform into separate multiobjective decision making problems, and solving it by using ε-constraint method.

Keywords: Three-Level programming; interactive algorithm; multiobjective decision-making problem;chance-constrained technique.

INTRODUCTION

Stochastic or probabilistic programming deals with situations where some or all of the parameters of the optimization problem are described by stochastic (or random or probabilistic) variables rather than by deterministic quantities[10,15].The sources of random variables my be several, depending on the nature and the type of problem. Decision problems of stochastic or probabilistic optimization arise when certain coefficients of an optimization model are not fixed or known but are instead, to some extent, stochastic (or random or probabilistic) quantities. In recent years methods of multiobjective stochastic optimization have become increasingly important in scientifically based decision making involved in practical problem arising in economic, industry, health care, transportation, agriculture, military purposes and technology.

A bi- level programming problem is formulate for a problem in which two decision makers (DMs) make decisions successively[1,3,4,8,12,13,14]. For example, in a decentralized firm, top management, an executive board or headquarters makes a decision such as a budget of the firm, and then each division determines a production plan in the full knowledge of the budget.

Three-level programming problem and their solution method have been presented as in[2,5,6,7,9,11].

Many researchers have developed various interactive algorithms for solving multicriteria decision making (MCDM) problem[11,12,13,14]. This study has proposed an interactive algorithm for solving three-level multiobjective stochastic linear programming problem with random parameters in the left hand side of constraints. The algorithm uses the concepts of satisfactoriness to multiobjective optimization at every level until a preferred solution concepts, the proposed solution method proceed from the first-level decision maker(FLDM) to the third-level decision maker (TLDM) passing through the second-level decision maker(SLDM).The (FLDM) gets the preferred or satisfactory solutions that are acceptable in rank order to the (SLDM).The(SLDM) will search for the preferred solution of the (FLDM) then he will gets this solution to the (TLDM) who will search for the preferred solution of the (SLDM)until the preferred solution is reached.

Problem Formulation and Solution Concept: Let $x_i \in R^{n_i}, (i=1,2,3)$ be a vector indicating the first decision level's choice, the second decision level's choice, and the third decision level's choice, $n_i \geq 1\ (i=1,2,3)$

Let $F_i : R^{n_1} \times R^{n_2} \times R^{n_3} \to R^{N_i}, (i=1,2,3)$ e the first-level objective functions, the second-level objective functions, and the third-level objective functions $N_i \geq 3, (i=1,2,3)$

Let the first-level decision maker (FLDM), the second level decision maker (SLDM) and the third level decision maker (TLDM) have N_1, N_2 and N_3 objective functions, respectively and G be the set of feasible choices $\{(x_1, x_2, x_3)\}$.So the three-level multiobjective stochastic linear programming(TL-MSLP) problem formulated as follows:

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[1st Level]

$$\underset{x_1}{Max}\, F_1(x_1,x_2,x_3) = \underset{x_1}{Max}(f_{11}(x_1,x_2,x_3),\ldots\ldots,f_{1N_1}((x_1,x_2,x_3))$$

[2st Level]

$$\underset{x_2}{Max}\, F_2(x_1,x_2,x_3) = \underset{x_2}{Max}(f_{21}(x_1,x_2,x_3),\ldots\ldots,f_{2N_2}((x_1,x_2,x_3))$$

[3st Level]

$$\underset{x_3}{Max}\, F_3(x_1,x_2,x_3) = \underset{x_3}{Max}(f_{31}(x_1,x_2,x_3),\ldots\ldots,f_{3N_3}((x_1,x_2,x_3))$$

subject to

$$G = \left\{ \begin{array}{l} (x_1,x_2,x_3) \mid P\{\sum_{j=1}^{n} a_{ij}x_j \le b_i\} \ge 1-\alpha_i, i=1,2,\ldots,m, \\ x_j \ge 0, j=1,2,\ldots,n \end{array} \right\}$$

P means probability and α_i is a specified probability value where, $0 \le \alpha_i \le 1$, Assume that the decision variables x_j are deterministic and also b_i.consider special case where only α_{ij} are random variables have normally distributed with known mean $E\{\alpha_{ij}\}$ and variance $var\{\alpha_{ij}\}$.Further the covariance of α_{ij} and $a_{i'j'}$ is given by cov $\{\alpha_{ij}, a_{i'j'}$ The basic idea in treating (TL-MSLP) problem is to convert the probabilistic nature of this problem into a deterministic form. Here, the idea of employing deterministic version will be illustrated by using the technique of chance-constrained programming[10,15].

Consider the ith constraint

$$P\{\sum_{j=1}^{n} a_{ij}x_j \le b_i\} \ge 1-a_i \quad ,$$

and define

$$h_i = \sum_{j=1}^{n} a_{ij}x_j$$

Then h_i is normally distributed with

$$E\{h_i\} = \sum_{j=1}^{n} E\{a_{ij}\}x_j \text{ and } var\{h_i\} = X^T D_i X$$

Where

$X = (x_1,x_2,x_3)^T$

$$D_i = i\text{th covariance matrix} = \begin{pmatrix} var\{a_1\} & cov\{a_1,a_{i2}\} \ldots\ldots cov\{a_{i1},a_{in}\} \\ cov\{a_{i2},a_{i1}\} & var\{a_{i2}\} \ldots\ldots\ cov\{a_{i2},a_{in}\} \\ \vdots & \\ cov\{a_{in},a_{i1}\} & cov\{a_{in},a_{i2}\} \ldots\ldots var\{a_{in}\} \end{pmatrix}$$

Now,

$$P\{h_i \le b_i\} = P\left\{\frac{h_i - E(h_i)}{\sqrt{var(h_i)}}\right\} \le \left\{\frac{b_i - E(h_i)}{\sqrt{var(h_i)}}\right\} \ge 1-\alpha_i$$

Where $(h_i - E\{h_i\}) / \sqrt{\text{var}\{h_i\}}$ is standard normal with mean zero and variance one. This means

$$P\{h_i \le b_i\} = \varphi\left\{\frac{b_i - E(h_i)}{\sqrt{\text{var}(h_i)}}\right\}$$

Where φ represents the cumulative distribution function (CDF) of the standard normal distribution.
Let k_{α_i} be the standard normal value such that

$$\varphi(k_{\alpha_i}) = 1 - \alpha_i$$

Then the statement $P\{h_i \le b_i\} \ge 1 - a_i$ is realized if and only if,

$$\frac{b_i - E(h_i)}{\sqrt{\text{var}(h_i)}} \ge k_{\alpha_i}$$

This yields the following nonlinear constraint

$$\sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{X^T D_i X} \le b_i$$

which is equivalent to the original stochastic constraint. For the special case where the normal distributions are independent, $\text{cov}\{a_{ij}, a_{i'j'}\} = 0$ and the constraint reduces to

$$\sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{\sum_{j=1}^{n} \text{var}(a_{ij})x_j^2} \le b_i$$

These are the deterministic nonlinear constraints equivalent to the original stochastic linear constraints.

Thus, problem (TL-MSLP) is equivalent to the following three level multiobjective deterministic nonlinear programming (TL-MDNP) problem:

[1st Level]

$$\underset{x_1}{Max}\, F_1(x_1, x_2, x_3)$$

[2st Level]

$$\underset{x_2}{Max}\, F_2(x_1, x_2, x_3)$$

[3st Level]

$$\underset{x_3}{Max}\, F_3(x_1, x_2, x_3)$$

subject to

$$G = \left\{\begin{matrix}(x_1, x_2, x_3) \mid \sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{\sum_{j=1}^{n} \text{var}(a_{ij})x_j^2} \le b_i, \\ i = 1, 2, \ldots, m \;,\; x_j \ge 0, j = 1, \ldots, n.\end{matrix}\right\}$$

Definition 1: For any x_1 ($x_1 \in G'_1 = \{x_1 \mid (x_1, x_2, x_3) \in G'\}$ given by FLDM, and x_2 ($x_2 \in G'_2 = \{x_2 \mid (x_1, x_2, x_3) \in G'\}$ given by SLDM, if the decision making variable x_3 ($x_3 \in G'_3 = \{x_3 \mid (x_1, x_2, x_3) \in G'\}$ is the non-inferior solution of the TLDM, then (x_1, x_2, x_3) is a feasible solution of (TL-MDLP) problem[9,13].

Definition 2: If (x_1^*, x_2^*, x_3^*) s a feasible solution of the (TL-MDLP) problem; no other feasible solution (x_1, x_2, x_3) $\in G'$ exists, such that (x_1^*, x_2^*, x_3^*) with at least one j (j = 1,......,n) is strict inequality, then (x_1^*, x_2^*, x_3^*) s the non-inferior solution of (TL-MDLP) problem[9,13].

Definitions and Theorems[9,13]: To obtain the solution of (TL-MDNP) problem solving (FLDM) problem and the lower levels problem each one separately. In this way, we can quantitatively present satisfactoriness and the preferred solution in view of singular-level multiobjective decision-making problem, and introduce several theorems with the help of the quality of ε-consentraint method multiobjective decision- making.

Consider a multiobjective decision making (MODM) problem as follows:

$$\text{Max } (f_1(x), \ldots, f_n(x))$$

subject to:

$$h_j(x) \geq 0, \; j = 1, \ldots\ldots, q,$$

where x denotes the decision making variable and $f_i(x)$, (i = 1,2,…,n) denotes the objective function of the multiobjective decision making problem.

Let $\Omega = \{x \mid \geq (x) \geq 0, \; j = 1, \ldots\ldots, q\}$, $\quad a_i = \underset{x \in \Omega}{Min} f_i(x)$, $b_i = \underset{x \in \Omega}{Max} f_i(x)$

On $u_i = [a_i, b_i]$ define $A_i \, \varepsilon \, f(u_i)$, whose membership function $\mu_{A_i} f_i(x))$ meets (i) and (ii) as below:

When the objective value $f_i(x)$ approaches or equals the decision maker's ideal value, $\mu_{A_i}(f_i(x))$ approaches or equals 1. Otherwise, 0.

If $\quad f_i(x) > f_i(x^*)$, then

$\mu_{A_i} f_i(x)) \; \mu_{A_i} f_i(x^*))$, i = 1,…, n.

Definition 3: If x^* is a non-inferior solution, then $\mu_{A_i}(f_i(x^*))$ is defined as the satisfactoriness of x^* to objective $f_i(x)$.

Definition 4: $\mu(x^*) = \text{Min } \mu_{A_i}(f_i(x^*))$ is defined as the satisfactoriness of non-inferior solution x^* to all the objectives.

Definition 5: If x, x^* are two non-inferior solution to the objective $f_i(x)$, then x^* is more preferred than x if $\mu_{A_i} f_i(x^*)) > \mu_{A_i} f_i(x))$.

Definition 6: With a certain value s_o given in advance by the decision maker, if non- inferior solution x^* satisfies $\mu(x^*) \geq s_o$, then x^* is the preferred solution corresponding to the satisfactoriness s_o.

The membership function $\mu_{A_i} f_i(x))$ is given as below:

$$\mu_{A_i}(f_i(x)) = 1 - \frac{b_i - f_i(x)}{b_i - a_i} \quad (1)$$

It is decided according to the decision maker's requirements. Obviously, (1) meets the two requirements for $\mu_{A_i} f_i(x))$.

The ε-constraint method[10] is effective for solving multiobjective decision making problems. The formulation of P (ε_{-1}) is as follows:

$$\text{Max } f_1(x)$$

subject to

$f_i(x) \leq \varepsilon_i$, $i = 2,\ldots, n$

$x \in \Omega$

Assume

$\varepsilon_{-1} = (\varepsilon_2, \ldots, \varepsilon_n)$,

$x^{/} (\varepsilon_{-1}) = \{ x | f_i(x) \leq \varepsilon_i , i = 2,\ldots, n, x \in \Omega\}$, and

$E_1 = \{ \varepsilon_{-1} \mid x^{/} (\varepsilon_{-1}) \neq \varphi$ (empty set)$\}$.

Theorem 1: If $\varepsilon_{-1} = (\varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n) \in E_1$, then the optimal solution to $P(\varepsilon_{-1})$ exists and includes a non-inferior solution of (MODM) problem.

Corollary: If x^1 is the only optimal solution to P (e_{-1}), then x^1 is the non-inferior solution of (MODM) problem. Given satisfactoriness s, if $\mu_{A_i} f_i (x)) \geq$ s, then by solving (I), obtain that:

$$f_i (x) = (b_i - a_i) \mu_{A_i} f_i (x)) + a_i \geq (b_i - a_i) s + a_i .$$

Let $\delta_i = (b_i - a_i) s + a_i$, (i = 1,2,…,n), ε_{-1} (s) = $(\delta_2 , \ldots, \delta_n)$.

Therefore, we can obtain P $(\varepsilon_{-1}(s))$, the ε constraint problem including satisfactoriness is as follows P $(\varepsilon_{-1}$ (s)):

$$\text{Max} \quad f_1(x)$$

subject to

$f_i(x) \geq \delta_i$, i = 2,3,…,n ,

$x \in \Omega$

Theorem 2: If P $((\varepsilon_{-1}$ (s)) has no solution or has the optimal solution $\bar{x}$ and $f_1(\bar{x}) \leq \delta_1$, then no non-inferior solution x^* exists, such that $\mu(x^*) \geq$ s.

Theorem 3: Assume s < s_1, if there is no preferred solution to s, then go to s_1.

Theorem 4: Assume $\bar{x}$ is an optimal solution of P$((\varepsilon_{-1}$ (s)), and $f_i(\bar{x}) \geq \delta_i$, (i = 1,2,…,n). Let $f_i(\bar{x}) = \varepsilon_i$ (i = 1,2,…. n) , and $\varepsilon_{-1} = (\varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n)$, then $\bar{x}$ is still an optimal solution of P (ε_{-1}).

- If $\bar{x}$ is the only optimal solution of P(ε_{-1}) , the $\bar{x}$ is non-inferior solution;
- If other optimal solution $x^{/}$ of P (ε_{-1}) exists, and L $\in$ {1,2,…,n} exists, such that $f_L (x^{/}) \geq \varepsilon_L$, then $\bar{x}$ is inferior solution.

An interactive Models for (TL-MDLP): To solve the (TL-MDLP) problem by adopting the three-planner stackelberg game[16], the FLDM gives the preferred or satisfactory solutions that are acceptable in rank order to the SLDM, and then the SLDM take the satisfactory solutions one by one to seek the solutions by ε-constraint method[5], and to arrive at the solution that gradually approaches the preferred solution of the FLDM, these variables are delivered to the TLDM who will seek the solutions by ε-constraint method, and to arrive at the solution that gradually approaches the preferred solution or satisfactory solution to the SLDM. Finally, the FLDM and the SLDM decides the preferred solution of the (TL-MDLP) problem according to they satisfactoriness.

The First-level Decision Maker (FLDM) Problem: The first-level decision maker (FLDM) problem of the (TL-MDNP) problem is as follows:

$$\underset{x_1}{Max} F_1 (x_1, x_2, x_3) = \underset{x_1}{Max} \ (f_{11} (x_1, x_2, x_3), \ldots\ldots, (f_{1N_1} (x_1, x_2, x_3))$$

subject to

$$G' = \left\{ \begin{array}{l} (x_1, x_2, x_3) \mid \sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{\sum_{j=1}^{n} \operatorname{var}(a_{ij})x_j^2} \le b_i, \\ i = 1, 2, \ldots, m \ , \ x_j \ge 0, j = 1, \ldots, n. \end{array} \right\}$$

To obtain the preferred solution of the (FLDM) problem, transform this problem into the following single objective decision-making problem which are introduced

$Max\ f_{11}\ (x_1,\ x_2,\ x_3)$

subject to:

$$f_{1j}(x_1, x_2, x_3) \ge \delta_{1j}, \quad j = (1, 2, \ldots, N_1) \qquad (2)$$

$$G' = \left\{ \begin{array}{l} (x_1, x_2, x_3) \mid \sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{\sum_{j=1}^{n} \operatorname{var}(a_{ij})x_j^2} \le b_i, \\ i = 1, 2, \ldots, m \ , \ x_j \ge 0, j = 1, \ldots, n. \end{array} \right\}$$

According to the definitions and theorems which are introduced in section 3, the algorithm steps for solving the above problem are as follows:

The algorithm for solving (FLDM) problem:
Step 1: Set the satisfactoriness. Let $s = s_o$ at the beginning, and set $s = s_1, s_2, \ldots$, respectively.

Step 2: Set the ε-constraint problem $P_1\ (\varepsilon_{-1}(s))$, if $P_1(\varepsilon_{-1}(s))$ has no solution or has a non-inferior solution making $f_{11}(\bar{x}_1, \bar{x}_2, \bar{x}_3) < \delta_{11}$, then go to step 1, to adjust $s = s_{j+1} < s_j$. Otherwise, go to step 3.

Step 3: Assuming that $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ an optimal solution of $P(\varepsilon_{-1}(s))$, judge by theorem 4 whether or not $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ is a non-inferior solution. If $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ is a non-inferior solution, turn to step 4. If $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ s inferior solution, there must be a $(\bar{x}'_1, \bar{x}'_2, \bar{x}'_3)$ such that $f_{11}(\bar{x}'_1, \bar{x}'_2, \bar{x}'_3) \ge f_{11}(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ and at least one " >"; Repeat step 3 with $(\bar{x}'_1, \bar{x}'_2, \bar{x}'_3)$

Step 4: If the decision – maker is satisfied with $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ then $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ s a preferred solution. Otherwise, go to step 5.

Step 5: Adjust the satisfactoriness. Let $s = s_{j+1} > s_j$, and go to step 2.

The Second-level Decision-maker (SLDM) Problem: According to the interactive mechanism of the (TL-MDNP) problem, the (FLDM) variables x_1^F should be given to the SLDM; hence, the (SLDM) problem can be written as follows:

$$\underset{x_2}{Max} F_2(x_1^F, x_2, x_3) = \underset{x_2}{Max}(f_{21}(x_1^F, x_2, x_3), \ldots\ldots, (f_{2N_2}(x_1^F, x_2, x_3))$$

subject to:

$$G' = \left\{ \begin{array}{l} (x_1^F, x_2, x_3) \mid \sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{\sum_{j=1}^{n} \operatorname{var}(a_{ij})x_j^2} \le b_i, \\ i = 1, 2, \ldots, m \ , \ x_j \ge 0, j = 1, \ldots, n. \end{array} \right\}$$

The (SLDM) problem will be converted into the following single objective function as follows $P_2(\varepsilon_{-1}(s))$:

$$Max\, f_{21}(x_1^F, x_2, x_3)$$

subject to

$$f_{2j}(x_1^F, x_2, x_3) \geq \delta_{2j}, \quad j = (2,\ldots,N_2) \qquad (3)$$

$$G' = \left\{ \begin{array}{l} (x_1^F, x_2, x_3) \mid \sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{\sum_{j=1}^{n} \operatorname{var}(a_{ij})x_j^2} \leq b_i, \\ i = 1,2,\ldots,m \;,\; x_j \geq 0, j = 1,\ldots,n. \end{array} \right\}$$

Our basic though on solving (3) is to find the second-level non-inferior solution (x_1^F, x_2^s, x_3^s) that is closest to the FLDM preferred solution (x_1^F, x_2^F, x_3^F) Now, we will test whether (x_1^F, x_2^s, x_3^s) is preferred solution to the (FLDM) problem or it may be changed, by the following test:

If $$\frac{\left\| F_1(x_1^F, x_2^F, x_3^F) - F_1(x_1^F, x_2^s, x_3^s) \right\|}{\left\| F_1(x_1^F, x_2^s, x_3^s) \right\|_2} < \delta^F$$

So (x_1^F, x_2^s, x_3^s) a preferred solution to the FLDM, where δ^F is a small positive constant given by the FLDM.

The third-level decision maker (TLDM) problem: In the same way, the SLDM variables x_2^S should give to the TLDM problem is as follows:

$$\underset{x_3}{Max}\, F_3(x_1^F x_2^s, x_3) = \underset{x_3}{Max}\left(f_{31}(x_1^F x_2^s, x_3), \ldots\ldots, f_{3N_3}(x_1^F x_2^s, x_3)\right)$$

subject to

$$(x_1^F x_2^s, x_3) \quad G'$$

The (TLDM) problem will be converting into the following single- objective function:

$$\underset{x_3}{Max}\, f_{31}(x_1^F, x_2^s, x_3)$$

subject to

$$f_{3j}(x_1^F, x_2^s, x_3) \geq \delta_{3j}, \quad j = (2,\ldots,N_3) \qquad (4)$$

$$G' = \left\{ \begin{array}{l} (x_1^F, x_2^s, x_3) \mid \sum_{j=1}^{n} E(a_{ij})x_j + k_{\alpha_i}\sqrt{\sum_{j=1}^{n} \operatorname{var}(a_{ij})x_j^2} \leq b_i, \\ i = 1,2,\ldots,m \;,\; x_j \geq 0, j = 1,\ldots,n. \end{array} \right\}$$

Our basic though on solving (4) is to find the TLDM non-inferior solution closest to the SLDM preferred solution (x_1^F, x_2^s, x_3^T)

Now we will test whether (x_1^F, x_2^s, x_3^s) is a preferred solution to the (SLDM) problem or it may be changed by the following test.

If $$\frac{\left\| F_2(x_1^F, x_2^s, x_3^s) - F_2(x_1^F, x_2^s, x_3^T) \right\|_2}{\left\| F_2(x_1^F, x_2^s, x_3^T) \right\|_2} < \delta^s$$

So (x_1^F, x_2^s, x_3^T) is a preferred solution to the SLDM which means (x_1^F, x_2^s, x_3^T) is a preferred solution of the (TL-MDNP) problem , where δ^S is a small positive constant given by the SLDM .

Interactive Algorithm for Solving (TL-MDLP) problem:

Step 1:

- Transform the three-level multiobjective stochastic linear programming
- (TL-MSLP) problem into equivalent three- level multiobjective deterministic nonlinear programming (TL-MDNOP) problem.

Step 2:

- Set k = 0, solve the 1^{st} level decision –making problem to obtain a preferred solutions that are acceptable to the FLDM. The FLDM puts the solutions in order in the format as follows:
- Preferred solution,

$$(x_1^k, x_2^k, x_3^k), \ldots\ldots\ldots\ldots, (x_1^{k+p}, x_2^{k+p}, x_3^{k+p})$$

- Preferred ranking (satisfactory ranking),

$$(x_1^k, x_2^k, x_3^k) > (x_1^{k+1} x_2^{k+1} x_3^{k+1}) > \ldots\ldots\ldots\ldots > (x_1^{k+p}, x_2^{k+p}, x_3^{k+p})$$

Step 3: Given $x_1 = x_1^F$ the SLDM solve the SLDM problem to obtain x_2.

Step 4:

If
$$\frac{\left\| F_1(x_1^F, x_2^F, x_3^F) - F_1(x_1^F, x_2^s, x_3^s) \right\|}{\left\| F_1(x_1^F, x_2^s, x_3^s) \right\|_2} < \delta^F$$

Where δ^F is a fairly small positive number given by the FLDM , then go to step 5. Otherwise, go to step 8.

Step 5: Given $x_1 = x_1^F$ $x_2 = x_2^s$ to the TLDM,solve the third -level decision making problem to obtain x_3.

Step 6:

If
$$\frac{\left\| F_2(x_1^F, x_2^s, x_3^s) - F_2(x_1^F, x_2^s, x_3^T) \right\|_2}{\left\| F_2(x_1^F, x_2^s, x_3^T) \right\|_2} < \delta^s$$

Where δ^S is a fairly small positive number given by the SLDM in the view of the FLDM ,then go to step 7. Otherwise, go to step 8.

Step 7: (x_1^F, x_2^s, x_3^T) is the preferred solution to the (TL-MSLP) problem, go to step 9.

Step 8: Set $k = k + 1$, and go to step 3.

Step 9: Stop.

REFERENCES

1. Abo-Sinna, M.A., 2001. "A bi-level non linear multiobjective decision-making under fuzziness"Journal of Operational Research Society of India(OPSEARCH), 38(5): 484-495.
2. Anandolingam, G. and V Apprey, 1991. "Multi-level programming and conflict resolution" European Journal of Operations Research, 51: 233-247.
3. Bard, J.F., J. Plummer and J.C. Sourie, 2000. "A bi-level Programming approach to determining tax credits for biofuel production" European Journal of Operational Research, 120: 30-46.
4. Calvete, H.I. and C. Gale, 1999. "The bi-level linear/ linear fractional programming problem"

European Journal of Operational Research, 114: 188-179.
5. Chakang, V. and V.Y. Haimes, 1983. “Multi-objective decision-making theory and methodology” North Holland, series in system science and Engineering.
6. Hsu, S.S., Y.J. Lai and L.E. Stanley, 1996. “Fuzzy approach for multi-level programming problems” Computers and Operation Research, 32(1): 73-91.
7. Hsu, S.S. and L.E. Stanley, 2001, “Fuzzy and Multi-level decision making” springer- verlag London Limited.
8. Marcotte, P., G. Savard and D.L. Zhu, 2001. “A trust region algorithm for nonlinear bi-level programming” Operations research letters, 29: 171-179.
9. Osman, M.S., M.A Abo-Sinna, A.H Amer and O.E Emam, 2003. "Interactive decision-making for three-Level multiobjective nonlinear Programming Problems", The 38th Annual conference on Statistics, Computer Sciences and Operation Research, 95-110.
10. Rao, S.S, 1977. "Optimization theory and application ", Indian Institute of technology, Kanpur.
11. Sakawa, M., 1993. "Fuzzy sets and interactive multi-objective optimization" plenum press. New Yourk.
12. Sakawa, M. and I. Nishizaki, 2001. "Interactive fuzzy programming for two-level linear fractional programming problem "Fuzzy Sets and System, 119: 31-40.
13. Shi, X and H. Xia, 1997. "Interactive bi-level multiobjective decision-making" Journal of Operational Research Society, 48: 943-949.
14. Shi, X and H. Xia, 1997. ”Interactive bi-level multi objective decision-making” Journal of Operational Research Society, 48: 943-949.
15. Shi, X and H. Xia, 2001. "Model and interactive algorithm of bi-level multiobjective decision making with multiple interconnected decision makers" Journal of Multi-criteria Decision Analysis, 10: 27-34.
16. Taha, H.A., 1976. "Operations Research An introduction", Macmillan publishing Co. INC, New York.
17. Thomas, L. and J. Walter, 1981. "Optimality in Parametric System", John Wiley and Sons, New York, London.