Design and Analysis of a Bipolar Charge Pump for Thermoelectric Applications

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Abstract

Background: For any generic energy-harvesting systems, a switching DC-DC converter is used to utilize energy from ambient energy sources. Among others, a high step-up converter is required for the energy-harvesting system utilizing thermoelectric energy, because the thermoelectric generator (TEG) provides only the small output voltage when the temperature difference between two module sides (hot side and cool side) is small. Objective: For the energy-harvesting system utilizing thermoelectric energy, a high step-up switching converter is designed by combining novel positive/negative charge pumps using a saving power technique. Furthermore, to obtain handy theoretical formulas for the proposed charge pump, an analysis method considering the on-resistance of transistor switches is proposed. Results: The simulation program with integrated circuit emphasis (SPICE) simulation showed the following results: (1) By the proposed three-stage bipolar charge pump, about a 9.1V output was obtained from a 1.5V input, (2) The proposed positive charge pump improved power efficiency more than 5% when the output load RL is 100kΩ. On the other hand, the proposed negative charge pump improved power efficiency more than 8% when the output load RL is 100kΩ, (3) The theoretical results were in good agreement with the SPICE simulated results, and (4) The standard deviation of the output voltage and power efficiency was less than 0.1 when capacitors and on-resistances have 10% tolerance with Gaussian distribution. Conclusion: In this paper, we proposed a bipolar charge pump and its analysis method for the energy-harvesting system utilizing thermoelectric energy. The conclusion of this research is as follows: (1) By combining positive/negative charge pumps, the proposed bipolar charge pump can achieve high step-up gain, (2) By reusing a part of the electric charge in stray parasitic capacitances, the proposed charge pump can achieve high power efficiency when the output load RL is a large value, (3) The proposed analysis method will be helpful to estimate the maximum power efficiency and the maximum output voltage of the charge pump, and (4) The proposed bipolar charge pump is robust to the fluctuation of circuit components.

Introduction

Recently, energy harvesting to take advantage of renewable energy attracts many researchers' attention. Among others, we focus on the energy harvesting system utilizing thermoelectric energy, where a thermoelectric generator (TEG) is used to extract energy from waste heat. For any generic energy harvesting systems, a switching DC-DC converter is used to convert electrical energy extracted from ambient energy sources. For example, Kim et al. (2013) designed a DC-DC boost converter for thermoelectric energy harvesting applications. By employing the maximum power point tracking technique, the boost converter can extract the energy from the TEG effectively. However, the boost converter requires a magnetic element. On the other hand, Doms et al. (2009) proposed a capacitive power management circuit for wireless sensor systems, where a positive charge pump with variable number of stages (Doms et al. (2009), Huang et al. (2012), Hwang et al. (2009), and Palumbo et al. (2002)) was used to convert the energy delivered by a TEG. Unlike the inductor-based converter such as a boost converter, the capacitor-based converter such as a positive charge pump can be implemented into an integrated circuit (IC) form, because the capacitor-based converter requires no magnetic components. Therefore, the energy harvesting system using charge pumps can achieve light weight, thin circuit composition, no flux of magnetic induction, and so on. However, to realize more efficient thermoelectric energy harvesting system, the capacitor-based converter which can realize not only IC-implementable structure but also...
high step-up gain is desirable. The voltage produced by the TEG depends on the temperature difference between two of its sides (hot side and cool side). The hot side is mounted on waste heat sources (e.g., external surface of steam pipe or heat exchanger) whereas the cool side is exposed to open air. Therefore, the capacitor-based converter with high step-up gain is desirable when the temperature difference between two module sides is small.

In this paper, a high step-up charge pump and its analysis method are proposed for the energy-harvesting system utilizing thermoelectric energy. Unlike the conventional positive charge pump, the proposed bipolar charge pump consists of novel positive/negative charge pumps using a power saving technique. In the proposed positive/negative charge pumps, a part of the electric charge in stray parasitic capacitances is reused to reduce parasitic power losses. By combining the outputs of these charge pumps, the proposed charge pump achieves not only high step-up gain but also high power efficiency. Furthermore, the analysis method considering the on-resistance of transistor switches is proposed to obtain handy theoretical formulas, because the theoretical analysis considering internal losses has not been performed in previous studies of the charge pump (Allasasmeh et al. (2010), Doms et al. (2009), Huang et al. (2012), Hwang et al. (2009), and Palumbo et al. (2002)). To confirm the validity of the proposed charge pump, simulation program with integrated circuit emphasis (SPICE) simulations and theoretical analysis are performed concerning the proposed charge pump with three stages.

The rest of this paper is organized as follows. In Section 2, the structure of the conventional charge pump and the proposed charge pump are presented. In Section 3, the characteristic of the proposed charge pump is analyzed theoretically. Simulation results are shown in Section 4. Finally, conclusion and future work are drawn in Section 5.

Charge Pump:

Conventional Charge Pump:

Figure 1 shows the circuit topology of the conventional charge pump (Doms et al. 2009) for micropower thermoelectric generators (TEGs). The conventional charge pump of Figure 1 consists of $3N + 1$ ($N = 1, 2, \ldots$) transistor switches $S_{pk}$ ($k = 1, 2, \ldots, N$), and an output capacitor $C_{out}$. In Figure 1, $C_{pk}$ denotes the stray parasitic capacitance between top plate and substrate, and $C_{sk}$ denotes the stray parasitic capacitance between bottom plate and substrate. The transistor switch is driven by non-overlapped two-phase clock pulse $\Phi_1$ and $\Phi_2$. By controlling power switches, the conventional positive charge pump achieves $(N + 1)\times$ step-up conversion as follows (Palumbo et al. 2002):

$$V_{out} = (N + 1)V_{in} - \frac{NI_{out}}{fC},$$

where $f$ is the pumping frequency, $C$ is the size of the main capacitor, and $I_{out}$ is the output current. However, most of the previous studies have not taken into account the influence of stray parasitic capacitances $C_{pk}$ and $C_{sk}$ ($k = 1, 2, \ldots, N$). In the conventional charge pump of Figure 1, energy stored in $C_{sk}$ is consumed idly when the main capacitor $C_{k}$ is connected to the ground through $S_{pk}$. Of course, the power efficiency of the charge pump consisting of discrete components is mainly limited by capacitor charging and discharging losses and resistive conduction losses. However, the energy loss due to stray parasitic capacitances cannot be ignored in the small power application such as energy harvesting systems.

To alleviate the influence of stray parasitic capacitances, several techniques have been proposed. For example, Allasasmeh et al. (2010) proposed the positive charge pump using a charge reusing technique, where two symmetrical converter blocks are operated in parallel by using complementary control signals. Lauterbach et al. (2000), Huang et al. (2012), and Hwang et al. (2009) proposed the positive charge pump using a charge sharing clock scheme, where the transistor switch is driven by the clock pulse which combines two-step adiabatic switching. However, in previous studies, these techniques have not been applied for the negative charge pump. Furthermore, as (1) shows, the theoretical analysis considering internal losses such as the on-resistance of the transistor switch has not been performed in previous studies of the charge pump (Allasasmeh et al. 2010, Doms et al. 2009, Huang et al. 2012, and Hwang et al. 2009 and Palumbo et al. 2002). To clarify the characteristics of the charge pump, handy theoretical formulas are necessary.

Proposed Charge Pump:

Figure 2 shows the circuit topology of the proposed bipolar charge pump. Unlike the conventional charge pump, the proposed charge pump consists of a positive charge pump and a negative charge pump. By combining positive/negative charge pumps, the proposed charge pump generates the following stepped-up voltage:

$$V_{out} \equiv V_{op} - V_{on},$$

$$\equiv (N + 1)\frac{V_{in}}{fC} - (N)\frac{V_{in}}{fC} = (2N + 1)\frac{V_{in}}{fC}.$$
In general, the amount of electric power produced by the TEG depends on the temperature difference between two of its sides (hot side and cool side). For this reason, the TEG provides only the small output voltage when the temperature difference between two module sides is small.

As Figure 2 and (2) show, the proposed charge pump employs a bipolar structure to achieve high step-up gain. To alleviate the influence of stray parasitic capacitances, the proposed positive/negative charge pumps are controlled by non-overlapped three-phase clock pulses $\Phi_1$, $\Phi_2$, and $\Phi_3$. As Figure 2 shows, the switches $S_{p3}$ and $S_{n3}$ are driven by $\Phi_3$ after $\Phi_1$ and $\Phi_2$ were turned on. In State-$T_3$, the electric charges stored in $C_{bk}$ ($k = 1, 2, \ldots, N$) are equalized through $S_{p3}$ and $S_{n3}$ before the electric charge stored in $C_{bk}$ is consumed idly. In other words, the power dissipation of the input can be reduced by the equalization process. Therefore, due to the equalization process of the electric charge in $C_{bk}$, the power dissipation of the proposed charge pump can be reduced.

Fig. 1: Conventional positive charge pump.

Fig. 2: Proposed bipolar charge pump; (a) Block diagram, (b) Positive charge pump block, and (c) Negative charge pump block.
Theoretical Analysis:
In this section, the property of the proposed charge pump is analyzed theoretically. Unlike the conventional method, the theoretical analysis considering the on-resistance of transistor switches is discussed in this analysis.

To evaluate the maximum output voltage, the theoretical analysis is performed under conditions that (1) Parasitic elements are negligibly small and (2) Time constant is much larger than the period of clock pulses.

Positive Charge Pump:
Figure 3 shows the instantaneous equivalent circuits of the positive charge pump. In Figure 3, $R_{on}$ is the on-resistance of the transistor switch. Firstly, the equivalent circuit of the converter block is derived, because the positive charge pump has a symmetric structure. In the steady state, the differential value of electric charges in $C_{pk}$ ($k=1, 2, \ldots, N$) satisfies the following equations:

$$
\Delta q_{T1}^{pk} + \Delta q_{T2}^{pk} + 2 \Delta q_{T3}^{pk} = 0,
$$

where $\Delta q_{Ti}^{pk}$ ($i=1, 2, 3$) and ($k=1,2, \ldots, N$) denote the electric charges of the $k$-th capacitor in the case of State-$T_i$. The interval of State-$T_i$ satisfies the following conditions:

$$
T = T_i + T_i + 2T_i, \quad T_i = T_2 = \frac{(1-2\delta)t}{2}, \quad \text{and} \quad T_i = \delta T
$$

where $T$ is the period of clock pulses, $T_i$ ($i=1, 2, 3$) is the pulse width of $\Phi_i$, and $\delta$ is the parameter to determine the time of State-$T_3$. In State-$T_i$, the differential values of electric charges in the input $V_{in}$ and the output $V_{op1}$, $\Delta q_{T1,in}$ and $\Delta q_{T1,op1}$, are expressed as

$$
\Delta q_{T1,in} = \begin{cases} 
\Delta q_{T1}^{(e)} & \text{if } N \text{ is an even number}, \\
\Delta q_{T1}^{(o)} & \text{if } N \text{ is an odd number}, 
\end{cases}
$$

and

$$
\Delta q_{T1,op1} = \begin{cases} 
\Delta q_{T1}^{(e)} + \Delta q_{T1}^{(o)} & \text{if } N \text{ is an even number}, \\
\Delta q_{T1}^{(e)} & \text{if } N \text{ is an odd number}. 
\end{cases}
$$
In State-$T_2$, the differential values of electric charges in $V_{in}$ and $V_{opt}$, $\Delta q_{T2,in}$ and $\Delta q_{T2,opt}$, are expressed as

$$\Delta q_{T2,in} = \begin{cases} \sum_{i=1}^{N/2} \Delta q_{T2,in}^{(2i-1)} & \text{if } N \text{ is an even number}, \\ \sum_{i=1}^{(N/2)^2} \Delta q_{T2,in}^{(2i)} & \text{if } N \text{ is an odd number}. \end{cases}$$

and

$$\Delta q_{T2,opt} = \begin{cases} \Delta q_{T2,opt}^{(2i)} \quad & \text{if } N \text{ is an even number}, \\ \Delta q_{T2,opt}^{(2i+1)} + \Delta q_{T2,opt}^{(2i+2)} \quad & \text{if } N \text{ is an odd number}. \end{cases}$$

In State-$T_3$, $\Delta q_{T3,in}$ and $\Delta q_{T3,opt}$, are expressed as

$$\Delta q_{T3,in} = 0 \quad \text{and} \quad \Delta q_{T3,opt} = \Delta q_{T3}^{opt}.$$  

Furthermore, the following equations are obtained:

$$\Delta q_{T1}^{p1} = \Delta q_{T1}^{p2} = \Delta q_{T1}^{p3} = \cdots = \Delta q_{T1}^{p(N-1)} \quad \text{and} \quad \Delta q_{T1}^{p2} = \Delta q_{T1}^{p4} = \Delta q_{T1}^{p6} = \cdots = \Delta q_{T1}^{pN}$$

and

$$\Delta q_{T1}^{p1} = \Delta q_{T1}^{p2} = \Delta q_{T1}^{p3} = \cdots = \Delta q_{T1}^{pN} \quad \text{if } N \text{ is an even number}. \quad \text{(10)}$$

$$\Delta q_{T1}^{p1} = \Delta q_{T1}^{p3} = \Delta q_{T1}^{p5} = \cdots = \Delta q_{T1}^{p(N-1)} \quad \text{and} \quad \Delta q_{T1}^{p2} = \Delta q_{T1}^{p4} = \Delta q_{T1}^{p6} = \cdots = \Delta q_{T1}^{pN}$$

and

$$\Delta q_{T1}^{p1} = \Delta q_{T1}^{p2} = \Delta q_{T1}^{p3} = \cdots = \Delta q_{T1}^{pN} \quad \text{if } N \text{ is an odd number}. \quad \text{(11)}$$

Using (5)-(9), the average input current and the average output current can be expressed as

$$\bar{I}_{in} = \frac{\Delta q_{V_{in}}}{T} = \frac{\Delta q_{T2,in} + \Delta q_{T1,in} + 2\Delta q_{T3,in}}{T}$$

and

$$\bar{I}_{opt} = \frac{\Delta q_{opt}}{T} = \frac{\Delta q_{T2,opt} + \Delta q_{T1,opt} + 2\Delta q_{T3,opt}}{T}. \quad \text{(12)}$$

In (12), $\Delta q_{V_{in}}$ and $\Delta q_{V_{opt}}$ are electric charges in $V_{in}$ and $V_{opt}$, respectively. Substituting (3)-(11) into (12), we have the relation between the average input current and the average output currents as follows:

$$\bar{I}_{in} = -(N+1)\bar{I}_{opt}. \quad \text{(13)}$$

where

$$\Delta q_{V_{in}} = -(N+1)\Delta q_{T1}^{pN} \quad \text{and} \quad \Delta q_{V_{opt}} = \Delta q_{T1}^{pN} \quad \text{if } N \text{ is an even number.}$$

$$\Delta q_{V_{in}} = (N+1)\Delta q_{T1}^{pN} \quad \text{and} \quad \Delta q_{V_{opt}} = -\Delta q_{T1}^{pN} \quad \text{if } N \text{ is an odd number.}$$

Next, let us consider the consumed energy in one period. Using (3)-(11), the consumed energy $W_T$ can be expressed as

$$W_T = W_{T_1} + W_{T_2} + 2W_{T_3}, \quad \text{(14)}$$

where

$$W_{T_1} = \frac{2R_2}{T_2} \left( \Delta q_{T1}^{p1} \right)^2 + \frac{3R_3}{T_2} \sum_{i=2}^{(N/2)^2} \left( \Delta q_{T1}^{p(2i-1)} \right)^2 + \frac{2R_2}{T_1} \left( \Delta q_{T1}^{pN} \right)^2,$$ 

$$W_{T_2} = \frac{3R_3}{T_2} \sum_{i=1}^{N/2} \left( \Delta q_{T1}^{p(2i)} \right)^2, \quad \text{and} \quad W_{T_3} = 0 \quad \text{if } N \text{ is an even number.} \quad \text{(15)}$$

and
\[ W_{T_i} = \frac{2R_m}{T_1} (\Delta q_{T_i}^{(1)})^2 + \frac{3R_m}{T_1} (\Delta q_{T_i}^{(2)})^2 + \sum_{i=1}^{(N-1)/2} \left( \Delta q_{T_i}^{(2i+1)} \right)^2, \]
\[ W_{T_i} = \frac{3R_m}{T_2} \sum_{i=1}^{(N-1)/2} (\Delta q_{T_i}^{(2i)})^2 + \frac{2R_m}{T_2} (\Delta q_{T_i}^{(N)})^2, \] and \[ W_{T_i} = 0 \] if \( N \) is an odd number. \hfill (16)

**Fig. 4:** General equivalent circuit of capacitor-based converters.

Here, it is known that a general equivalent circuit of capacitor-based converters can be expressed by the circuit shown in Figure 4 (Eguchi et al. 2012, 2013), where \( R_{SC} \) is called the SC resistance and \( M \) is the ratio of an ideal transformer. In the general equivalent circuit of capacitor-based converters, the consumed energy can be defined as

\[ W_c := \left( \frac{\Delta q_{in}}{T} \right)^2 R_{in} T. \] \hfill (17)

Substituting (14)-(16) into (17), the SC resistance of the converter block of the positive charge pump, \( R_{SCp} \), can be obtained as

\[ R_{SCp} = \left( \frac{6N + 2}{1 - 2\delta} \right) R_m. \] \hfill (18)

By combining (13) and (18), the equivalent circuit of the converter block as follows:

\[
\begin{bmatrix}
\frac{V_{in}}{I_{in}} \\
\frac{V_{op}}{I_{op}}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 1 & R_{SCp} & V_{op} \\
0 & N+1 & N+1 & 0 & 1 & -I_{op}
\end{bmatrix}
\begin{bmatrix}
\frac{V_{in}}{I_{in}} \\
\frac{V_{op}}{I_{op}}
\end{bmatrix}
\] \hfill (19)

From (19), the equivalent circuit of the positive charge pump is expressed by the following determinant, because the positive charge pump consists of two converter blocks connected in parallel.

\[
\begin{bmatrix}
\frac{V_{in}}{I_{in}} \\
\frac{V_{op}}{I_{op}}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 1 & R_{SCp} & V_{op} \\
0 & N+1 & N+1 & 0 & 1 & -I_{op}
\end{bmatrix}
\begin{bmatrix}
\frac{V_{in}}{I_{in}} \\
\frac{V_{op}}{I_{op}}
\end{bmatrix}
\] \hfill (20)

From (20), the maximum output voltage and the maximum power efficiency \( \eta \) are obtained as

\[ \frac{V_{op}}{V_{in}} = \frac{2(N+1)R_L}{2R_L + R_{SCp}} \] and \[ \eta = \frac{2R_L}{2R_L + R_{SCp}}. \] \hfill (21)
Negative Charge Pump:

Figure 5 shows the instantaneous equivalent circuits of the negative charge pump. Firstly, the equivalent circuit of the converter block is derived. In the steady state, the differential value of electric charges in \( C_k \) (\( k=1, 2, \ldots, N \)) satisfies the following equations:

\[
\Delta q_{T_1}^{(i,k)} + \Delta q_{T_2}^{(i,k)} + 2 \Delta q_{T_3}^{(i,k)} = 0,
\]

where \( \Delta q_{T_i}^{(i,k)} \) (\( i=1, 2, 3 \) and \( k=1,2, \ldots, N \)) denote the electric charges of the \( k \)-th capacitor in the case of State-\( T_i \). In Figure 5, the differential values of electric charges in the input \( V_{in} \) and the output \( V_{out} \), \( \Delta q_{T1,vin} \) and \( \Delta q_{T1,vo} \), are expressed as

\[
\Delta q_{T1,vin} = \begin{cases} 
\sum_{i=1}^{N/2} \Delta q_{T1}^{(i,2i-1)} & \text{if } N \text{ is an even number}, \\
\sum_{i=1}^{N/2} \Delta q_{T1}^{(i,2i-1)} & \text{if } N \text{ is an odd number}.
\end{cases}
\]

and

\[
\Delta q_{T1,vo} = \begin{cases} 
-\Delta q_{T1}^{(i,2i)} + \Delta q_{T1}^{(i,2i)} & \text{if } N \text{ is an even number}, \\
-\Delta q_{T1}^{(i,2i)} & \text{if } N \text{ is an odd number}.
\end{cases}
\]

In State-\( T_2 \), the differential values of electric charges in \( V_{in} \) and \( V_{out} \), \( \Delta q_{T2,vin} \) and \( \Delta q_{T2,vo} \), are expressed as

\[
\Delta q_{T2,vin} = \begin{cases} 
\sum_{i=1}^{N/2} \Delta q_{T2}^{(i,2i)} & \text{if } N \text{ is an even number}, \\
\sum_{i=1}^{N/2} \Delta q_{T2}^{(i,2i)} & \text{if } N \text{ is an odd number}.
\end{cases}
\]

and

\[
\Delta q_{T2,vo} = \begin{cases} 
-\Delta q_{T2}^{(i,2i)} & \text{if } N \text{ is an even number}, \\
-\Delta q_{T2}^{(i,2i)} - \Delta q_{T2}^{(i,2i)} & \text{if } N \text{ is an odd number}.
\end{cases}
\]

In State-\( T_3 \), \( \Delta q_{T3,vin} \) and \( \Delta q_{T3,vo} \), are expressed as

\[
\Delta q_{T3,vin} = 0 \text{ and } \Delta q_{T3,vo} = \Delta q_{Ti}^{(i,2i)}.
\]

Furthermore, the following equations are obtained:

\[
\begin{align*}
\Delta q_{T1}^{(i)} &= \Delta q_{T2}^{(i)} = \Delta q_{T3}^{(i)} = \cdots = \Delta q_{T1}^{(N-1)} \quad \text{and} \quad \Delta q_{T2}^{(i)} = \Delta q_{T4}^{(i)} = \cdots = \Delta q_{T1}^{(N)}, \\
\Delta q_{T1}^{(i)} &= \Delta q_{T2}^{(i)} = \Delta q_{T3}^{(i)} = \cdots = \Delta q_{T1}^{(N)} \quad \text{if} \ N \text{ is an even number.}
\end{align*}
\]

\[
\begin{align*}
\Delta q_{T1}^{(i)} &= \Delta q_{T2}^{(i)} = \Delta q_{T3}^{(i)} = \cdots = \Delta q_{T1}^{(N)} \quad \text{and} \quad \Delta q_{T2}^{(i)} = \Delta q_{T4}^{(i)} = \cdots = \Delta q_{T1}^{(N-1)} \quad \text{if} \ N \text{ is an odd number.}
\end{align*}
\]
\[
\bar{I}_m = \frac{\Delta q_{V_m}}{T} = \frac{\Delta q_{V_{m},V_{i}} + \Delta q_{V_{m},V_{i}} + 2\Delta q_{V_{m},V_{i}}}{T}
\]
and
\[
\bar{I}_{\text{on}} = \frac{\Delta q_{V_{\text{on}}}}{T} = \frac{\Delta q_{V_{\text{on},V_{i}} + \Delta q_{V_{\text{on},V_{i}}}} + 2\Delta q_{V_{\text{on},V_{i}}}}{T}. \tag{30}
\]

In (30), \(\Delta q_{V_m}\) and \(\Delta q_{V_{\text{on}}}\) are electric charges in \(V_m\) and \(V_{\text{on}}\), respectively. Substituting (22)-(29) into (30), the relation between the average input current and the average output currents is obtained as
\[
\bar{I}_m = N\bar{I}_{\text{on}}. \tag{31}
\]

where
\[
\Delta q_{V_m} = -N\Delta q_{V_m}^N \quad \text{and} \quad \Delta q_{V_{\text{on}}} = -\Delta q_{V_{\text{on}}}^N \quad \text{if} \quad N \text{ is an even number.}
\]
\[
\Delta q_{V_m} = N\Delta q_{V_m}^N \quad \text{and} \quad \Delta q_{V_{\text{on}}} = \Delta q_{V_{\text{on}}}^N \quad \text{if} \quad N \text{ is an odd number.}
\]

Using (22)-(29), the consumed energy \(W_I\) can be expressed as
\[
W_I = W_{I_1} + W_{I_2} + 2W_{I_3}, \tag{32}
\]

Where
\[
W_{I_1} = \frac{2R_m}{T_1} \left( \Delta q_{V_{I_1}}^N \right)^2 + \frac{3R_m}{T_1} \sum_{i=1}^{(N-2)/2} \left( \Delta q_{V_{I_1}}^{n(2i+1)} \right)^2 + \frac{2R_m}{T_1} \left( \Delta q_{V_{I_1}}^{n(2i)} \right)^2,
\]

\[
W_{I_2} = \frac{3R_m}{T_2} \sum_{i=1}^{N/2} \left( \Delta q_{V_{I_2}}^{n(2i)} \right)^2, \quad \text{and} \quad W_{I_3} = 0 \quad \text{if} \quad N \text{ is an even number.} \tag{33}
\]

and
\[
W_{I_1} = \frac{2R_m}{T_1} \left( \Delta q_{V_{I_1}}^N \right)^2 + \frac{3R_m}{T_1} \sum_{i=1}^{(N-1)/2} \left( \Delta q_{V_{I_1}}^{n(2i-1)} \right)^2,
\]

\[
W_{I_2} = \frac{3R_m}{T_2} \sum_{i=1}^{(N-1)/2} \left( \Delta q_{V_{I_2}}^{n(2i-1)} \right)^2 + \frac{2R_m}{T_2} \left( \Delta q_{V_{I_2}}^{n(2i)} \right)^2, \quad \text{and} \quad W_{I_3} = 0 \quad \text{if} \quad N \text{ is an odd number.} \tag{34}
\]

Substituting (32)-(34) into (17), the SC resistance of the converter block of the negative charge pump, \(R_{\text{SCn}}\), can be obtained as
\[
R_{\text{SCn}} = \left( \frac{6N + 2}{1 - 2\delta} \right) R_m. \tag{35}
\]

because the general equivalent circuit of capacitor-based converters is given by (17). As (18) and (35) show, the SC resistance of the negative charge pump \(R_{\text{SCn}}\) is the same as \(R_{\text{SCp}}\). By combining (31) and (35), the equivalent circuit of the converter block as follows:
\[
\begin{bmatrix}
V_m \\
I_m
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{N} & 0 \\
0 & -N
\end{bmatrix} \begin{bmatrix}
1 & R_{\text{SCn}} \\
0 & 1
\end{bmatrix} \begin{bmatrix}
V_{\text{on}} \\
I_{\text{on}}
\end{bmatrix}. \tag{36}
\]

From (36), the equivalent circuit of the negative charge pump is expressed by the following determinant, because the negative charge pump consists of two converter blocks connected in parallel.
\[
\begin{bmatrix} \bar{V}_m \\ \bar{T}_m \end{bmatrix} = \begin{bmatrix} -\frac{1}{N} & 0 \\ 0 & -N \end{bmatrix} \begin{bmatrix} 1 & \frac{R_{SCn}}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{V}_m \\ -\bar{T}_m \end{bmatrix}.
\]

From (37), the maximum output voltage and the maximum power efficiency \( \eta \) are obtained as

\[
\bar{V}_{om} = \left( 2NR_{IL} \right) \bar{V}_m \quad \text{and} \quad \eta = \left( \frac{2R_{IL}}{2R_L + R_{SCn}} \right).
\]

Proposed Bipolar Charge Pump:

From (20) and (38), the equivalent circuit of the bipolar charge pump is expressed by Figure 6. From Figure 6, the maximum output voltage and the maximum power efficiency \( \eta \) are obtained as

\[
\bar{V}_{om} = \frac{R_L}{R_L + \frac{R_{SCp}/2}{2 + R_{SCn}/2}} \left\{ \left( N + 1 \right) \bar{V}_m - \left( -N \right) \bar{V}_m \right\}
\]

\[
= \frac{(2N+1)(1-\delta)R_L}{(1-2\delta)R_L + (6N+2)R_{om}} \bar{V}_m
\]

and

\[
\eta = \frac{R_L}{R_L + \frac{R_{SCp}/2}{2 + R_{SCn}/2}}
\]

\[
= \frac{(1-2\delta)R_L}{(1-2\delta)R_L + (6N+2)R_{om}}
\]

Fig. 6: Equivalent circuits of the proposed bipolar charge pump.

Simulation:

Confirmation of Theoretical Analysis:

To confirm the validity of the theoretical analysis, the SPICE simulations are performed under conditions that \( V_{in}=1.5V, \ C_{i}=200nF, \ C_{j}= C_{in}=0IF, \ R_{om}=1\Omega, \ T=100ns, \ T_1=T_2=45ns, \ T_3=5ns, \) and \( N=3. \) Figure 7 shows the comparison between theoretical results and simulated results, where Figure 7 (a) shows the maximum output voltage, and Figure 7 (b) shows the maximum power efficiency. As these figures show, the theoretical result is in good agreement with the SPICE simulated result. Therefore, the formulas obtained by the proposed analysis method will be helpful to estimate the maximum power efficiency and the maximum output voltage of the charge pump.

Characteristic Comparison:

To clarify the characteristics of the proposed charge pump, the SPICE simulations are performed under conditions that \( V_{in}=1.5V, \ C_{i}=200pF, \ C_{j}= C_{in}=200IF, \ R_{om}=1\Omega, \ T=100ns, \ T_1=T_2=45ns, \ T_3=5ns, \) and \( N=3. \) Figure 8 shows the comparison between the proposed positive charge pump and the conventional positive charge pump, where Figure 8 (a) shows the simulated output voltage, and Figure 8 (b) shows the simulated power efficiency. As Figure 8 shows, according to the increase of the output load \( R_L, \) the proposed positive charge pump can alleviate the influence of stray parasitic capacitances. Concretely, the proposed positive charge pump improves power efficiency more than 5% when the output load \( R_L \) is 100k\( \Omega. \)
Figure 7: Comparison between theoretical results and simulated results; (a) Maximum output voltage and (b) Maximum power efficiency.

Figure 9 shows the comparison between the proposed negative charge pump and the conventional negative charge pump, where Figure 9 (a) shows the simulated output voltage, and Figure 9 (b) shows the simulated power efficiency. As in the case of the positive charge pump, the proposed negative charge pump can alleviate the influence of stray parasitic capacitances according to the increase of the output load $R_L$. Concretely, the proposed negative charge pump improves power efficiency more than 8% when the output load $R_L$ is 100kΩ.

Figure 10 shows the simulated characteristics of the proposed bipolar charge pump, where Figure 10 (a) shows the simulated output voltage, and Figure 10 (b) shows the simulated power efficiency. Compared with Figures 8 and 9, the proposed bipolar charge pump can achieve high step-up gain.

Fig. 8: Comparison between the proposed positive charge pump and the conventional positive charge pump; (a) Simulated output voltage and (b) Simulated power efficiency.

Fluctuation Analysis:
As Figure 2 shows, the proposed bipolar charge pump has many circuit components. For this reason, to predict the influence of fluctuation of circuit components, Monte Carlo simulations were performed under conditions that (1) $C_k$, $C_{bk}$, $C_{tk}$, and $R_{on}$ have 10% tolerance with Gaussian distribution and (2) the output load is 10kΩ. Figure 11 shows the result of the Monte Carlo simulation. In Figure 11, the Monte Carlo simulation was performed 100 times. In the output voltage of Figure 11 (a), the mean value is 9.174V and the standard deviation is $3.50 \times 10^{-3}$. On the other hand, in the power efficiency of Figure 11 (b), the mean value is 84.52% and the standard deviation is $5.86 \times 10^{-2}$. As these results show, the proposed bipolar charge pump is robust to the fluctuation of circuit components.
Fig. 9: Comparison between the proposed negative charge pump and the conventional negative charge pump; (a) Simulated output voltage and (b) Simulated power efficiency.

Fig. 10: Comparison between the proposed negative charge pump and the conventional negative charge pump; (a) Simulated output voltage and (b) Simulated power efficiency.

Fig. 11: Result of the Monte Carlo analysis; (a) Output voltage and (b) Power efficiency.

Conclusion:
For energy-harvesting systems utilizing thermoelectric energy, a bipolar charge pump and its analysis method have been proposed in this paper. By combining positive/negative charge pumps using power saving techniques, the proposed charge pump achieves not only high step-up gain but also high power efficiency.
Furthermore, the analysis method considering the on-resistance of transistor switches is proposed to obtain handy theoretical formulas.

The SPICE simulation showed the following results: (1) The proposed analysis method will be helpful to estimate the maximum power efficiency and the maximum output voltage of the charge pump, because the theoretical results were in good agreement with the SPICE simulated results. (2) The proposed charge pump can reduce parasitic power losses when the output load $R_i$ is a large value. Concretely, the proposed positive charge pump improved power efficiency more than 5% when the output load $R_i$ is 100kΩ. On the other hand, the proposed negative charge pump improved power efficiency more than 8% when the output load $R_i$ is 100kΩ. (3) The proposed bipolar charge pump will be useful to convert energy from TEGs, because the proposed bipolar charge pump showed high step-up gain. Concretely, by the proposed three-stage bipolar charge pump, about a 9.1V output was obtained from a 1.5V. (4) The proposed bipolar charge pump is robust to the fluctuation of circuit components. Concretely, the standard deviation of the output voltage and power efficiency was less than 0.1 when capacitors and on-resistances have 10% tolerance with Gaussian distribution.

The IC implementation of the proposed charge pump is left to a future study.

REFERENCES


