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# Designing an Optimal PID Controller for the Inverted Pendulum Using Imperialist Competitive Algorithm

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ABSTRACT

One of the most important problems today is robotics and its control, due to the vast Application of inverted pendulum in robots. In this paper, by using a combination In this paper, a colonial competitive algorithm is applied to the problem of designing a PID controller for inverted pendulum. We have tried to optimally control the inverted pendulum by nonlinear equations. The results of this simulation have been mentioned in the conclusion. It seems that the results be acceptable results.



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## INTRODUCTION

The inverted pendulum system is a standard problem in the area of control systems. They are often useful to demonstrate concepts in linear control such as the stabilization of unstable systems. Since the system is inherently nonlinear, it has also been useful in illustrating some of the ideas in nonlinear control. In this system, an inverted pendulum is attached to a cart equipped with a motor that drives it along a horizontal track. The user is able to dictate the position and velocity of the cart through the motor and the track restricts the cart to movement in the horizontal direction. Sensors are attached to the cart and the pivot in order to measure the cart position and pendulum joint angle, respectively. Measurements are taken with a quadrature encoder connected to a MultiQ-3 general purpose data acquisition and control board. Matlab/Simulinkis used to implement the controller and analyze data.

The inverted pendulum system inherently has two equilibriums, one of which is stable while the other is unstable. The stable equilibrium corresponds to a state in which the pendulum is pointing downwards. In the absence of any control force, the system will naturally return to this state. The stable equilibrium requires no control input to be achieved and, thus, is uninteresting from a control perspective. The unstable equilibrium corresponds to a state in which the pendulum points strictly upwards and, thus, requires a control force to maintain this position. The basic control objective of the inverted pendulum problem is to maintain the unstable equilibrium position when the pendulum initially starts in an upright position. The control objective for this project will focus on starting from the stable equilibrium position (pendulum pointing down), swinging it up to the unstable equilibrium position (pendulum upright), and maintaining this state.

*Design of PID Controller:*

Industrial PID controllers are usually available as a packaged, and it's performing well with the industrial process problems. The PID controller requires optimal tuning. Figure 4 shows the diagram of a simple closed-loop control system. In this structure, the controller Gc(s) has to provide closed-loop stability, smooth reference tracking, shape of the dynamic and the static qualities of the disturbance response, reduction of the effect of supply disturbance and attenuation of the measurement noise effect (M. sheybani, M. meibody2006).

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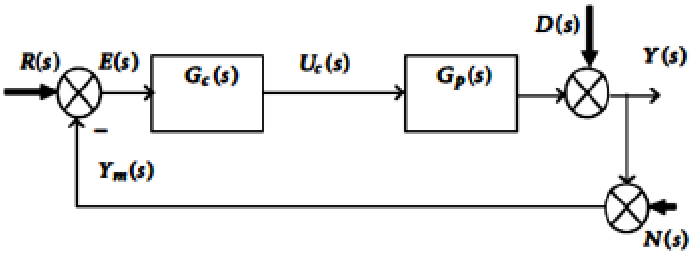



**Fig.1:** diagramofa closed-loop controlsystem.

In this study reference tracking, load disturbance rejection, and measurement noise attenuation are considered. Closed-loop response of the system with set point R(s), load disturbance D(s), and noise N(s) can be expressed as Equations (1)and( 2).

$$\mathbf{Y(s)} = \left[\frac{\mathbf{G_p(s)G_C(s)}}{\mathbf{1 + G_p(s)G_C(s)}}\right]\mathbf{R(s)} + \left[\frac{\mathbf{1}}{\mathbf{1 + G_p(s)G_C(s)}}\right]\mathbf{D(s)} - \left[\frac{\mathbf{G_p(s)G_C(s)}}{\mathbf{1 + G_p(s)G_C(s)}}\right]\mathbf{N(s)} \quad (1)$$

$$\mathbf{Y(s)} = [\mathbf{T(s) * R(s) - N(s)}] + (\mathbf{S(s) * D(s)}] \quad (2)$$

Where the complementary sensitivity function and sensitivity function of the above loop are represented in 3and 4, respectively.

$$\mathbf{T(s)} = \frac{\mathbf{Y(s)}}{\mathbf{R(s)}} = \left[\frac{\mathbf{G_p(s)G_C(s)}}{\mathbf{1+G_p(s)G_C(s)}}\right] \quad (3)$$

$$\mathbf{S(s)} = \left[\frac{\mathbf{1}}{\mathbf{1 + G_p(s)G_C(s)}}\right] \quad (4)$$

The final steady state response of the system for the set point tracking and the load disturbance rejection is given in (5) and (6), respectively:

$$\boldsymbol{y_R(\infty) = \lim_{s\to 0} s\,Y_R(s) = \lim_{s\to 0} s\left[\frac{G_p(s)G_C(s)}{1 + G_p(s)G_C(s)}\right]\left(\frac{A}{s}\right) = A} \quad (5)$$

$$\boldsymbol{y_D(\infty) = \lim_{s\to 0} s\,Y_D(s) = \lim_{s\to 0} s\left[\frac{1}{1+G_p(s)G_C(s)}\right]\left(\frac{L}{s}\right) = 0} \quad (6)$$

Where A is amplitude of the reference signal and L is disturbance amplitude. To achieve a satisfactory yR(∞) and yD(∞), it is necessary to have optimally tuned PID parameters. From the literature it is observed that to get a guaranteed robust performance, the integral Controller gain “Ki” should have an optimized value. In this study, a no interacting form of PID (GPID) Controller Structure is considered. For real control applications, the Feedback Signal is the sum of the measured output and Measurement noise Component. A low pass filter is used with the derivative term to reduce the effect of measurement noise. The PID structures are defined as the following or Equation (7)

$$\mathbf{G_{PID} = K_P e(t) + K_d \frac{de(t)}{dt} + K_i \int_0^t e(\tau)d(\tau) = K_p\left[1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d s}{N} + 1}\right]} \quad (7)$$

$$\mathbf{\frac{K_P}{T_i} = K_i \qquad , K_P * T_d = K_d \qquad and\ N = filter\ constant}$$

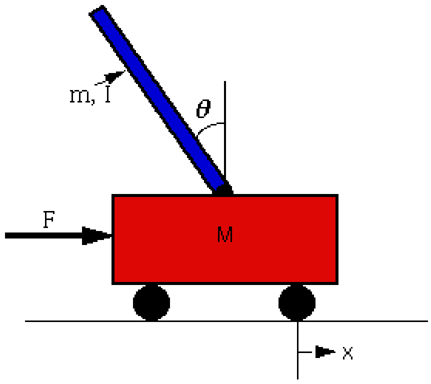
Cost Function in Optimization Algorithms as Equation

$$\mathbf{Cost\ Function = w_1 T_r + w_2 M_p + w_3 T_s} \quad (8)$$

In Cost Function, w1, w2, w3 respectively are the weight or important coefficient of system performance. Tr, Mp, Ts are the Rise Time, Maximum Over Shot and Settling Time. Suggested Cost Function makes all Coefficients and parameters be same, and caused more impact of characteristics (V. Rajinikanth and K. Latha2012).

*Modeling an Inverted Pendulum:*

The cart with an inverted pendulum, shown below, is "bumped" with an impulse force, F. Determine the dynamic equations of motion for the system, and linearize about the pendulum's angle, theta = 0 (in other words, assume that pendulum does not move more than a few degrees away from the vertical, chosen to be at an angle of 0). Find a controller to satisfy all of the design requirements given below. For this example, let's assume that.




**ig. 2:** The structure of an Inverted Pendulum

**Table 1:** physical parameters of Inverted Pendulum

| | | |
|---|---|---|
| M | mass of the cart | 0.5 kg |
| m | mass of the pendulum | 0.2 kg |
| b | friction of the cart | 0.1 N/m/sec |
| l | length to pendulum center of mass | 0.3 m |
| I | inertia of the pendulum | 0.006 kg*m^2 |
| F | force applied to the cart | |
| x | cart position coordinate | |
| theta | pendulum angle from vertical | |

This system is tricky to model in Simulink because of the physical constraint (the pin joint) between the cart and pendulum which reduces the degrees of freedom in the system. Both the cart and the pendulum have one degree of freedom (X and theta, respectively). We will then model Newton's equation for these two degrees of freedom.

$$\frac{d^2x}{dt^2} = \frac{1}{M}\sum_{cart} F_x = \frac{1}{M}\left(F - N - b\frac{dx}{dt}\right) \tag{9}$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{I}\sum_{pend} \tau = \frac{1}{I}(NL\cos(\theta) + PL\sin(\theta)) \tag{10}$$

It is necessary, however, to include the interaction forces N and P between the cart and the pendulum in order to model the dynamics. The inclusion of these forces requires modeling the x and y dynamics of the pendulum in addition to its theta dynamics. Generally, we would like to exploit the modeling power of Simulink and let the simulation take care of the algebra. Therefore, we will model the additional x and y equations for the pendulum.

$$m\frac{d^2x_p}{dt^2} = \sum_{pend} F_x = N \tag{11}$$

$$\Rightarrow N = m\frac{d^2x_p}{dt^2} \tag{12}$$

$$m\frac{d^2y_p}{dt^2} = P - mg \tag{13}$$

$$\Rightarrow P = m\left(\frac{d^2y_p}{dt^2} + g\right) \tag{14}$$

However, xp and yp are exact functions of theta. Therefore, we can represent their derivatives in terms of the derivatives of theta.

$$x_p = x - L\sin(\theta) \tag{15}$$

$$\frac{dx_p}{dt} = \frac{dx}{dt} - L\cos(\theta)\frac{d^2\theta}{dt^2} \tag{16}$$

$$\frac{d^2x_p}{dt^2} = \frac{d^2x}{dt^2} + L\sin(\theta)\left(\frac{d\theta}{dt}\right) - L\cos(\theta)\frac{d^2\theta}{dt^2} \tag{17}$$

$$y_p = L\cos(\theta) \tag{18}$$

$$\frac{dy_p}{dt} = -L\sin(\theta)\frac{d\theta}{dt} \tag{19}$$

$$\frac{d^2y_p}{dt^2} = -L\cos(\theta)\left(\frac{d\theta}{dt}\right)^2 - L\sin(\theta)\frac{d^2\theta}{dt^2} \tag{20}$$

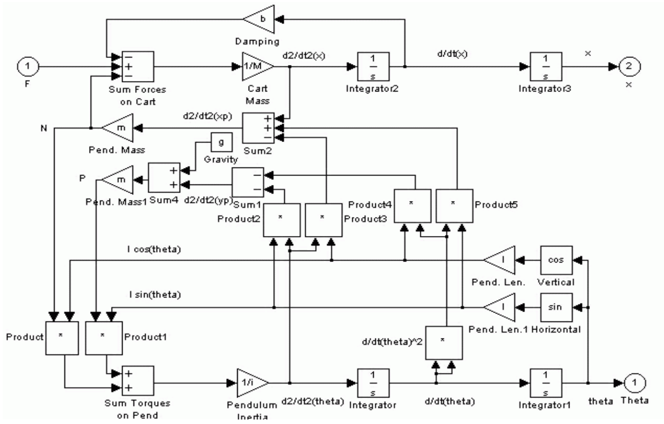



**Fig. 3:** The block diagram of an Inverted Pendulum

These expressions can then be substituted into the expressions for N and P. Rather than continuing with algebra here, we will simply represent these equations in Simulink. Simulinkcan work directly with nonlinear equations,

*Imperialist Competitive Algorithm (ICA):*

Imperialist Competitive Algorithm is a new evolutionary optimization method which is inspired by imperialistic competition and has been applied in some different fields (R. Rajabioun, E. Atashpaz ,C. Lucas 2008, B.Oskouyi, E. Atashpaz-Gargari, N. Soltani and C. Lucas.2009). Like other evolutionary algorithms, it starts with an initial population which is called country and is divided into two types of colonies and imperialists which together form empires. Imperialistic competition among these empires forms the proposed evolutionary algorithm. During this competition, weak empires collapse and powerful ones take possession of their colonies (M. Jasour, E. Atashpaz, C. Lucas.2008). Imperialistic competition converges to a state in which there exists only one empire and colonies have the same cost function value as the imperialist. After dividing all colonies among imperialists and creating the initial empires, these colonies start moving toward their relevant imperialist state which is based on assimilation policy. Figure 6 shows the movement of a colony towards the imperialist. In this movement, θ and x are random numbers with uniform distribution as illustrated in Equation 32 and d is the distance between colony and the imperialist(Khabbazi, E. Atashpaz and C. Lucas.2009).

$$X \sim U(0, \beta \times d),\ \theta \sim U(-\gamma,\gamma) \qquad (21)$$

Where β and γ are parameters that modify the area that colonies randomly search around theImperialist.

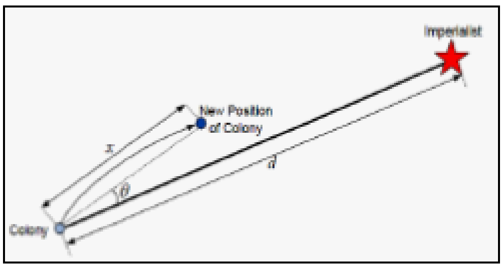



**Fig. 4:** Motion ofcoloniestowardtheir relevantimperialist

The total power of an empire depends on both the power of the imperialist country and the power of its colonies which is shown in Equation (22).

$$\mathbf{T.C.}_n = \mathbf{Cost\ (imperialist}_n) + \zeta * \mathbf{mean\ \{Cost(coloniesofimpire}_n)\}} \qquad (22)$$

Fig.4 shows a big picture of the modeled imperialistic competition. Based on their total power, in This competition, each of the empires will have a likelihood of taking possession of the mentioned colonies. The more powerful an empire, the more likely it will possess the colonies. In other words these colonies will not be certainly possessed by the most powerful empires, but these empires will be more likely to possess them. Any empire that is not able to succeed in imperialist competition and cannot increase its power (or at least prevent decreasing its power) will be eliminated. ( M. Kohansal, M. J. Sanjari and G. B. Gharehpetian2013)

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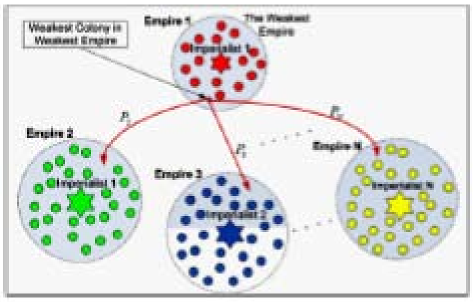


**Fig. 5:** Imperialistic Competition adapted from

With the implementation of optimization algorithms, the coefficients of the PID controller become optimal. In order to compare, based on cost function or Equation21, we will display the performance (response) of these algorithm in Figure 8. According to the responses, Rise and Settle Time, easily we can select the ICA Algorithm has better performance and it will be optimized Algorithm in PID Controller

*Simulation Results:*

Terms of the objective function (performance profile system) in the paper are: Rise Time, Over Shot and Settling Time. Three parameters; Kp, Ki ,Kd belongs to the controller are considered as variables in the designing. Optimization is performed for the objective function and the results come in the form as curves. According to the simulation results, based on design constraints, Imperialist Competitive Algorithm (ICA) has the best performance.

**Table 2:** Parameter of PID

| | KP | KI | KD |
|---|---|---|---|
| PID | 100 | 0.4332 | 4.7739 |
| PID_D | 55.3849 | 0.9570 | .5000 |
| PID_N | 200 | 4.5812 | 9.7429 |

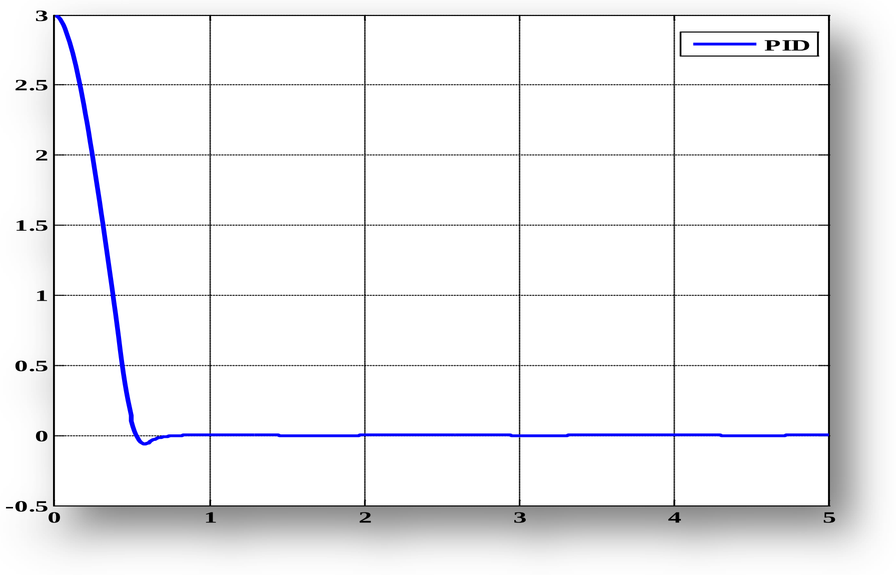


**Fig. 6:** Response of system to PID controller designed

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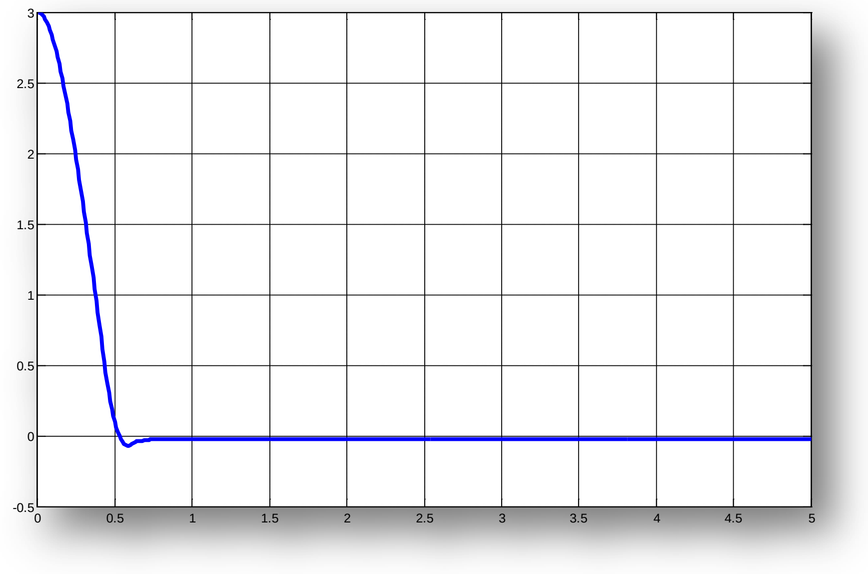


**Fig. 7:** Response of system to PID controller designed with noise

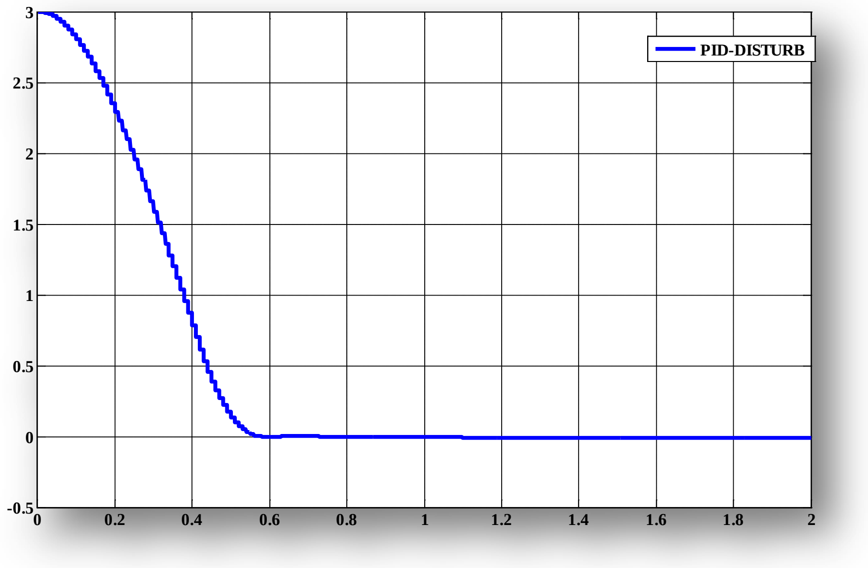


**Fig. 8:** Response of system to PID controller designed with disturbance

*Conclusion:*

The PID controller is used, based on the model presented in this paper, in order to control the angle in inverted pendulum. Controller coefficients are obtained from the optimization procedure, based on Design Adverbs 21 by using MATLAB ® Simulink toolbox. Furthermore, the results of the coefficients are used in PID controller. In curves shown better results of the optimization based on design Adverbs. Achieve these results by using single-objective optimization is possible.

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