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Applying Game Theory in Price and Quality Decisions of Supply Chains with a Manufacturer and Two Competitive Retailers

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ABSTRACT

This paper considers a two-echelon supply chain with a manufacturer who sells a single product to customers through two competitive retailers. We assume that product quality and unit price charged by the retailers influence the demand of the product being sold. The manufacturer invests in the technology in order to improve quality to develop the market for the product so she determines product quality and the wholesale price for the product. On the other hand, the retailers set the retail price and associated order quantity. We model the supply chain with four scenarios game-theoretic framework and analyze the effects of the manufacturer and retailers different competitive and cooperative behaviors on the optimal decisions of the manufacturer and the retailers. Numerical examples are also presented in this paper to compare the results between different models considered and the optimal configuration from each firm's perspective is discussed. Sensitivity analysis of some key parameters including , and is also done at the end of the paper.

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INTRODUCTION

In the last two decades, both academicians and practitioners have shown keen interest on the supply chain management (SCM) subjects. Globalization of market, increased competition, reducing gap between products in terms of quality and performance are compelling the academicians and industry to rethink about how to manage business operations efficiently and effectively. Thomas and Grifin (1996) have mentioned effective supply chain management requires planning and coordination among the various channel members including manufacturers, retailers and intermediaries. In order to effectively model and analyze decision making in such multi person situation where the outcome depends on the choice made by every party, game theory is a natural choice. Each firm can be seen as players in a game defined by a common goal, but with separate constraints and conflicting objectives. Game theory provides a mathematical background for modeling the system and generating solutions in competitive or conflicting situations and has become an essential tool in analysis of supply chains with multiple agents, often with conflicting objectives. In supply chain, firms may both compete and cooperate with each other in order to maximize their profits. Through cooperative relationships, the firms work to influence product demand by investing in demand-enhancing efforts. These efforts may include investment in technology by one firm to improve product quality/design, as well as investment in selling effort by the other firm to develop the market for the product.

Pricing is important business behavior and competing firms often play a price war to attract customers. Similarly, the buyer, e.g., a retailer, has the opportunity to influence final demand by choosing the appropriate retail price. In general, the supplier's wholesale pricing and the buyer's resale pricing can be affected by the non-cooperative and cooperative behaviors between members of a supply chain. Significant amount of research has been done in the area of supply chain coordination. Much of it has assumed that the demand in a retail market varies with the retail price. Lau and Lau (2002) studied effects of different demand curves on the optimal solution. They found out that under a downward-sloping price-versus-demand relationship the manufacturer's profit is the double of the retailer's. Lau and Lau (2005) also considered the retailer-Stg system with a single manufacturer and a single retailer and investigated the effect of the demand-curve's shape on the optimal solution. Other related two-echelon price-sensitive-demand Stackelberg models mainly focus on incorporating stochastic demand, information asymmetry and other factors. Cachon (2001) developed game-theoretic models for the competitive cases of continuous review and periodic review

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and periodic review models. Moses and Seshadri (2000) considered a periodic review model with lost sales. Esmaeili *et al*., (2009) proposed several game models of seller–buyer relationship to optimize pricing and lot sizing decisions. Game-theoretic approaches are employed to coordinate pricing and inventory policies. Granot and Yin (2007) constructed a Stackelberg game to analyze sequential commitment in supply chain with one manufacturer and one retailer under price-dependent demand, where the manufacturer's decisions are the wholesale price and the buyback price, and the retailer's decisions are the order quantity and the sale price. SeyedEsfahani *et al*., (2011) considered vertical co-op advertising along with pricing decisions in a supply chain consists of one manufacturer and one retailer where demand is influenced by both price and advertisement. Four game-theoretic models are established in order to study the effect of supply chain power balance on the optimal decisions of supply chain members. Huang *et al*., (2011) developed a two-period pricing and production decision model in a one- manufacturer-one-retailer dual-channel supply chain that experiences a disruption in demand during the planning horizon. In this paper, they examine how to adjust the prices and the production plan so that the potential maximal profit is obtained under a disruption scenario and obtained the manufacturer's and the retailer's individual optimal pricing decision, as well as the manufacturer's optimal production quantity in a decentralized decision-making setting.

Most of the two-echelon supply chain models listed above considered a single retailer but few of them involve competition between different manufacturers/suppliers in upstream market or different retailers/buyers in downstream market in a two-echelon supply chain. More interesting issues would be to relax the classical two channel members situation to a three channel members situation (either two manufacturers and one retailer or two retailers and one manufacturer) to move one step towards the understanding of the role of the competition and the cooperation. This kind of study has been done more in the field of pricing too. For example, Trivedi (1998) found that the competitions at both retailer and manufacturer levels have significant impacts on the members' profits and prices. Choi (1996) focused on the intra-channel and inter-channel price competitions. Van Mieghem and Dada (1999) studied the production capacity decision in price competition and extended simultaneous price postponement duopoly to oligopoly. Dai *et al*., (2005) considered the pricing strategies of multiple firms providing the same service and owing finite capacity in a revenue management context. Coughlan and Wernerfelt (1989) investigated a two-echelon system with homogeneous retailers. Ingene and Parry (1995) further extended the work of Coughlan and Wernerfelt (1989) to the situation with non homogeneous retailers. In their model, they only explored impact of two retailers' Cournot behavior on the channel under the assumption that retail demand functions are linear. By assuming that retailers' demand is a deterministic constant, Wang and Wu (2000) developed a similar model and proposed a supplier's pricing policy that is superior for the supplier when there are many different retailers/buyers. Yang and Zhou (2006) considered the pricing decisions of a two-echelon system with a manufacturer and two competitive retailers, and analyzed the effects of the duopolistic retailers' different competitive behaviors—Cournot, Collusion and Stackelberg—on the optimal decisions of the manufacturer and the duopolistic retailers themselves. Huang *et al*., (2011) have considered the coordination of suppliers and components selection, pricing, and replenishment decisions in a multi-level supply chain composed of multiple suppliers, one single manufacturer and multiple retailers. The problem was modeled as a three-level dynamic non-cooperative game. Leng and parlar (2010) considered a multiple-supplier, single manufacturer assembly supply chain. In this single-period problem the suppliers determine their production quantities and the manufacturer chooses the retail price. They analyze both simultaneous-move and leader–follower games to respectively determine the Nash and Stackelberg equilibria, and find the globally-optimal solution that maximizes the system-wide expected profit.

Wu *et al*., (2012) investigated the pricing decisions in a non-cooperative supply chain that consists of two retailers and one common supplier. The retailers order from the common supplier and compete in the same market. They have characterized the equilibrium decisions in the presence of horizontal and vertical competition by using six game models.

All these studies are focused on the field of pricing. However, in many real-life situations, besides price, from the perspective of customers' behavior, product's quality influences the customers' preferences and their purchasing decisions, and hence market demand. The manufacturer invests in quality improvement efforts that may include new high-precision equipment with high reliability, fast or flexible equipment, organizational training and restructuring, etc., which improve the demand potential of the product. Another example of manufacturer initiated efforts to improve the demand potentials is the investment in brand name (see Lal, 1990). A larger investment in technology improves the quality of the product which results later in an increased demand potential for the product. For instance, in the electricity markets, power generating firms can invest in different dimensions of power quality such as environmentally-friendly green power or premium power for sensitive computing, as opposed to lower quality interruptible power for flexible producers (Savitski, 2002). In the fast food retail business, final demand is not only affected by the retail price and the value added by the buyer, it also depends on the investments made by the franchisor in its brand name (Lal, 1990). In the automotive industry, Japanese firms made dramatic gains in market share in the 1980s as compared to the US firms by investing in quality-improvement efforts (Garvin, 1988).

Gurnani *et al.*, (2007) considered a supply chain with two firms which the product supplier invests in the technology to improve quality, and the purchasing firm (buyer) invests in selling effort to develop the market for the product. They studied the Impact of product pricing and timing of investment decisions on supply chain.

To the best of our knowledge, very few works consider two-echelon supply chain under the price and quality decision structure of tree members in chain that the market demand varies with the product quality level as well as retail. Thus it is an interesting and important research issue. Hence, In this paper, we considers two-echelon supply chain consists of a manufacturer who sells a single product to customers through two competitive retailers to address the above issues, where the product quality and unit price charged by the retailers influence the demand of the product being sold. The manufacturer invests in technology to improve quality to develop the market for the product and two retailers compete in retail price. We analyze the effects of various behaviors between the members on the optimal solution and channel profit-split of a two echelon supply chain. In order to analyze this, It considers four possible cases: In case 1, each retailer independently determines their retail price and order quantity to maximize its profit simultaneously. A Stackelberg game exists between two retailers and the manufacturer which manufacturer acts as a leader and retailers are followers; In case 2, two retailers plays Stackelberg game and also in vertical channels exist Stackelberg game; In case 3, Two retailer cooperate with side-payment contract and they determine their sale prices and order quantity jointly to maximize the total profit of the downstream retail market and between two retailers and the manufacturer exist Stackelberg game ; In case 4, The manufacturer and one of the retailers play cooperative game through side-payment contract and between them and another retailer exist Stackelberg game.

The rest of the paper is organized as follows. In Section 2, formulation of the model is done and we determine the optimal expressions for four various decision making structures. In Section 3, we compare the results of four scenarios and examine the effects of different competitive and cooperative behaviors on the optimal solutions of the models. In Section 4, the effects of change of parameters β , λ and η on the optimal solutions examined. Finally, we conclude the paper in Section 5.

The Basic Model:

In this section, we will first develop the two-echelon supply chain models with a manufacturer and two competitive retailers, which can tell both the manufacturer and the retailers how to make their decisions. It is mainly considered that the situations where the retailers and the manufacturer implement the four scenarios below:

1) Each retailer independently determine their retail price and order quantity to maximize their profit, simultaneously and between two retailers and the manufacturer exist Stackelberg game, in which manufacturer acts as a leader and retailers are followers.

2) Two retailers Playing Stackelberg game and also a Stackelberg structure is assumed between two echelons in the two-echelon chain, in which the manufacturer acts as a leader and the retailers act as followers

3) Two retailers cooperate through side-payment contract and they determine their sale prices and order quantities jointly to maximize the total profit of the downstream retail market and the interaction mechanism between two echelons is assumed to be the manufacturer-Stg process in which the manufacturer acts as a leader and the retailers act as followers.

4) The manufacturer and one of the retailers cooperate through side-payment contract and a Stackelberg structure is assumed between them and another retailer, in which manufacturer and retailer 1 act as a leader and retailer 2 acts as a follower.

Following assumptions and notations are considered (i = 1, 2):

p_i : The sale price charged to customers by retailer-i

π_{ri} : Retailer-i’s profit

π_M : The manufacturer's profit

w_i : The wholesale price per unit charged to the retailer-i by the manufacturer

q : The quality level selected by the manufacturer

c : Unit manufacturing cost

D_i: Deterministic demand faced by retailer-i or quantity ordered by retailer-i

RM: The ratio of the manufacturer's profit to the total profit of the duopolistic retailers

This paper will consider a two-echelon supply chain consisting of a manufacturer selling its product through two duopolistic retailers. The manufacturer determines the wholesale price and the investment in product quality/design and two retailers determine retail prices and corresponding order quantities.

In Section 1, we have pointed that retail price and quality level are two important factors affecting the market demand. Thus, we assume that retailer-i faces the following downward-sloping demand function:

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$$D_i = \alpha - P_i + \beta P_j + \lambda q \qquad i, j = 1,2 \tag{1}$$

Where α is the market size (i.e., the maximum possible demand), λ measures the impact of product quality on demand and β is the substitutability coefficient. The substitutability coefficient is a measure of the sensitivity of the i^{th} retailer's sales to the change of j^{th} retailer's price, which reflects the impact of the marketing mix decision of the retailers on customer demand. Several researchers, like McGuire and Staelin (1983) and Ingene and Parry (1995), have successively used this type of demand curve. Thus, the retailers' profit function is

$$\max_{(P)} \pi_{ri}(p) = (p_i - w)(\alpha - p_i + \beta p_j + \lambda q) \quad i = 1,2 \tag{2}$$

The manufacturer invests in quality improvement efforts that may include new high-precision equipment with high reliability, fast or flexible equipment, organizational training and restructuring, etc., which improve the demand potential of the product.

The quality level selected by the manufacturer, q, affects his total expected costs in two ways: First, investment in quality improvement programs increases fixed costs, $\eta q^2/2$. Similar to the existing marketing literature, we use convex cost of investment to model the diminishing return of investment in influencing demand. The quality level also has an impact on the variable costs. We let the variable costs as $c(1+vq)$, where v may be less than or greater than zero. Allowing v to be negative allows us to model the case when the variable production costs actually decline due to improvement in quality (Banker *et al*., 1998). Then, the manufacturer's profit function is

$$\max_{(w,q)} \pi_M(w,q) = (w - c(1+vq))(D_1 + D_2) - \frac{\eta q^2}{2} \tag{3}$$

Case 1:

In this case, the manufacturer acts as a leader and determines the investment in product quality/design and sets the wholesale price for the retailers. Then, the retailers who act as followers independently determine their retail prices and order quantities to maximize their profit, simultaneously. For any wholesale price and quality level declared by the manufacturer, the profit function of each retailer, which equals his total revenue less his purchasing cost, is

$$\pi_{r1} = (p_1 - w)D_1 = (p_1 - w)(\alpha - p_1 + \beta p_2 + \lambda q)$$
$$\pi_{r2} = (p_2 - w)D_2 = (p_2 - w)(\alpha - p_2 + \beta p_1 + \lambda q)$$

Thus, retailer 1 will maximize π_{r1} with respect to p_1, treating p_2 as a parameter, and retailer 2 will maximize π_{r2} with respect to p_2, treating p_1 as a parameter. It is easy to see that the retailers' profit function is concave in p_i .Hence, for any given w and q, the optimal retail prices (p_1^* and p_2^*) of retailers are given by (obtained by solving $\frac{\partial \pi_{r1}}{\partial p_1} = 0$ and $\frac{\partial \pi r_{r2}}{\partial p_2} = 0$)

$$\frac{\partial \pi_{r1}}{\partial p_1} = 0 \Rightarrow p_1 = \frac{\alpha + \beta p_2 + \lambda q + w}{2} \tag{4}$$

$$\frac{\partial \pi_{r2}}{\partial p_2} = 0 \Rightarrow p_2 = \frac{\alpha + \beta p_1 + \lambda q + w}{2} \tag{5}$$

Solving (4) and (5) for p_1 and p_2, we get:

$$P^*{}_i = \frac{\alpha + \lambda q + w}{2 - \beta} \tag{6}$$

Substituting (6) into (1), one can easily obtain the optimal order quantities of the retailers as below.

$$D^*{}_i = \alpha + \lambda q + \frac{(\beta - 1)(\alpha + \lambda q + w)}{2 - \beta} = \frac{\alpha + \lambda q + (\beta - 1)w}{2 - \beta} \tag{7}$$

(6), (7) show the optimal reaction functions of the two duopolistic retailers for any w and q set by the manufacturer. From the manufacturer side, the manufacturer knows the retailers' reaction functions for any w and q -values he sets. Hence the manufacturer's profit will be

$$\pi_M = (w - c(1+vq))(D^*{}_1 + D^*{}_2) - \frac{\eta q^2}{2} = (w - c(1+vq))\left[2(\alpha + \lambda q) + \frac{2(\beta - 1)(\alpha + \lambda q + w)}{2 - \beta}\right] - \frac{\eta q^2}{2} \tag{8}$$

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Lemma 1: we can show that if $\frac{4(1-\beta)[4c\nu\lambda+\eta(2-\beta)]}{2-\beta} > \left[\frac{2\lambda-2c\nu(\beta-1)}{2-\beta}\right]^2$, then π_M is a concave function of w and q. So solving $\frac{\partial \pi_M}{\partial w}=0$ and $\frac{\partial \pi_M}{\partial q}=0$ will give the optimal wholesale price and quality level set by the manufacturer as:

Proof. All proofs are in the Appendix.

$$\frac{\partial \pi_M}{\partial w}=0 \Rightarrow w^* = \frac{c(\beta-1)(1+\nu q)-(\alpha+\lambda q)}{2(\beta-1)} \tag{9}$$

$$\frac{\partial \pi_M}{\partial q}=0 \Rightarrow q^* = \frac{2w\lambda-2c(\lambda+\nu\alpha+\nu(\beta-1)w)}{4c\nu\lambda+\eta(2-\beta)} \tag{10}$$

Solving (9) and (10) for w and q, we can obtain Nash equilibrium for w and q as:

$$9{,}10 \Rightarrow \begin{cases} W^N = \dfrac{R(L-\alpha)-2kL(\lambda+\nu\alpha)}{2(\beta-1)(k^2+R)} & (11) \\ q^N = \dfrac{K(\alpha-L)-2L(\lambda+\nu\alpha)}{k^2+R} & (12) \end{cases}$$

Which

$R=(\beta-1)[4c\nu\lambda+\eta(2-\beta)]$

$K=c\nu(\beta-1)-\lambda$

$L=c(\beta-1)$

By Substituting (11) and (12) into (6) and (7), we can obtain Nash equilibrium for retail prices and order quantities of the retailers as below

$$P_i^N = \frac{A(k+2\lambda(\beta-1))+\alpha B+(\alpha-L)(2(\beta-1)\lambda K-R)}{(2-d)B} \tag{13}$$

$$D_i^N = \frac{\alpha d+(\alpha+L)(2k\lambda-R)+(2\lambda+K)A}{(2-\beta)d} \tag{14}$$

Which

$A=-2L(\lambda+\nu\alpha)$

$B=2(\beta-1)(k^2+R)$

$d=2(k^2+R)$

Case 2:

In this case, one of the two retailers (e.g., retailer 1) acts as a Stackelberg leader and the other (i.e., retailer 2) acts as a Stackelberg follower and the interaction mechanism between two echelons is assumed to be the manufacturer-Stg process in which the manufacturer acts as a leader and the retailers who act as followers set their sale prices and associated order quantities. The retailer 2 (the follower) observes her reaction function and adjusts her retail price to maximize her profit, given the pricing decision of the leader.

It can be easily observed from (2) that retailer 2's profit is a concave function of p_2 ($\frac{\partial^2 \pi_{r2}}{\partial p_2^2}=-2<0$).

Then, from the first order conditions, we have

$$\frac{\partial \pi_{r2}}{\partial p_2}=0 \Rightarrow p_2 = \frac{\alpha+\beta p_1+\lambda q+w}{2} \tag{15}$$

Retailer 1 maximizes her profit, given the follower's reaction function. Substituting retailer 2's reaction function into her profit function gives her profit as:

$$\pi_{r1} = (p_1-w)\left(\alpha-p_1+\frac{\beta(\alpha+\beta p_1+\lambda q+w)}{2}+\lambda q\right) \tag{16}$$

Lemma 2: we can show that π_{r1} is a concave function in p_1. So solving $\frac{\partial \pi_{r1}}{\partial p_1}=0$ will give the optimal retail price set by the retailer 1

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Proof, all proofs are in the Appendix.

$$\frac{\partial \pi_{r1}}{\partial p_1} = 0 \Rightarrow p_1^* = \frac{(\alpha + \lambda q + w)(\beta + 2) - w\beta^2}{2(2-\beta^2)} \tag{17}$$

Retailer 2's optimal sale price and order quantity can be determined by substituting retailer 1's optimum leadership retail price in retailer 2's reaction function. The relevant expressions are

$$P_2^* = \frac{2(1+\beta)(\alpha + \lambda q + w) - w\beta^3}{2(2-\beta^2)} \tag{18}$$

$$D_2^* = \alpha + \lambda q - \frac{(2+\beta)(\alpha + \lambda q + w)}{2(2-\beta^2)} \tag{19}$$

Since the manufacturer knows the retailers' reaction functions given by (17) and (18) for any w and q-values she sets, her profit will be

$$\pi_M = (w - c(1+vq))(D_1^* + D_2^*) - \frac{\eta q^2}{2} = (w - c(1+vq))(2(\alpha + \lambda q) + (\beta - 1)(p_1^* + p_2^*)) - \frac{\eta q^2}{2}$$

Therefore

$$\pi_M = (w - c(1+vq))\left(2(\alpha + \lambda q) + (\beta - 1)\left(\frac{(\alpha + \lambda q + w)(\beta + 2) - w\beta^2}{2(2-\beta^2)} + \frac{2(1+\beta)(\alpha + \lambda q + w) - w\beta^3}{2(2-\beta^2)}\right)\right) - \frac{\eta q^2}{2} \tag{20}$$

Lemma 3: We can show that the manufacturer's profit function is concave as the Hessian is negative definite if,

$$\frac{(\beta-1)[(3\beta+4) - \beta^2(1+\beta)]}{2-\beta^2} \times \left[-4cv\lambda - \frac{2cv(\beta-1)(3\beta+4)}{2(2-\beta^2)} - \eta\right] > \left(2\lambda + \frac{(\beta-1)\lambda(3\beta+4)}{2(2-\beta^2)} - cv(\beta-1)\left[\frac{(3\beta+4) - \beta^2(1+\beta)}{2(2-\beta^2)}\right]\right)^2$$

We shall assume that the above condition is valid, Therefore, when the duopolistic retailers play Stackelberg game, the manufacturer will set her optimal wholesale price and quality level as (obtained by solving $\frac{\partial \pi_M}{\partial w} = 0$ and $\frac{\partial \pi_M}{\partial q} = 0$)

Proof, all proofs are in the Appendix.

$$\frac{\partial \pi_M}{\partial w} = 0 \Rightarrow w^* = \frac{(\alpha + \lambda q)(4 + \beta(1-\beta)) + c(1+vq)((\beta^2 - 2)^2 - \beta)}{2[(\beta^2 - 2)^2 - \beta]} \tag{21}$$

$$\frac{\partial \pi_M}{\partial q} = 0 \Rightarrow q^* = \frac{[\lambda(w-c) - cv\alpha](4 + \beta(1-\beta)) + cvw((\beta^2 - 2)^2 - \beta)}{2cv\lambda(4 + \beta(1-\beta)) + 2\eta(2-\beta^2)} \tag{22}$$

Solving (21) and (22) simultaneously for w and q, we get:

$$W^S = \frac{(\lambda E - cvF)(\lambda E + cvF)(\alpha E - cF)}{2F[2FG - (\lambda E + cvF)^2]} + \frac{\alpha E + cF}{2F} \tag{23}$$

$$q^S = \frac{(\lambda E - cvF)(\alpha E - cF)}{2FG - (\lambda E + cvF)^2} \tag{24}$$

Which

$$E = 4 + \beta(1-\beta)$$

$$F = (\beta^2 - 2)^2 - \beta$$

$$G = 2cv\lambda E + 2\eta(2-\beta^2)$$

Thus, combining (23) and (24) with (2), (3) and (17)–(19), one can derive the optimal pricing policies and the corresponding maximum profits of the manufacturer and duopolistic retailers.

Case 3:

In this case two retailers cooperate through side-payment contract, and they determine their sale prices and order quantity jointly to maximize the total profit of the downstream retail market and the interaction mechanism between two echelons is assumed to be the manufacturer-Stg process in which the manufacturer acts

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as a leader and sets the product quality level and a wholesale unit price to the two retailers who act as followers. Two retailers cooperate through side-payment, therefore, the total profit of downstream retail market is

$$\pi_r = \pi_{r1} + \pi_{r2}$$

$$\pi_r = (p_1 - w)(\alpha - p_1 + dp_2 + \lambda\theta) + (p_2 - w)(\alpha - p_2 + dp_1 + \lambda\theta) \tag{25}$$

Lemma 4: The total profit of the downstream retail market is a concave function of the p_1 and p_2. Proof of Lemma 4 is given in the appendix.

From Lemma 4, the optimal sale prices of the retailers can be obtained by solving, $\frac{\partial \pi_r}{\partial P_2} = 0$ and $\frac{\partial \pi_r}{\partial P_1} = 0$, which give:

$$\frac{\partial \pi_r}{\partial P_1} = 0 \Rightarrow P_1 = \frac{\alpha + \lambda q + 2\beta p_2 + w(1-\beta)}{2} \tag{26}$$

$$\frac{\partial \pi_r}{\partial P_2} = 0 \Rightarrow p_2 = \frac{\alpha + \lambda q + 2\beta p_1 + w(1-\beta)}{2} \tag{27}$$

Solving (26) and (27) for p_1 and p_2, we get:

$$\begin{cases} P_1^* = \dfrac{\alpha + \lambda q + w(1-\beta)}{2(1-\beta)} & (28) \\ P_2^* = \dfrac{\alpha + \lambda q + w(1-\beta)}{2(1-\beta)} & (29) \end{cases}$$

Substituting (28) and (29) into (1), one can easily obtain the optimal order quantities of the retailers as below

$$D_i^* = \frac{\alpha + \lambda q + w(\beta - 1)}{2} \tag{30}$$

(28)– (30) show the optimal reaction functions of the two duopolistic retailers for any w and q set by the manufacturer. From the manufacturer side, the manufacturer knows the retailers' reaction functions for any w and q-values he sets. Hence the manufacturer's profit will be

$$\pi_M = (w - c(1+vq))(D_1^* + D_2^*) - \frac{\eta q^2}{2} = (w - c(1+vq))(\alpha + \lambda q + w(\beta - 1)) - \frac{\eta q^2}{2} \tag{31}$$

Lemma 5: We can show that if $(2-2\beta)(2cv\lambda + \eta) > (\lambda - cv\beta + cv)^2$, the manufacturer's profit π_M, is a concave function of w and q. We shall assume that the above condition is valid, Then, from the first order conditions and noting that $\frac{\partial \pi_M}{\partial w} = 0$ and $\frac{\partial \pi_M}{\partial q} = 0$, we get:

Proof, all proofs are in the Appendix.

$$\frac{\partial \pi_M}{\partial w} = 0 \Rightarrow w^* = \frac{\alpha + \lambda q + c(1+vq)(1-\beta)}{2(1-\beta)} \tag{32}$$

$$\frac{\partial \pi_M}{\partial q} = 0 \Rightarrow q^* = \frac{\lambda(w-c) - cv(\alpha + w(\beta-1))}{\eta - 2cv\lambda} \tag{33}$$

Solving (32) and (33) simultaneously for w and q, we get:

$$w^h = \frac{N(\alpha - L) + 2ML(\lambda + \alpha v)}{2(N - M^2)(1-\beta)} \tag{34}$$

$$q^h = \frac{M(a - L) + 2L(\lambda + \alpha v)}{(N - M^2)} \tag{35}$$

Which

$$M = \lambda + cv(1-\beta)$$

$$N = 2(1-\beta)(\eta + 2cv\lambda)$$

$$L = c(\beta - 1)$$

By Substituting (34)and (35) into (28)–(30), we can obtain equilibrium for retail prices and order quantities of the retailers as below

$$P_i^h = \frac{2\alpha\left(N - M^2\right) + (\alpha - L)(2\lambda M + N) + 2L(\lambda + \alpha v)(2\lambda + M)}{4(1-\beta)\left(N - M^2\right)} \tag{36}$$

$$D_i^h = \frac{2\alpha\left(N - M^2\right) + (\alpha - L)(2\lambda M - N) + 2L(\lambda + \alpha v)(2\lambda - M)}{4\left(N - M^2\right)} \tag{37}$$

Case 4:

In this case the manufacturer and one of the retailers (e.g., retailer 1) cooperate through side-payment contract and a Stackelberg structure is assumed between them and another retailer, in which manufacturer and retailer 1 act as leader and retailer 2 acts as a follower. The retailer 2 (the follower) adjusts her retail price to maximize her profit, given the pricing and quality decisions of the leader (manufacturer and retailer 1). The retailer 2's profit, $\pi_{r2} = (P_2 - w)(\alpha - p_2 + \beta p_1 + \lambda q)$ is also a concave function of p_2.Then, from the first order conditions, we have:

$$\frac{\partial \pi_{r2}}{\partial p_2} = 0 \Rightarrow p_2 = \frac{\alpha + \beta p_1 + \lambda q + w}{2} \tag{38}$$

The manufacturer and retailer 1 cooperate through side-payment, so the manufacturers and retailer 1's integrated profit function is:

$$\pi_{rM} = \pi_{r1} + \pi_M$$

$$\pi_{rM} = (P_1 - w)(\alpha - P_1 + \beta P_2 + \lambda q) + \left(w - c(1 + vq)\right)(D_1 + D_2) - \frac{\eta q^2}{2} \tag{39}$$

Since the manufacturer and retailer 1 know the retailer 2 ' reaction functions given by (38) for any w, q and p_1 -values they set, their profit will be

$$\pi_{rM} = (P_1 - w)(\alpha - P_1 + \beta P_2 + \lambda q) + \left(w - c(1 + vq)\right)\left(2(\alpha + \lambda q) + (\beta - 1)(P_1 + P_2)\right) - \frac{\eta q^2}{2} \tag{40}$$

Therefore

$$\pi_{rM} = (P_1 - w)(\alpha - P_1 + \beta \frac{\alpha + \beta p_1 + \lambda q + w}{2} + \lambda q) +$$
$$\left(w - c(1 + vq)\right)\left(2(\alpha + \lambda q) + (\beta - 1)(P_1 + \frac{\alpha + \beta p_1 + \lambda q + w}{2})\right) - \frac{\eta q^2}{2} \tag{41}$$

Lemma 6: we can show that if

$2(\beta^2 - 1)(cv\lambda(\beta + 3) + \eta) + \frac{1}{2}(\beta(\beta + 4) + 3)(\lambda - cv(\beta - 1))^2 < 0$, then π_{rM} is a concave function of p_1, w and q. So solving $\frac{\partial \pi_M}{\partial w} = 0$, $\frac{\partial \pi_M}{\partial q} = 0$ and $\frac{\partial \pi_{rM}}{\partial p_1} = 0$ will give the optimal wholesale price, quality level and retail price of retailer 1 as :

Proof. all proofs are in the Appendix.

$$\frac{\partial \pi_{rM}}{\partial p_1} = 0 \Rightarrow p_1^* = \frac{2\beta w + (\beta + 2)(\alpha + \lambda q) + (1 - \beta)(\beta + 2)c(1 + vq)}{2\left(2 - \beta^2\right)} \tag{42}$$

$$\frac{\partial \pi_{rM}}{\partial w} = 0 \Rightarrow w^* = \frac{\alpha + \lambda q + 2\beta P_1 + (1 - \beta)c(1 + vq)}{2} \tag{43}$$

$$\frac{\partial \pi_{rM}}{\partial q} = 0 \Rightarrow q^* = \frac{\lambda(\beta + 2)P_1 + \lambda w - c(\beta + 3)(\lambda + \alpha v) - cv(\beta - 1)(w + P_1(\beta + 2))}{2(\beta + 3)cv\lambda + 2\eta} \tag{44}$$

Solving (42) to (44) simultaneously for w and q and p_1, we get:

$$P_1 = \frac{\alpha + \lambda q + c(1 + vq)(1 - \beta)}{2(1 - \beta)} \tag{45}$$

$$w = \frac{\alpha + \lambda q + c(1 + vq)(1 - \beta)}{2(1 - \beta)} \tag{46}$$

$$q = \frac{(\beta+3)(\lambda + c\nu(\beta-1))(\alpha + c(\beta-1))}{2(1-\beta)y - (\lambda + c\nu(1-\beta))^2(\beta+3)} \tag{47}$$

Which

$$y = 2(\beta+3)c\nu\lambda + 2\eta$$

By Substituting (45)–(47) into (38), we can obtain equilibrium for retail price of the retailer 2.

Comparison of the optimal solutions for the four cases:

In the previous section, we derived the expressions for the optimal parameters for the four cases. In this section, we now compare the optimal solutions of four models presented above and discuss the potential of conflicting choices between the manufacturer and retailers.

Based on the analysis shown in the previous section it is complicated to show analytical results. So we will directly present the numerical results. Doing so, we show seven numerical examples. The assumed parameter values for these examples are listed in Table1. In Table1, for example 1 setting $\beta = 0.3$ is to depict that high degree of competition between two retailers. In example 2 setting $\beta = 0.7$ is to depict that low degree of competition between two retailers and in other examples setting $\beta = 0.5$ is to represent a moderate degree of competition between two retailers.

In example 3 setting $\lambda = 0.3$ is to show that the effect of quality level on the demand potential for the product is low. In example 4 setting $\lambda = 0.7$ is to show that the effect of quality level on the demand potential for the product is high and in other examples setting $\lambda = 0.5$ is to represent a moderate influence of quality level on the demand potential. In examples 5 and 6, we also change the value of cost of quality to investigate the effect of cost of quality on the results.

By using the proposed two-echelon supply chain models, we obtain the optimal solutions of the two-echelon system under the four scenarios for each numerical example.

Table 1: data for numerical examples

example	α	β	λ	η	ν	c
1	200	0.3	0.5	2	0.05	10
2	200	0.7	0.5	2	0.05	10
3	200	0.5	0.3	2	0.05	10
4	200	0.5	0.7	2	0.05	10
5	200	0.5	0.5	5	0.05	10
6	200	0.5	0.5	15	0.05	10
7	200	0.5	0.5	2	0.05	10

Table 2 presents the results. Columns 3–11 in Table 2 represent the optimal pricing policy, order quantities, quality level and the maximum profit of retailer1, retailer 2 and the manufacturer under different competitive and cooperative behaviors. The 12th and 13th rows show respectively the change of the total channel profit under the four kinds of behaviors and the variation of the ratio of the manufacturer's profit to the retailers' profits.
Our results can be summarized as follows:

Comparison of the Retailers' Retail Price:

In case 3 retailers charge the higher sale prices while in case 1 the retailers charging the lower sale price. These changes in the retailers' pricing policies directly result in the variation of the profit split between the downstream markets of the two-echelon system discussed in this paper and thus, retailers gain more profit from cooperation and prefer case 3.

In case 2, when retailers play Stackelberg game in the downstream market, always the follower's charge the higher sale price than the leader's.

In case 4, the retail price that charge by retailer 2 is higher as compared to retailer 1's retail price.

if we compare the retailers' retail price in four case, results show that, In all cases, the retail price charge by retailer 2 in case 2 is highest and afterwards, the retailers' retail price in case 3, the retailers' retail price in case 1 when the value of λ is not high, the retailer 2's retail price in case 4, the retailer 1's retail price in case 2 when the value of β is not high and the retailer 1's retail price in case 4 are higher, respectively.

Comparison of the Retailers' Order Quantity:

In case 2, when retailers play Stackelberg game, the leader's order quantity is higher than the follower's.

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In case 4, the order quantity of retailer 1 (leader's) is higher as compared to retailer 2's (follower's) order quantity.

The retailer 2's order quantity in case 2 is significantly lower compared to other cases and we note that the gap between retailer 2's order quantity in case 2 and others increase as the value of β increases.

if we compare the retailers' order quantity in four case, results show that, the retailer 1's order quantity in case 4 is the largest when the value of β is not high and afterwards, retailer 1's order quantity in case 2, retailers' order quantity in case 1, retailer 2's order quantity in case 4, retailers' order quantity in case 3 and retailer 2's order quantity in case 2 are higher, respectively.

Comparison of the Wholesale Price:

The results show that the different competitive and cooperative behaviors of the channel's members make the manufacturer charge different wholesale prices.

The manufacturer charge a highest wholesale price to two retailers in case 4 and a lowest wholesale price is charged in case 2 and the results show that it is significantly lower compared to other cases. We also note that the wholesale price charged by the manufacturer to two retailers in case 1 is higher than the price charged in case 3.

Comparison of the Quality Level:

Results show that among four kinds of behavior, in case 4 the manufacturer invests most in product quality, and in case 2 the manufacturer invests lowest in product quality, also in case 1 manufacturer makes higher investment in product quality while in case 3 makes lower investment in product quality.

Comparison of the Retailers' Profits:

The retailers' profits are higher in case 3 as compared to case 1. It means that when the retailers cooperate through side-payment contract gain more profit as compared as their non-cooperation.

In case 4, when the retailer 1 cooperates with the manufacturer through side-payment contract, the retailer 1's profit is zero. Thus, he doesn't have any inclination for cooperation, so the manufacturer should share this obtained profit with retailer 1 to induce her for cooperation. If the retailers play Stackelberg game in the downstream market, always the leader's profit is greater than the follower's, thus retailer prefers to be the leader and make her decisions first. It means that when the retailers play Stackelberg game, the leader obtains more profit as compared to the follower. This is just consistent to intuitive expectation. On the other hand in case2, as β increases, the leader's profit increases while follower's profit decreases.

If we compare the retailers' profits in four cases, results show that, the profit of retailer 1 in case 2 is significantly higher compared to other cases and afterwards, the retailers' profits in case 3 and case 1 are higher, respectively. The profit of retailer 2 in case 2 is higher as compared to retailer 2's profits in case 4 when the value of β is low; else, the profit of retailer 2 in case 4 is higher.

Comparison of the Manufacturer's Profit:

The manufacturer's profit is the largest under the case 4, It means that when the manufacturer cooperate with the retailer 1 through side-payment contract gain more profit compared to other cases and thus, prefers case 4 to other cases, but the retailer 1's profit is zero in this case, thus the manufacturer can share obtained profit from cooperation with retailer 1 to convince her to accept this configuration.

When two retailers move simultaneously and decide on their retail prices and order quantities in case 1, the manufacturer's profit is higher compared to case 2 and case 3.

The manufacturer's profit is higher in case 3 as compared to case 2. Therefore, the manufacturer, as a Stackelberg leader, should find a way to induce the retailers to cooperate through side-payment contract.

Comparison of the Total Channel's Profit:

In all cases, the total channel profit of the two-echelon system is the highest under case 4 and afterwards in case 1 the channel profit is higher than two other cases.

The comparison between case 2 and case 3 depends on the value of model's parameters; the channel profit sometimes is higher in case 2 and sometimes is higher in case 3.

Comparison of the Total Ratio of the Manufacturer's Profit to the Retailers' Profits:

Known from Table 2, the manufacturer's profit is just double of the total profit of the retailers in the downstream market in case 3 which retailers cooperate through side payment contract, but in the other tree cases the profit of the manufacturer is more than the double of the total profit of the duopolistic retailers. It implies that in a two-echelon system with a manufacturer and two retailers, the profit gained by the upstream market

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does not always double the profit obtained by the downstream market rather than depends on the competitive and cooperative behavior between the channel's members.

The value of MR in case 4 is higher compared to other cases and afterwards, the value of MR in case 1 is higher.

In case 2 the value of MR is higher as compared to the value of MR in case 3 when the value of β is high (In case 2, the value of MR increases as the value of β increases); else, the value of MR in case 3 is higher.

Table 2: The Optimal Solutions of Four Models.

		P_1^*	P_2^*	D_1^*	D_2^*	W^*	q^*	$\pi_{r_1}^*$	$\pi^*_{r_2}$	π_M^*	π_C^*	MR
Case1	Example 1	212.61	212.61	57.30	57.30	155.31	12.27	3284.03	3284.03	15800.21	22368.27	2.40
	Example 2	541.82	541.82	89.88	89.88	451.93	104.86	8079.48	8079.48	59025.09	75184.05	3.65
	Example 3	273.68	273.68	65.10	65.10	208.58	6.51	4239.11	4239.11	25392.32	33870.55	2.99
	Example 4	344.39	344.39	75.14	75.14	269.24	67.63	5646.69	5646.69	29306.35	40599.75	2.59
	Example 8	281.01	281.01	66.10	66.10	214.91	13.22	4369.43	4369.43	25779.66	34518.52	2.95
	Example 9	273.63	273.63	65.36	65.36	208.26	4.35	4272.33	4272.33	25491.62	34036.29	2.98
	Example 7	298.26	298.26	67.82	67.82	230.43	33.91	4600.37	4600.37	26452.17	35652.93	2.87
Case2	Example1	200.34	229.11	71.80	34.40	134.60	6.83	4720.91	3252.05	12824.43	20797.39	1.60
	Example 2	447.37	593.19	199.15	-48.7	328.77	62.58	23620.50	-12888.2	39323.92	50056.24	3.64
	Example 3	252.61	311.67	103.18	14.58	170.78	-0.14	8443.55	2055.29	18943.41	29442.26	1.80
	Example 4	296.42	366.01	116.19	11.81	205.98	42.30	10508.57	1890.42	20589.89	32988.88	1.66
	Example 5	258.82	319.39	104.78	13.92	176.04	7.81	8673.13	1996.69	19094.96	29764.79	1.78
	Example 6	254.72	314.29	103.72	14.35	172.57	2.59	8520.47	2035.10	18993.65	29549.22	1.79
	Example 7	268.20	331.04	107.20	12.94	184.00	19.78	9027.63	1903.52	19326.92	30258.08	1.76
Case3	Example1	223.67	223.67	48.64	48.64	154.18	10.42	3379.90	3379.90	13519.62	20279.43	2
	Example2	590.48	590.48	54.84	54.84	407.65	63.99	10028.11	10028.11	40112.45	60168.67	2
	Example 3	305.30	305.30	48.81	48.81	207.68	4.88	4765.03	4765.03	19060.12	28590.18	2
	Example 4	359.86	359.86	54.24	54.24	251.37	48.81	5884.39	5884.39	23537.56	35306.34	2
	Example 5	311.13	311.13	49.36	49.36	212.40	9.87	4874.21	4874.21	19496.88	29245.31	2
	Example 6	305.35	305.35	48.95	48.95	207.44	3.26	4792.98	4792.98	19171.93	28757.90	2
	Example 7	324.51	324.51	50.32	50.32	223.87	25.16	5064.72	5064.72	20258.90	30388.35	2
Case4	Example1	158.35	207.25	112.46	48.89	158.35	17.28	0	2391.04	22245.25	24636.30	9.30
	Example 2	480.30	541.01	163.93	60.71	480.30	131.04	0	3686.50	73760.52	77447.03	20.00
	Example 3	209.70	258.55	122.14	48.85	209.70	8.54	0	2386.99	33344.82	35731.81	13.96
	Example 4	293.64	352.89	148.12	59.24	293.64	93.31	0	3510.32	40436.76	43947.09	11.51
	Example 8	218.08	267.92	124.60	49.84	218.08	17.44	0	2484.05	34015.97	36500.03	13.69
	Example 9	209.29	258.40	122.77	49.10	209.29	5.72	0	2411.60	33516.26	35927.87	13.89
	Example 7	238.84	290.41	128.92	51.57	238.84	45.12	0	2659.49	35196.69	37856.18	13.23

Sensitivity Analysis of Model's Key Parameters:

The results indicate that in the presented two-echelon system, none of the four cases strictly dominates the others for all parameter combinations from the perspectives of the manufacturer and the retailers. It is concluded that the optimal quality, selling price, and profits are quite sensitive to model's key parameters in all cases. so sensitivity analysis of parameters including β, λ and η is done. in this section to examine the effect of variation of them on the optimal solutions of the two-echelon models.

Effects of Variation of β on the Optimal Solutions of the Two-Echelon Models:

In order to examine the impact of variation of β on the optimal solutions of the two-echelon models presented in the previous section, we change β from 0.1 to 0.9 in step sizes of 0.2 and the values of other parameters set as $\alpha = 200$; $\lambda = 0.5$; $\nu = 0.05$; $\eta = 2$ and $c = 10$. For each of the four scenarios, we observe respectively the pricing policies of the manufacturer and the retailers as well as the profits of the manufacturer, the retailers and the total channel and the quality level for different β-values. The results are shown in Fig.1. From Fig.1, it is not difficult to see that for each scenario, both the sale prices charged to customers by the retailers and the wholesale price offered to the retailers by the manufacturer increase as the value of β increases. This change of the pricing policies of the manufacturer and the retailers directly results in the profits of the manufacturer; the retailers and the total channel also increase as β increases. That is to say, the bigger value of β (i.e. the more sensitive retailer-i's sales to changes of retailer-j's sale price or the more drastic retail market competes), the higher manufacturer's and duopolistic retailers' pricing should be. This is just contrary to the retailer's pricing behavior in practice. This is because competitive retailers usually attempt to reduce their retail prices to attract customers when competition becomes more drastic.

In addition, observe in Fig.1 that as β increases, the product quality level increases, it means that, when the retailers compete more drastically, the manufacturer attempt more to improve product quality.

Effects of Variation of λ on the Optimal Solutions of the Two-Echelon Models:

In order to examine the impact of variation of λ on the optimal solutions of the two-echelon models presented in the previous section, we change λ from 0.1 to 0.9 in step sizes of 0.2 and the values of other

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parameters set as $\alpha = 200$; $\beta = 0.5$; $\nu = 0.05$; $\eta = 2$ and $c = 10$. For each of the four scenarios, we observe respectively the quality level and the pricing policies of the manufacturer and the retailers for different λ-values. The results are shown in Fig.2. From this Fig, we can observe that for each scenario, the quality level selected by manufacturer increases as the value of λ increases. This is just consistent to intuitive expectation. This is due to the fact that, as the impact of product quality on demand increases, the manufacturer invests more in quality improvement to develop the market of product to gain more profit. On the other hand this increase in quality improvement causes that the wholesale price offered to the retailers by the manufacturer increases and subsequently the sale prices charged to customers by the retailers increase too.

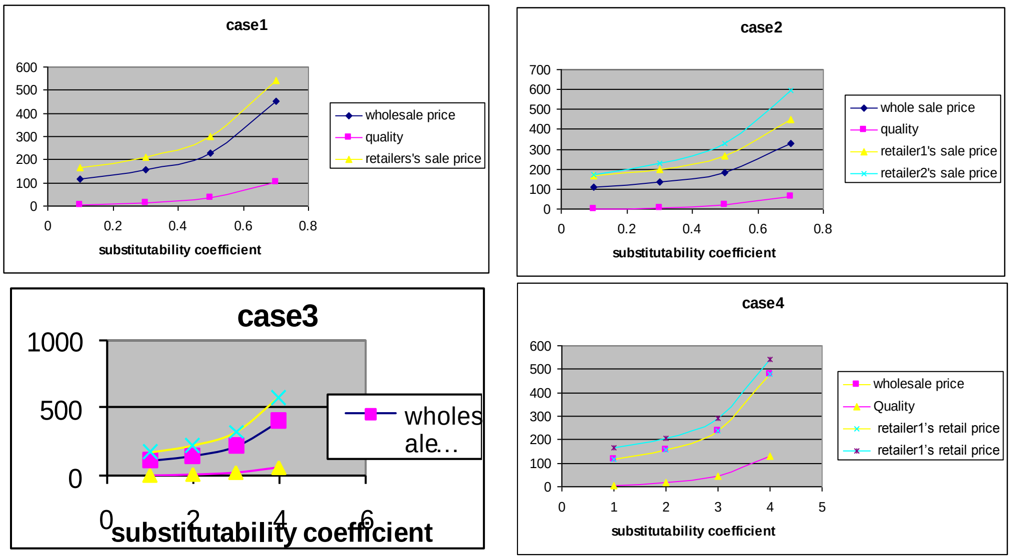

Fig. 1: Effects of β on the optimal solutions of the two-echelon model under the four scenarios.

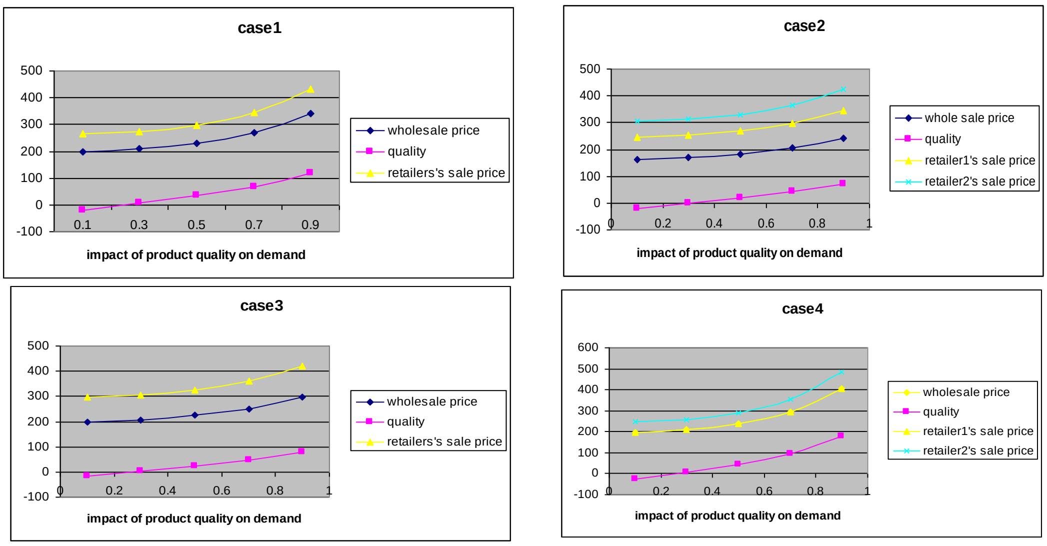

Fig. 2: Effects of λ on the optimal solutions of the two-echelon model under the four scenarios.

Effects of Variation of η on the Optimal Solutions of the Two-Echelon Models:

In order to examine the impact of variation of η on the optimal solutions of the two-echelon models presented in the previous section, we change η from 0.1 to 0.9 in step sizes of 0.2 and the values of other parameters keep set as $\alpha = 200$; $\beta = 0.5$; $\lambda = 0.5$; $\nu = 0.05$ and $c = 10$. For each of the four scenarios, we observe respectively the product quality level, the pricing policies of the manufacturer and the retailers as well as the profits of the manufacturer, the retailers and the total channel for different η-values. The results are shown in Fig.3. From Fig.3, it is not difficult to see that for each scenario, the quality level selected by manufacturer decreases as the value of η increases. This is just consistent to intuitive expectation. It means that when the cost of quality increases the manufacturer invests less in quality improvement efforts. This deduction in quality improvement causes that the wholesale price offered to the retailers by the manufacturer decreases and subsequently the sale prices charged to customers by the retailers decrease too.

This change of the pricing policies of the manufacturer and the retailers directly results in the profits of the manufacturer; the retailers (except retailer 2's profit in case 2) and the total channel also decrease as η increases. In addition, observe in Fig.4 that the slope of each curve decreases as the value of η increases. It means that with the increasing cost of quality, its effect on the optimal quality decreases.

Conclusion:

In supply chains, firms both compete and cooperate with each other in order to maximize their profits. In this paper, we model a supply chain consists of a manufacturer and two retailers with four scenarios game-theoretic framework. We consider that product quality and unit price charged by the retailers influence the demand of the product being sold. The optimal pricing, quantity and quality level decisions are provided for the manufacturer and the duopolistic retailers for the four kinds of behaviors. Also, we Analyze the effects of β-value's change, λ-value's change and η-value's change on the quality level and on the optimal pricing policies of the manufacturer and the retailers.

Our results show the effect of different competitive and cooperative behavior between two echelons in the supply chain on the quality investment decisions and on the product pricing decisions and on the profits for the three firms which can be summarized as follows:

(i) Among the four scenarios, in case 4 the manufacturer invests most in product quality, charge the highest wholesale price to two retailers and gain more profit compared to other cases and thus, prefers case4 than other cases. While she invests lowest in product quality, charge lowest wholesale price and her profit is the lowest under the case2. Furthermore, the product quality level, the wholesale price charged by the manufacturer to two retailers and the manufacturer's profits are higher in case 1 as compared to case3 ; (ii) The profit of retailer1 in case2 is significantly higher compared to other cases and afterwards, the retailers' profits in case3 and case1 are higher, respectively. the profit of retailer2 in case2 is higher as compared to retailer2's profits in case4 when the value of β is low ; else, the profit of retailer2 in case4 is higher ; (iii) In all cases, the total channel profit of the two-echelon system is the highest under case4 and afterwards in case1 the channel profit is higher than two other cases. The comparison between case2 and case3 depends on the value of model's parameters (the channel profit sometimes is higher in case2 and sometimes is higher in case3) ; (iv) In a two-echelon system with a manufacturer and two retailers, the profit gained by the upstream market does not always double the profit obtained by the downstream market rather than depends on the competitive and cooperative behavior between the channel's members (the manufacturer's profit is just the double of the total profit of the retailers in the downstream market in case 3, but in the other tree cases the profit of the manufacturer is more than the double of the total profit of the duopolistic retailers).

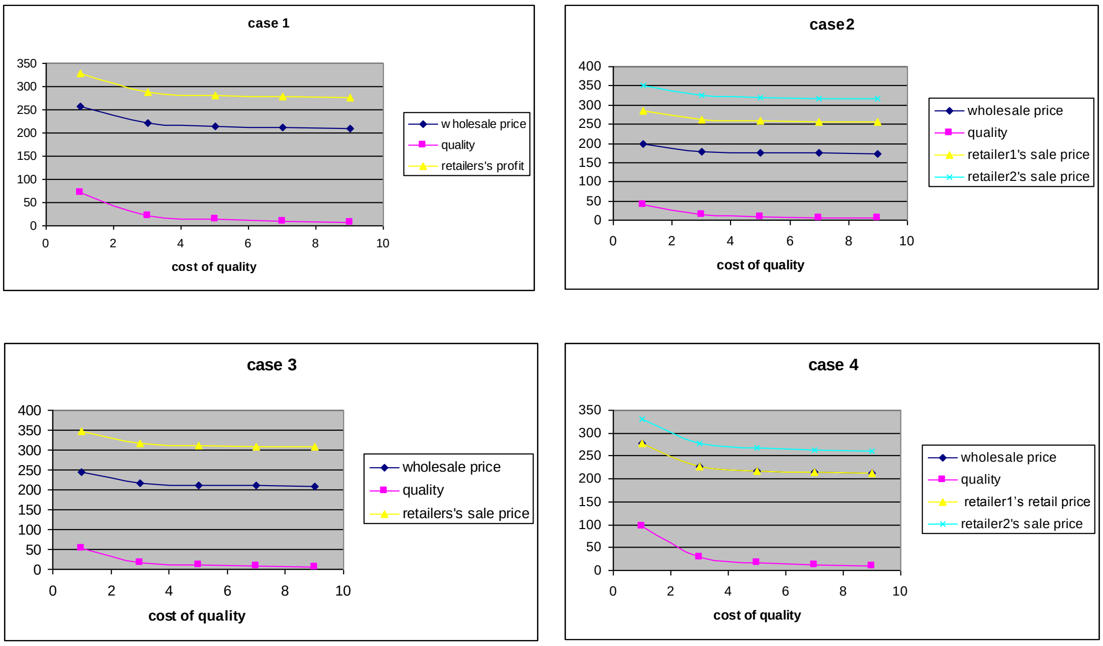

Fig. 3: Effects of η on the optimal solutions of the two-echelon model under the four scenarios.

There are several directions that this research could continue. For examples, we can relax some assumptions such as retailers compete both in price and service level. Another possible avenue for future research is to investigate the competition among different manufacturers. Manufacturers' Competition in product quality will have a significant impact on the channel structure. Also, even though the manufacturer or retailers presumably knows her own costs and price, it is unlikely that her opponent would be privy to such information. This would lead to incomplete knowledge on the part of the participants and result in bargaining models with incomplete information along the line of models. The more general case with more than two competing retailers is still another important problem to be studied.

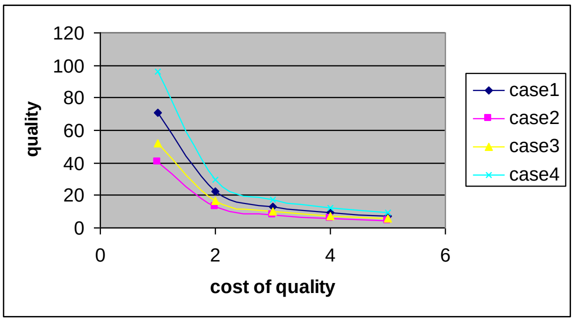

Fig. 4: Effects of cost of quality on the optimal quality of the four scenarios.

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Appendix

Proof of Lemma 1

Proof. Hessian matrix of π_M is

$$H = \begin{vmatrix} \dfrac{4(\beta-1)}{2-\beta} & \dfrac{2\lambda - 2c\nu(\beta-1)}{2-\beta} \\ \dfrac{2\lambda - 2c\nu(\beta-1)}{2-\beta} & -4c\nu\lambda - \eta(2-\beta) \end{vmatrix}$$

The manufacturer's profit function, π_M , is a concave function on (w,q) if and only if Hessian matrix H is negatively definite.

IF $\dfrac{4(1-\beta)[4c\nu\lambda + \eta(2-\beta)]}{2-\beta} > \left[\dfrac{2\lambda - 2c\nu(\beta-1)}{2-\beta}\right]^2$, the Hessian matrix H is negative definite. Hence, The manufacturer's profit function, π_M , is a concave function on (w,q). The proof of Lemma 1 is completed.

Proof of Lemma 2

Proof. Taking the second-order partial derivatives of π_{r1} with respect to p_1, we obtain

$$\frac{\partial^2 \pi_{r1}}{\partial p_1^2} = -2 + \beta^2$$

Due to $0 < \beta < 1$ the second derivative is negative. Hence, the retailer 1's profit is a concave function of p_1. The proof of Lemma 2 is completed.

Proof of Lemma 3

Proof. Taking the second-order partial derivatives of π_M with respect to w and q, we obtain

$$\frac{\partial^2 \pi_M}{\partial w^2} = \frac{(\beta-1)[(3\beta+4) - \beta^2(1+\beta)]}{2-\beta^2} \text{ and } \frac{\partial^2 \pi_M}{\partial q^2} = -4c\nu\lambda - \frac{2c\nu(\beta-1)(3\beta+4)}{2(2-\beta^2)} - \eta \text{ and}$$

$$\frac{\partial \pi_M}{\partial w \partial q} = 2\lambda + \frac{(\beta-1)\lambda(3\beta+4)}{2(2-\beta^2)} - c\nu(\beta-1)\left[\frac{(3\beta+4) - \beta^2(1+\beta)}{2(2-\beta^2)}\right] \text{ and}$$

$$\frac{\partial \pi_M}{\partial q \partial w} = 2\lambda + \frac{(\beta-1)(3\beta+4)\lambda}{2(2-\beta^2)} - c\nu(\beta-1)\left[\frac{(3\beta+4) - \beta^2(1+\beta)}{2(2-\beta^2)}\right]$$

Let Δ_1 and Δ_2 denote respectively the first and second-order principal minors of Hessian matrix of the total profit of manufacturer's profit, π_M . Then, due to $0 < \beta < 1$, $\Delta_1 < 0$ and if

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$$\frac{(\beta-1)\left[(3\beta+4)-\beta^2(1+\beta)\right]}{2-\beta^2}\times\left[-4cv\lambda-\frac{2cv(\beta-1)(3\beta+4)}{2(2-\beta^2)}-\eta\right]>$$
$$\left(2\lambda+\frac{(\beta-1)\lambda(3\beta+4)}{2(2-\beta^2)}-cv(\beta-1)\left[\frac{(3\beta+4)-\beta^2(1+\beta)}{2(2-\beta^2)}\right]\right)^2$$

Then, we have $\Delta_2 > 0$. Hence, the Hessian matrix of the manufacturer's profit function, is negative definite. The proof of Lemma 3 is completed.

Proof of Lemma 4

Proof. Taking the second-order partial derivatives of π_r with respect to p_1 and p_2, we obtain

$$\frac{\partial^2\pi_r}{\partial p_1^2}=\frac{\partial^2\pi_r}{\partial p_2^2}=-2<0$$
$$\frac{\partial^2\pi_r}{\partial p_1\partial p_2}=\frac{\partial^2\pi_r}{\partial p_2\partial p_1}=2\beta$$

Let Δ_1 and Δ_2 denote respectively the first and second-order principal minors of Hessian matrix of the total profit of the downstream retail market, π_r. Then, we have

$\Delta_1 = -2 < 0$ and $\Delta_2 = 4-4\beta^2 > 0$.

Hence, the Hessian matrix of the total profit of the downstream retail market, is negative definite. The proof of Lemma 4 is completed.

Proof of Lemma 5

Proof. Hessian matrix of π_M is

$$H=\begin{vmatrix} 2\beta-2 & \lambda-cv\beta+cv \\ \lambda-cv\beta+cv & -2cv\lambda-\eta \end{vmatrix}$$

Since, $0<\beta<1$, IF $(2-2\beta)(2cv\lambda+\eta)>(\lambda-cv\beta+cv)^2$, the Hessian matrix H is negative definite. Hence, The manufacturer's profit function, π_M, is a concave function on (w, q). The proof of Lemma 5 is completed.

Proof of Lemma 6

Proof. Taking the second-order partial derivatives of π_{rM} with respect to p_1, w and q, we obtain

$\frac{\partial^2\pi_{rM}}{\partial {p_1}^2}=\beta^2-1$ and $\frac{\partial\pi_{rM}}{\partial p_1\partial w}=\beta$ and $\frac{\partial\pi_{rM}}{\partial p_1\partial q}=(1+\frac{\beta}{2})(\beta-cv(\beta-1))$ and

$\frac{\partial\pi_{rM}}{\partial w\partial p_1}=\beta$ and $\frac{\partial^2\pi_{rM}}{\partial {w_1}^2}=-1$ and $\frac{\partial\pi_{rM}}{\partial w\partial q}=\frac{1}{2}(\lambda-cv(\beta-1))$ and

$\frac{\partial\pi_{rM}}{\partial q\partial p_1}=(1+\frac{\beta}{2})(\beta-cv(\beta-1))$ and $\frac{\partial\pi_{rM}}{\partial q\partial w}=\frac{1}{2}(\lambda-cv(\beta-1))$ and $\frac{\partial^2\pi_{rM}}{\partial {q_1}^2}=-cv\lambda(\beta+3)-\eta$

Let Δ_1 and Δ_2 and Δ_3 denote respectively the first and second and third-order principal minors of Hessian matrix of the manufacturer's and retailer 1's integrated profit function, π_{rM}. Then, due to $0<\beta<1, \Delta_1<0$ and $\Delta_2>0$ and if $2(\beta^2-1)(cv\lambda(\beta+3)+\eta)+\frac{1}{2}(\beta(\beta+4)+3)(\lambda-cv(\beta-1))^2<0$ we have $\Delta_3<0$. Hence, the Hessian matrix of π_{rM} (the manufacturer's and retailer 1's integrated profit function), is negative definite. The proof of Lemma 6 is completed.