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Boundary layer flow of nanofluid over a nonlinear permeable stretching sheet with heat source/sink and suction/injection

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ABSTRACT

Nanofluid flow is one of the most important areas of research in the present time due to its wide applications in industry and many other fields. In this paper, flow and heat transfer analysis for boundary layer over a nonlinearly stretching sheet in a porous medium saturated by a nanofluid with internal heat generation/absorption and suction/blowing is studied numerically. Similarity transformations are used to convert the partial differential equation corresponding to the momentum equation into nonlinear ordinary differential equation. The solutions for the temperature and nanoparticle concentration distributions depend on Prandtl number Pr, Lewis number Le, suction/blowing parameter S, permeability parameter k_1, source/sink parameter λ, the Brownian motion parameter Nb, the thermophoresis parameter Nt. Numerical results are presented both in tabular and graphical forms illustrating the effects of these parameters on thermal and concentration boundary layers.

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INTRODUCTION

The flow over a stretching surface is an important problem in many engineering processes with applications in industries such as extrusion, melt-spinning, the hot rolling, wire drawing, glass-fiber production, manufacture of plastic and rubber sheets, cooling of a large metallic plate in a bath, which may be an electrolyte, etc. In industry, polymer sheets and filaments are manufactured by continuous extrusion of the polymer from a die to a windup roller, which is located at a finite distance away. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity through an ambient fluid (Khan, W.A., I. Pop, 2010). Crane (1970) studied the steady two-dimensional incompressible boundary layer flow of a Newtonian fluid caused by the stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. This problem is particularly interesting since an exact solution of the two-dimensional Navier–Stokes equations has been obtained by Crane (1970). Elbashbeshy and Bazid (2004) considered flow and heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection. The boundary layer equations were converted into a set of ordinary differential equations and solved numerically for the flow and heat transfer characteristics. Layek (2007) has reported heat and mass transfer boundary layer stagnation-point flow of an incompressible viscous fluid towards a heated porous stretching sheet embedded in a porous medium subject to suction/blowing with internal heat generation or absorption. The governing boundary layer equations were transformed by scaling group of transformation into a system of ordinary differential equations. Kuznetsov and Nield (2010) to study the natural convective flow of a nanofluid over a vertical plate. Their similarity analysis identified four parameters governing the transport process, namely a Lewis number Le, a buoyancy-ratio number Nr, a Brownian motion number Nb, and a thermophoresis number Nt. The same authors later extended the work to a nanofluid saturated porous medium (Nield, D.A., A.V. Kuznetsov, 2009). Hamad (Hamad, M.A.A., 2011) obtained the analytical solutions for convective flow and heat transfer of an viscous incompressible nanofluid past a semi-infinite vertical stretching sheet in the presence of magnetic field. Hayat *et al*., (2010) investigated the two-dimensional mixed convection boundary layer magnetohydrodynamic (MHD) stagnation-point flow through a porous medium bounded by a stretching vertical plate with thermal radiation. They considered that the stretching velocity and the surface temperature are assumed to vary linearly with the distance from the stagnation-point. Cortell (2007) examined flow and heat transfer on a nonlinear stretching sheet for two different types of thermal

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boundary conditions on the sheet, constant surface temperature (CST case) and prescribed surface temperature (PST case). Sultana, Saha and Rahman (10) have analyzed the effects of internal heat generation, radiation and suction or injection on the heat transfer in a porous medium over a stretching surface. Ali (1994) studied flow and heat transfer characteristics on a stretched surface subject to a power-law velocity and temperature distributions for three different boundary conditions. Kelson and Farrell (2001) studied self-similar boundary layer flow a micropolar fluid driven by a porous stretching sheet and obtained analytical results for the shear stress and the microrotation at the surface for the limiting cases of large suction or injection. Rajeswari, Jothiram and Nelson (2009) have studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through vertical porous surface with heat source in the presence of suction.

The objective of the present study is to analyze the development of the steady boundary layer flow, heat transfer and nanoparticle fraction over a permeable nonlinearly stretching surface in a nanofluid. A similarity solution is presented. This solution depends on a Prandtl number Pr, a Lewis number Le, a Brownian motion number Nb, a thermophoresis number Nt, Heat source/sink parameter λ, Permeability parameter k_1, Nonlinearly parameter n and Suction/injection parameter S. The dependency of the local Nusselt and local Sherwood numbers on these parameters is numerically investigated.

Basic equation:

Consider steady, incompressible, laminar, two-dimensional boundary layer flow of a nanofluid past a flat sheet coinciding with the plane y = 0 and the flow being confined to y > 0. The flow is generated, due to non-linear stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the x-axis. Keeping the origin fixed, the sheet is then stretched with a velocity $u_w = bx^n$ where b is a constant, n is a nonlinear stretching parameter and x is the coordinate measured along the stretching surface, varying nonlinearly with the distance from the slit. The pressure gradient and external forces are neglected. The stretching surface is maintained at constant temperature and concentration, T_w and C_w, respectively, and these values are assumed to be greater than the ambient temperature and concentration, T_∞ and C_∞, respectively.

The basic steady conservation of mass, momentum, thermal energy and nanoparticles equations for nanofluids can be written in Cartesian coordinates x and y as, see Khan and Pop (2010),

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\{D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\} \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \tag{5}$$

Subject to the boundary condition

$u=u_x(x)=bx^n$, $v=v_w$, $T=T_w$, $C=C_w$ at $y=0$

$u\to u=ax$, $T\to T_\infty$, $C\to C_\infty$ as $y\to\infty$ (6)

Here u and v are the velocity components along the axes x and y, respectively, p is the fluid pressure, ρ_f is the density of the base fluid, α is the thermal diffusivity, υ is the kinematic viscosity, a,b is a positive constant, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and $\tau=(\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density, c is the volumetric volume expansion coefficient and ρ_p is the density of the particles.

We look for a similarity solution of Eqs. (1)–(5) with the boundary conditions (6) of the following form:

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$$u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \Psi}{\partial x}$$

$$\emptyset(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (7)$$

In seeking of the similarity solution (7), we have taken into account that the pressure in the outer (inviscid) flow is p = p_0 (constant). On substituting (7) into Eqs. (2)–(5), we obtain the following ordinary differential equations:

$$f''' + ff' - \left(\frac{2n}{n+1}\right) f'^2 + \frac{a^2}{b^2} + k_1\left(\frac{a}{b} - f'\right) = 0 \quad (8)$$

$$\frac{1}{Pr}\theta'' + f\theta' + \lambda\theta + Nb\theta'\emptyset' + Nt\theta'^2 = 0 \quad (9)$$

$$\emptyset'' + \frac{1}{2}Lef\emptyset' + \frac{Nt}{Nb}\emptyset'' = 0 \quad (10)$$

Subject to the boundary condition

$$f(0) = S, f'(0) = 1, \theta(0) = 1, \emptyset(0) = 1$$

$$f'(\infty) = \frac{a}{b}, \theta(\infty) = 0, \emptyset(\infty) = 0 \quad (11)$$

where $Pr = \upsilon/\alpha$ is the Prandtl number, $\lambda = Q_0/(\rho b c_p)$ is the heat source (λ > 0) or sink (λ < 0) parameter, $k_1 = v/(bk)$ is the permeability parameter of the porous medium, $S = -v_w/\sqrt{bv}$ (S > 0 corresponds to suction and S < 0 corresponds to blowing), $Le = v/D_B$ is the Lewis number, $Nb = \tau D_B(C_w - C_\infty)/v$ is Brownian motion parameter and $Nt = \tau D_B(C_w - C_w)/v$ is the thermophoresis parameter (see Khan and Pop [1]). For the linearly stretching boundary problem (i.e., n = 1) the exact solution for f is $f(\eta) = 1 - e^{-\eta}$, first obtained by Crane (1970) and this exact solution is unique, while for the nonlinearly stretching boundary problem (i.e., n ≠ 1) there is no exact solution. The quantities of practical interest, in this study, are the Nusselt number and the Sherwood number which are defined as

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \quad (12)$$

where q_w and q_m are heat flux and mass flux at the surface, Acording to Kuznetsov and Nield [5], the redused Nusselt number Nur and the redused sherwood number Shr which are defined respectively by

$$Nur = -\theta'(0), Shr = -\emptyset'(0) \quad (13)$$

RESULT AND DISCUSSION

Eqs.(8)-(10) subject to the boundary condition (11) have been solved numerically for some values of the governing parameter Pr, Le, λ, a/b, n and S using Matlab. The variation of the reduced Nusselt number Nur is presented in Table1.

Tables 2-4 represent the variation of tempreture θ(η) for three set of values of Nb, Pr, Le.

Figs. 1-10 show various feature of the tempreture function θ(η) and mass franction function $\emptyset(\eta)$ for the physical problem under consideration.

It is found from Fig. 1 and 2 that mass fraction function for λ >0 is the same λ <0 with the large or low values the Lewis number Le and the thickness of the boundary layer for the mass fraction function decrease when Le increase. It can also be seen that the thickness of the boundary layer for the large Le is smaller than the low Le.

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Table 1: Variation of Nur with Nb for (a) Pr=10, Le=15, (b) Pr=15, Le=25, (c) Pr=20, Le=35

(a) Pr=10	Le=15	(b) Pr=15	Le=25	(c) Pr=20	Le=35
Nb	Nur	Nb	Nur	Nb	Nur
0.1	1.643925601	0.1	1.562559071	0.1	1.343829517
0.2	0.873714735	0.2	0.515901373	0.2	0.242403023
0.3	0.40800399	0.3	0.106952174	0.3	-0.016867338
0.4	0.154962425	0.4	-0.020456911	0.4	-0.054124449
0.5	0.031127651	0.5	-0.049498044	0.5	-0.050192422

Table 2: Numerical values for θ(η) at Nb=0.1, Pr=10, Le=10

η	Θ(η)		Θ(η)		Θ(η)
0.1	0.81155238	0.6	0.11083327	1.1	0.004078955
0.2	0.61861859	0.7	0.06245164	1.2	0.001889501
0.3	0.44201482	0.8	0.03354519	1.3	0.000850861
0.4	0.29609228	0.9	0.01725260	1.4	0.000373367
0.5	0.18647339	1.0	0.00853555	1.5	0.000160031

Table 3: Numerical values for θ(η) at Nb=0.5, Pr=30, Le=15

η	Θ(η)	η	Θ(η)	η	Θ(η)
0.1	1.009997615	0.6	0.1692963	1.1	2.78E-05
0.2	1.013051416	0.7	0.04775537	1.2	2.93E-06
0.3	0.955997662	0.8	0.010079176	1.3	2.80E-07
0.4	0.747392664	0.9	0.001682381	1.4	2.44E-08
0.5	0.426487153	1.0	0.000232968	1.5	1.97E-09

Table 4: Numerical values for θ(η) at Nb=0.3, Pr=40, Le=50

η	Θ(η)	η	Θ(η)	η	Θ(η)
0.1	1.004452097	0.6	0.003038621	1.1	6.00E-09
0.2	0.899993055	0.7	0.000305963	1.2	2.85E-10
0.3	0.488974669	0.8	2.56E-05	1.3	1.21E-11
0.4	0.13817392	0.9	1.82E-06	1.4	4.65E-13
0.5	0.024021314	1.0	1.12E-07	1.5	1.62E-14

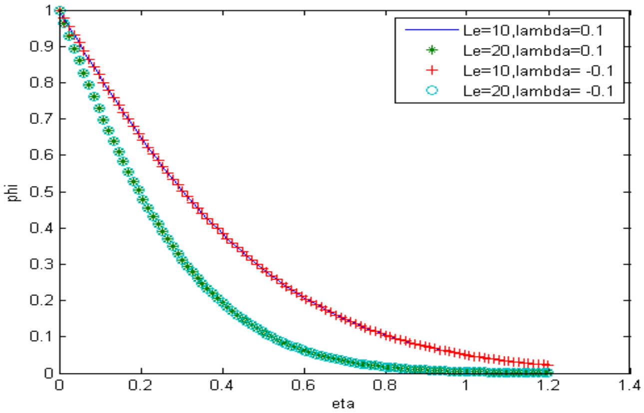

Fig. 1: Effect of λ and Le on concentration distribution for λ=(0.1,-0.1) when a/b,k_1,Nb=0.1, n=2, Nt=0, S=1, Pr=10

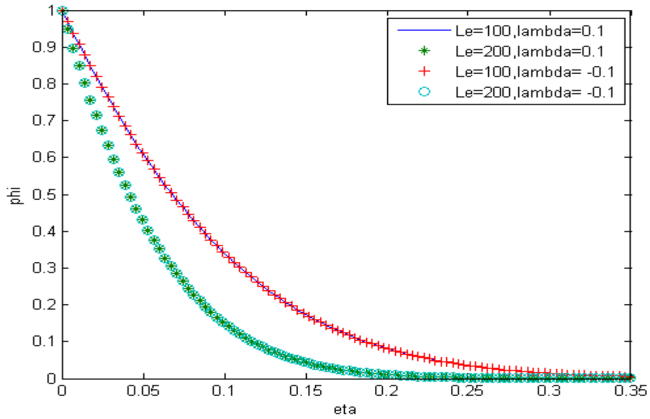

Fig. 2: Effect of λ and Le on concentration distribution for λ=(0.1,-0.1) when a/b,k_1,Nb=0.1, n=2, Nt=0, S=1, Pr=10

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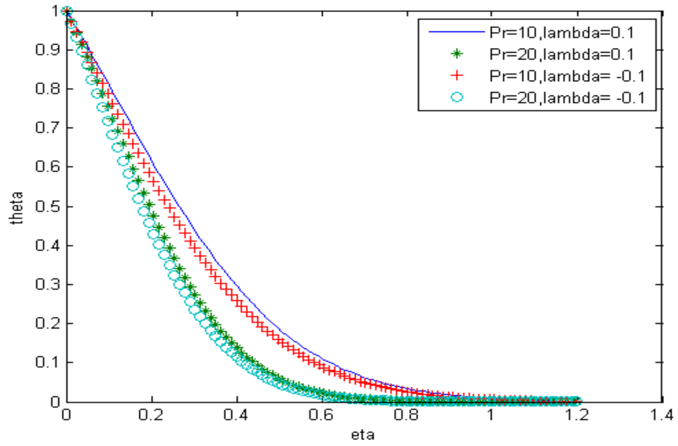

Fig. 3: Effect of λ and Pr on tempreture distribution for λ=(0.1,-0.1) when a/b,k_1,Nb=0.1, n=2, Nt=0, S=1, Le=10

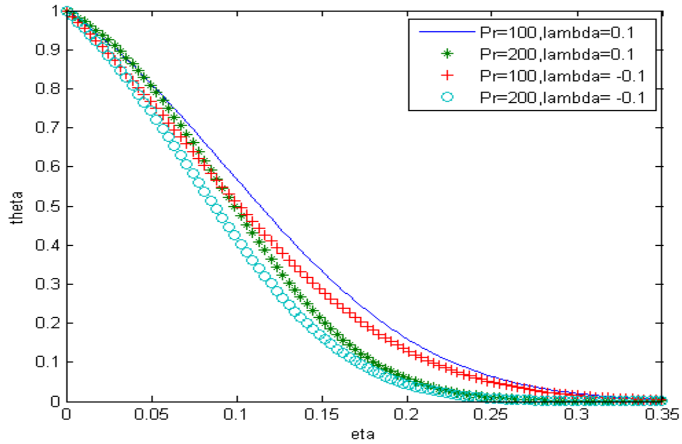

Fig. 4: Effect of λ and Pr on tempreture distribution for λ=(0.1,-0.1) when a/b,k_1,Nb=0.1, n=2, Nt=0, S=1, Le=10

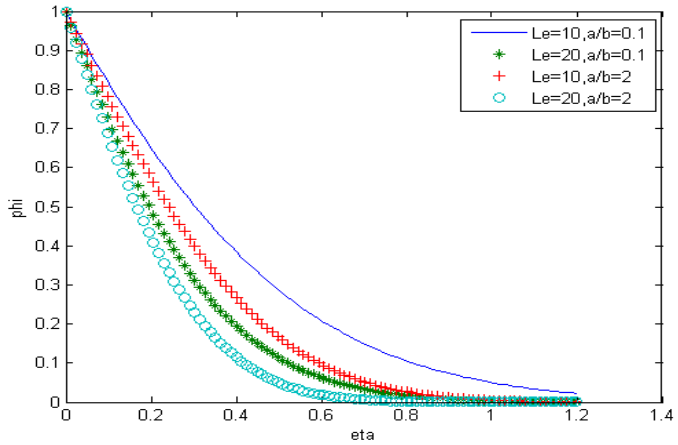

Fig. 5: Effect of a/b and Le on concentration distribution for a/b=(0.1,2) when λ,k_1,Nb=0.1, n=2, Nt=0, S=1, Pr=10

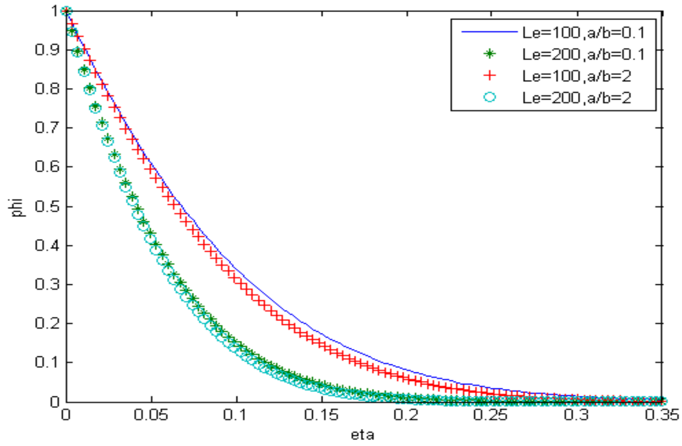

Fig. 6: Effect of a/b and Le on concentration distribution for a/b=(0.1,2) when λ,k_1,Nb=0.1, n=2, Nt=0, S=1, Pr=10

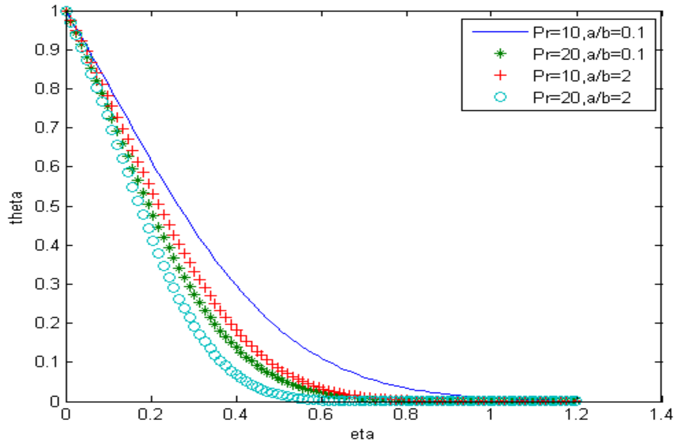

Fig. 7: Effect of a/b and Pr on tempreture distribution for a/b=(0.1,2) when λ,k_1,Nb=0.1, n=2, Nt=0, S=1, Le=10

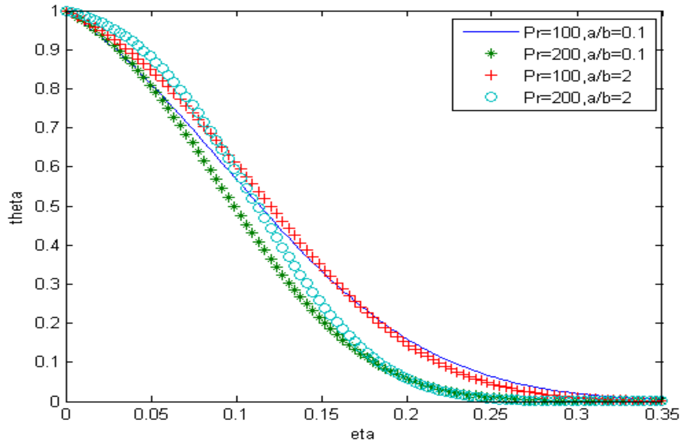

Fig. 8: Effect of a/b and Pr on tempreture distribution for n=(0.1,2) when λ,k_1,Nb=0.1, n=2, Nt=0, S=1, Le=10

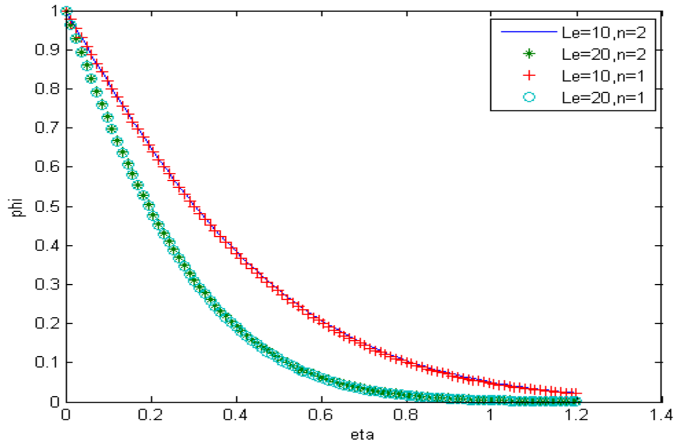

Fig. 9: Effect of n and Le on concentration distribution for n=(1,2) when a/b,λ,k_1,Nb=0.1, Nt=0, S=1, Pr=10

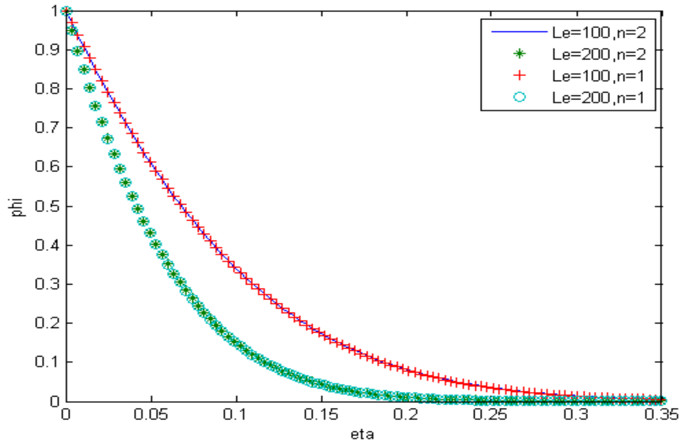

Fig. 10: Effect of n and Le on concentration distribution for n=(1,2) when a/b,λ,k_1,Nb=0.1, Nt=0, S=1, Pr=10

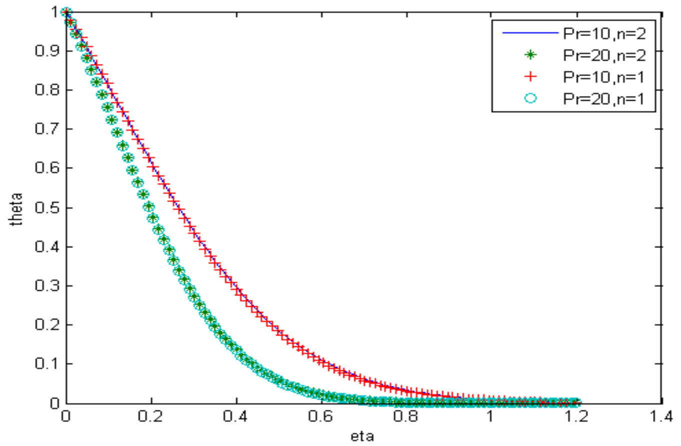

Fig. 11: Effect of n and Pr on tempreture distribution for n=(1,2) when a/b,λ,k_1,Nb=0.1, Nt=0, S=1, Le=10

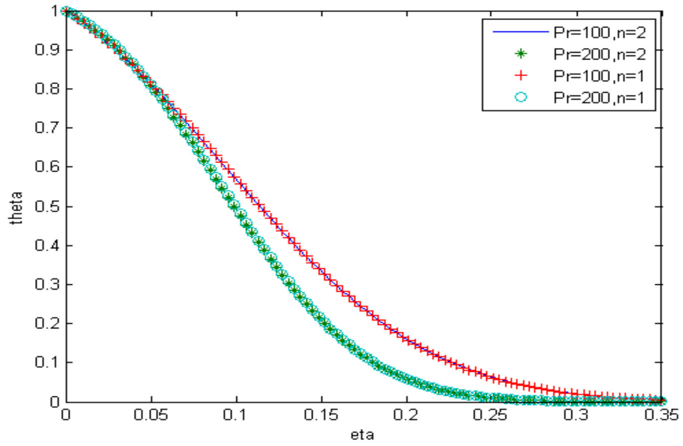

Fig. 12: Effect of n and Pr on tempreture distribution for n=(1,2) when a/b,λ,k_1,Nb=0.1, Nt=0, S=1, Le=10

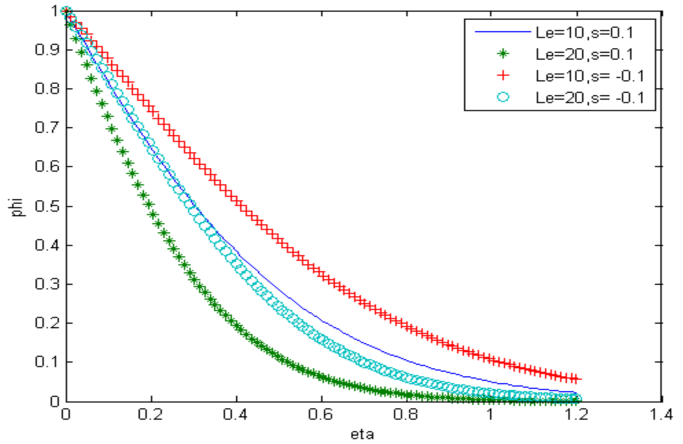

Fig. 13: Effect of S and Le on consentration distribution for S=(0.1,-0.1) when a/b,λ,k_1,Nb=0.1, n=2, Nt=0, Pr=10

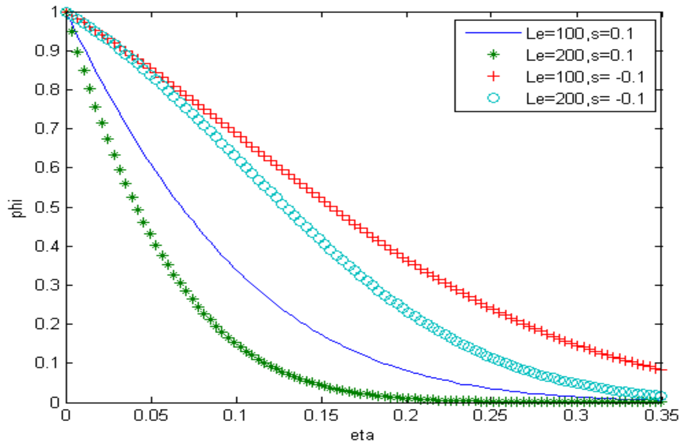

Fig. 14: Effect of S and Le on consentration distribution for S=(0.1,-0.1) when a/b,λ,k_1,Nb=0.1, n=2, Nt=0, Pr=10

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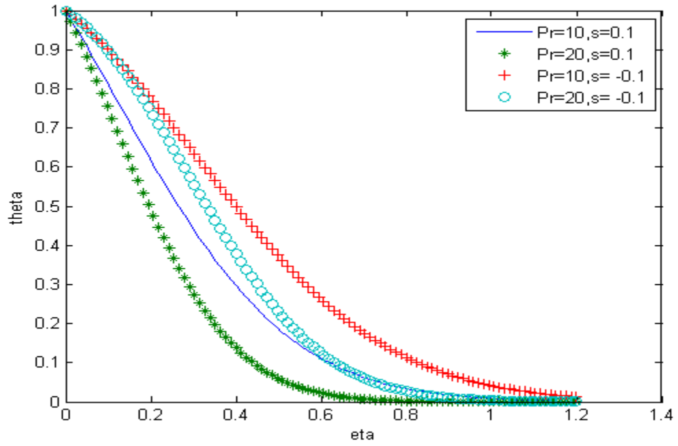

Fig. 15: Effect of S and Pr on tempreture distribution for S=(0.1,-0.1) when a/b,λ,k_1,Nb=0.1, n=2, Nt=0, Le=10

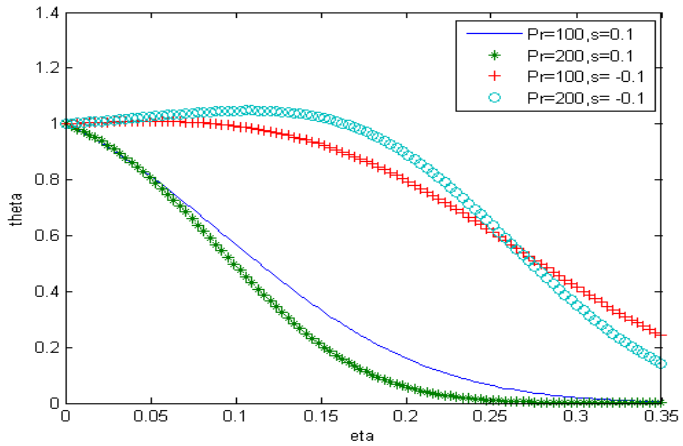

Fig. 16: Effect of S and Pr on tempreture distribution for S=(0.1,-0.1) when a/b,λ,k_1,Nb=0.1, n=2, Nt=0, Le=10

Fig. 3 and 4 show the effrct of Lewis number Le in heat source (λ>0) and heat sink (λ<0) cases on the tempreture for a/b,k_1,Nb=0.1, n=2, Nt=0, S=1, Le=10 and Pr=10,20,100,200.

Fig. 5 and 6 show the variation values of the Lewis number Le in a/b<1 and a/b>1 cases for λ,k_1,Nb=0.1, n=2, Nt=0, S=1, Pr=10 and Le=10,20,100,200.

Fig. 7 and 8 shows the effect of Prandtl number Pr on the tempreture distribution for a/b<1 and a/b>1. It is seen that for a low value of Pr the tempreture distribution for a/b<1 is greater than for a/b>1.

Fig. 9-12 show the variation of tempreture and concentration distribution with various values of the Lewis number Le and Prandtl number Pr in n=1 and n≠1 for a/b,λ,k_1,Nb=0.1, n=2, Nt=0, S=1, Le=10,100 and Pr=10,100.

Fig. 13-16 present the effect of Le and Pr on tempreture and concentration distribution in blowing (S<0) and suction (S>0) cases. It is seen that for various values of Le and low values of Pr tempreture and concentration for S<0 is greater than for S>0.

Conclusions:

we have obtained the similarity solutions of a two-dimensional laminar forced convection flow over a permeable nonlinearly stretching surface in a porous medium saturated by a nanofluid. This solution depends on Lewis number Le, Brownian motion parameter Nb, thermophoresis parameter Nt, permeability parameter k_1, suction/injection parameter S, heat generation/absorption parameter λ, Prandtl number Pr, nonlinearly parameter n and the parameter a/b. The inclusions of nanoparticles into the base fluid of this problem changes the flow pattern. The numerical results have been also presented in the form of graphs and tables.

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