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A Theoretical Investigation Internal Heat Generation Effect on SiO_2-Water Nanofluid Heat Transfer Over an Stretching Sheet

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ABSTRACT

In this study, we examine the combined effects of internal heat generation and SiO2 nanoparticle volume fraction and a convective boundary condition over a stretching sheet. It is assumed that lower surface of the stretching sheet is in contact with a hot fluid. Using a non-linear partial differential equations have been transformed into a set of coupled non-linear ordinary differential equations, which are solved numerically by applying shooting iteration technique together with fourth order Rung-Kutta integration scheme. The effects of local Biot number, the internal heat generation parameter, and nanoparticle volume fraction on the velocity and temperature profiles are illustrated and interpreted in physical terms, then compared with previous published results on the same case of the problems.

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INTRODUCTION

The flow induced by a moving boundary over a flat plate, known as stretching sheet, is important in industries.

Sakiadis was one of who carried out the pioneering work in this area (B.C. Sakiadis, 1961) and studied the boundary layer floe on a continuously stretching sheet with a constant speed. Work of Sakiadis was further verified by Tsou et al. experimentally (F.K. Tsou, *et al.*, 1967).

Recently, nanofluids are among the most intensively investigated options to improve heat transfer (L. Godson, *et al.*, 2010, S. Özerinç, *et al.,2010*).

Nanofluids are liquids in which the particles in the size of nanometer are suspended in a base fluid. Recently number of researchers investigated the flow and heat transfer of nanofluids over stretching sheet. Yacob *et al*. Rana and Bhargava numerically studied the flow and heat transfer of a nanofluid (P. Rana and R. Bhargava,2011). In a study numerically studied the effect of SiO2 nanoparticle volume fraction on the heat transfer (A. Noghrehabadi, *et al.*, 2011). In this study investigated numerically the effects of combined internal heat generated and SiO2 nanoparticle volume fraction over a stretching sheet, then compared with last study (A. Noghrehabadi, *et al.*, 2011).

Mathematical Analysis:

Consider an incompressible steady two-dimensional boundary layer flow past a stretching sheet in a water-based nanofluid which can contains different volume fraction of SiO2 nanoparticles while the lower surface of the stretching sheet is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f as shown in fig.1.

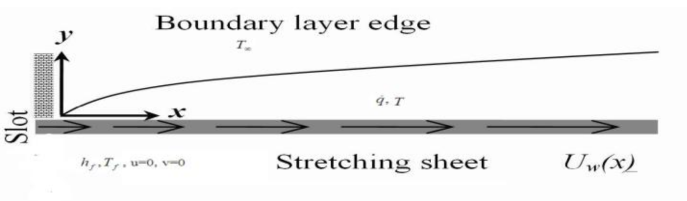

Fig. 1: Sheet configuration and coordinate system.

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It is assumed that the induced flow of nanofluid is laminar, and the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the fluid and nanoparticles are given in Table 1 (G.R. Kefayati, *et al*., 2011).

It is assumed that the lower stretching sheet fluid temperature is T_f,the heat transfer coefficient h_f and the temperature of the ambient fluid is T_∞. The fluid outside the boundary layer is quiescent, and the stretching sheet velocity is U(x)=cx where c is a constant.

The steady two-dimensional boundary layer equations for the nanofluid, in the Cartesian coordinates can be represent as (N.A. Yacob, *et al*., 2011),

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{\dot{q}}{(\rho C_p)_{nf}} \tag{3}$$

Subject to the following boundary conditions,

$$V(x,0)=0, \quad u(x,0)=U_w(x), \quad \text{at } y=0 \tag{4}$$

The thermal boundary condition at the stretching sheet lower surface,

$$-\frac{k_{nf}\partial T}{k_f\partial y}(x,0)=h_f[T_f-T(x,0)] \tag{5}$$

The boundary condition at the far stretching sheet upper surface are defined as,

$$u(x,\infty)=v(x,\infty)=0 \qquad T=T_\infty \ , \text{ as } y\longrightarrow\infty \tag{6}$$

where the subscript of nf denote the properties of nanofluid. The properties of nanofluids are defined as follows[7],

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}} \ , \tag{7}$$

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s , \tag{8}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{\frac{5}{2}}} , \tag{9}$$

Therefore,

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s \tag{10}$$

And,

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)} \tag{11}$$

where subscript of f and s represent the base fluid and nanoparticle suspension, respectively. Here, ϕ is the nanoparticle volume fraction.

To attain similarity solution of equations (1-3) subject to (4-6), the dimensionless variables can be posited in the following form,

$$\begin{bmatrix} \eta = \left(\frac{c}{\upsilon_f}\right)^{1/2} y, u = cxf'(\eta), v = -\sqrt{c\upsilon}\, f(\eta), \\ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \lambda_x = \frac{k_{nf}\dot{q}x^2}{k_f \,\mathrm{Re}_x (\mathrm{T}_f - T_\infty)}, \end{bmatrix} \quad (12)$$

Where, prime symbol denotes differentiation with respect to η and $Re_x = U_\infty x/\upsilon$ is the local Reynolds number. The local internal heat generation parameter λ_x is defined so that the internal heat generation $\dot{q}$ decays exponentially with the similarity variable η as stipulated in (R. Viskanta, 1988).

By applying the introduced similarity transforms of (12) on the governing equations of (1-3), the similarity equations are obtained as follows,

$$\frac{1}{(1-\phi)^{\frac{5}{2}}(1-\phi+\phi\rho_s/\rho_f)} f''' + ff'' - f'^2 = 0 \quad (13)$$

$$\frac{1}{\mathrm{Pr}} \frac{\frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}}{(1-\phi+\phi(\rho C_p)_s/(\rho C_p)_f)} \theta'' + f\theta' + \lambda_x e^{-\eta} = 0 \quad (14)$$

Subject to the following boundary conditions:

At $\eta = 0$: $\quad f = 0, \quad f' = 1, \quad \theta' = -Bi_x[1-\theta]$ (15)

At $\eta \longrightarrow \infty$: $\quad f' = 0, \quad \theta = 0$ (16)

$$Bi_x = \frac{k_f h_f}{k_{nf}} \sqrt{\frac{\upsilon x}{U_w(x)}}, \quad (17)$$

The solutions generated whenever Bi_x and λ_x are defined as in Eqs. (13)-(17) are the local similarity solutions. In order to have a true similarity solution the parameters Bi_x and λ_x must be constants and not depend on x. This condition can be met if the heat transfer coefficient h_f is constant and the internal heat generation $\dot{q}$ is proportional to x^{-1}. In this case, we assume

$$h_f = b, \quad \dot{q} = lx^{-1} \quad (18)$$

where b and l are constants but have the appropriate dimensions. Substituting Eq. (18) into Eqs. (12) and (17), we obtain

$$Bi = \frac{bk_f}{k_{nf}}\sqrt{\frac{\upsilon}{c}}, \quad \lambda = \frac{k_{nf} l\upsilon e^{\eta}}{k_f c(T_f - T_\infty)} \quad (19)$$

The Biot number (Bi) lumps together the effects of convection resistance of the hot fluid and the conduction resistance of the stretching sheet. The parameter λ is a measure of the strength of the internal heat generation. For practical purposes, the skin friction coefficient can be introduced as,

$$C_f = -\frac{\mu_{nf}}{\rho_f u_w^{\ 2}}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{(1-\phi)^{\frac{5}{2}}}\sqrt{\mathrm{Re}_x}\, f''(0),$$
$$\frac{C_f}{\sqrt{\mathrm{Re}_x}} = \frac{f''(0)}{(1-\phi)^{\frac{5}{2}}}, \tag{20}$$

and the reduced Nusselt number as,

$$Nu_x = \frac{x k_{nf}}{k_f (T_f - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\sqrt{\mathrm{Re}_x}\,\frac{k_{nf}}{k_f}\theta'(0)$$
$$Nur = \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\frac{k_{nf}}{k_f}\theta'(0) \tag{21}$$

Table 1: Thermophysical properties of water and SIO_2

Physical Properties	Fluid Phase (water)	SiO_2[9]
	(N. A. Yacob, *et al*., 2011)	
$C_p (J/kg.K)$	4179	765
$\rho (kg/m^3)$	997.1	3970
$k (W/m.K)$	0.613	36
$\alpha \times 107 (m^2/s)$	1.47	118.536

Results:

The nonlinear ordinary differential equations (13) and (14) subject to the boundary conditions (15),(16) and (17) were solved numerically by the Runge-Kutta-Fehlberg method with shooting technique. The value of η_∞ is taken as 15 to achieve the asymptotically far filed boundary condition. The equations are solved for variation of volume fraction of SiO2 nanoparticles, Biot number and internal heat generation. The Prandtl number of the base fluid (water) is kept constant at 6.2. As thr test of accuracy of numerical solution, the result of present method are compared with result of Noghrehabadi et al[4]. In that study[4] only investigated effect of variation of volume fraction of SiO2 nanoparticles but in this study investigated effect of internal heat generated and Biot number. Also see in fig.s (2) and (3) the results are equal with that study[4]. Fig. 2 shows the profiles of dimensionless velocity for selected values of SiO2 nanoparticle volume fraction. This figure reveals that increase of nanoparticle volume fraction increases the magnitude of velocity for comparatively high values of nanoparticles. The low volumes fractions of nanoparticles have not significant effect on the velocity profiles.

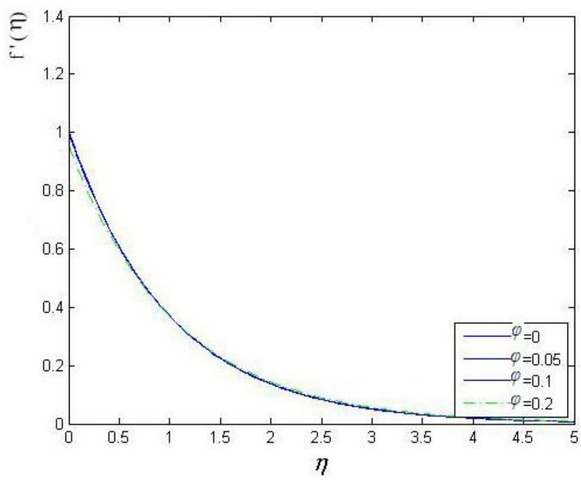

Fig. 2: Profiles of non-dimension velocity for selected values of volume fraction of ϕ

Fig. 3 shows profiles of f "(η) over the stretching sheet. This figure depicts that increase of nanoparticle volume fraction in low concentrations has not any significant effect on the shear stresses in the fluid, but comparatively large values of concentration results in increase of f "(η) near the wall. Also show that the internal heat generated and Biot number have no effect on the velocity profile and the variation of non-dimensional wall shear stress.

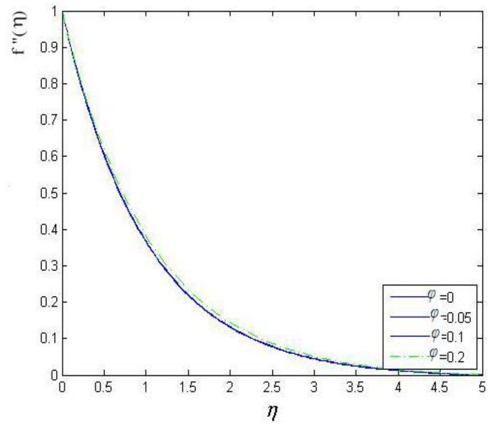

Fig. 3: Profiles of $f^{''}(\eta)$ for selected values of volume fraction of ϕ

Fig.s (4) and (5) show that the results of the effect of variation of volume fraction of SiO2 nanoparticles on the temperature profile $\theta(\eta)$ and $\theta^{'}(\eta)$ those brought from that study[4].

Fig. (4) reveals that increase of nanoparticle volume fraction in any concentration has significant effect on the temperature profile. As seen, the magnitude of temperature increases with increase of concentration. Furthermore, it is clear that increase of concentration increases the thickness of the thermal boundary layer. The combined effect of variation of volume fraction and internal heat generated cause to decrease the temperature profile near the wall and increase far from the wall, and with increase of internal heat generated has effect on temperature profile and increase it, see in fig. (6). Compare with fig.(6) and fig.(10) show that increase of Biot number cause to decrease temperature.

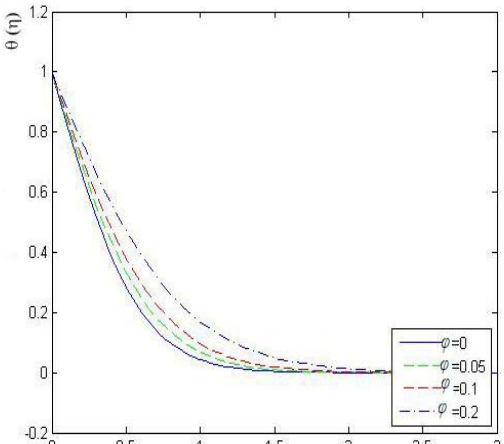

Fig. 4: Profiles of $\theta(\eta)$ for selected values of volume fraction of ϕ

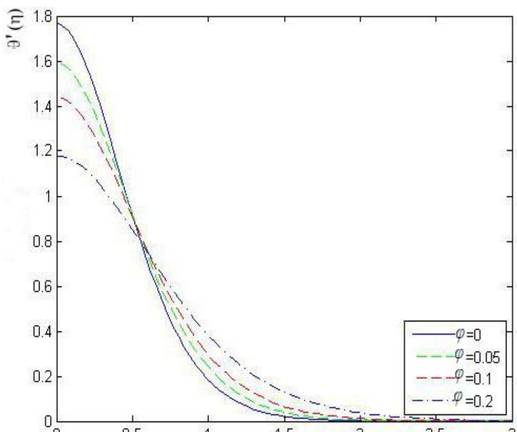

Fig. 5: Profiles of $\theta^{'}(\eta)$ for selected values of volume fraction of ϕ

Fig.s (8), (9) and (11) show that the effect of internal heat generation in variation of volume fraction, who compare with fig. (5) shows that when increase volume fraction first decrease the values of $\theta^{'}(\eta)$ near the wall and then increases them. Fig (8) and fig (9) show that the combined effect of internal heat generation and volume fraction who near the wall with increase of volume fraction cause to decrease $\theta^{'}(\eta)$ and then increase them, but compare between fig (3) and fig (4) show that by increase internal heat generation the values of

$\theta^{'}(\eta)$ too increase. Compare with the fig (8) and fig (11) show that the effect of Biot number on values of $\theta^{'}(\eta)$, whom increase Biot number cause to deduced the values of $\theta^{'}(\eta)$.

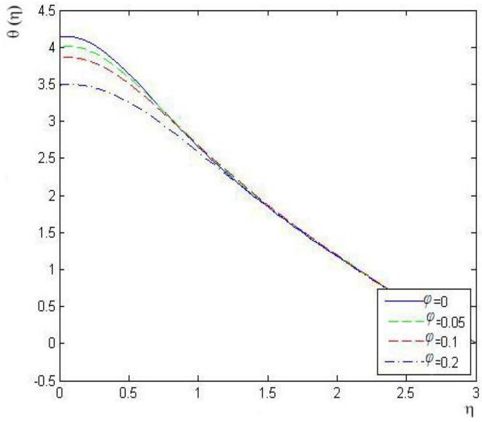

Fig. 6: Profiles of $\theta(\eta)$ for Bi=0.1, $\lambda_x = 1$, and selected values of volume fraction of ϕ

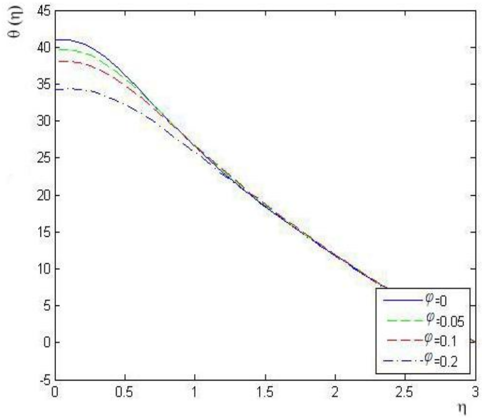

Fig. 7: Profiles of $\theta(\eta)$ for Bi=0.1, $\lambda_x = 10$, and selected values of volume fraction of ϕ

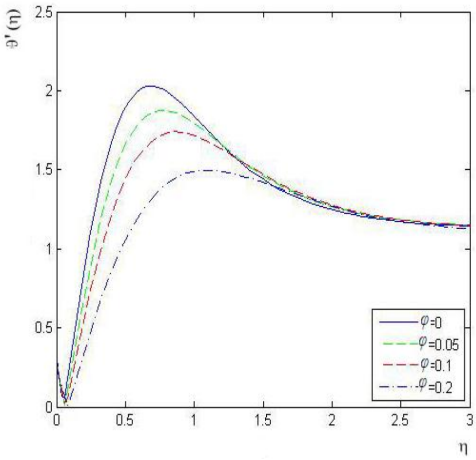

Fig. 8: Profiles of $\theta^{'}(\eta)$ for Bi=0.1, $\lambda_x = 1$, and selected values of volume fraction of ϕ

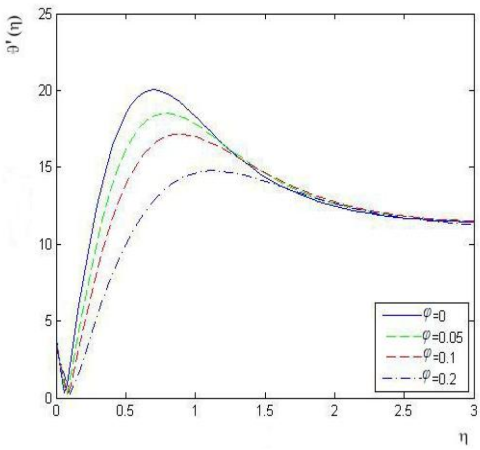

Fig. 9: Profiles of $\theta^{'}(\eta)$ for Bi=0.1, $\lambda_x = 10$, and selected values of volume fraction of ϕ

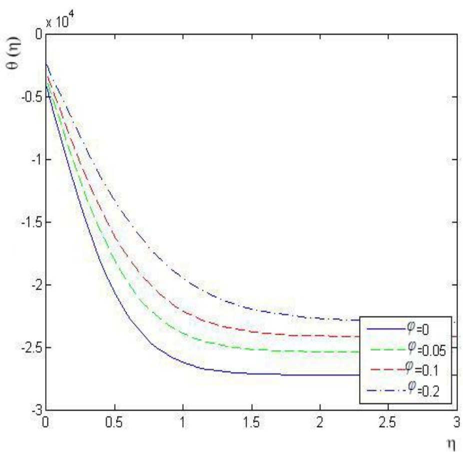

Fig. 10: Profiles of $\theta(\eta)$ for Bi=10, $\lambda_x = 0.1$ and selected values of volume fraction of ϕ

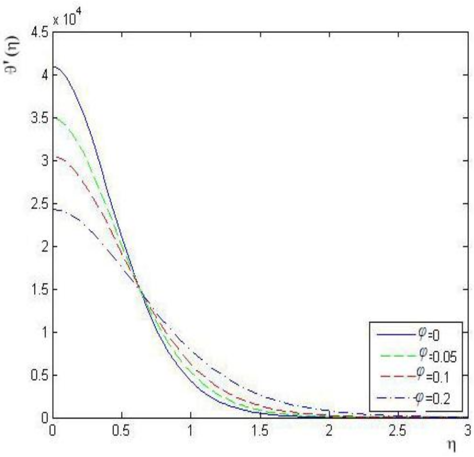

Fig. 11: Profiles of $\theta^{'}(\eta)$ for Bi=10, $\lambda_x = 0.1$, and selected values of volume fraction of ϕ

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Conclusions:

In this study investigated the combined effects of variation of volume fraction of SiO2 nanoparticles, and internal heat generation on the boundary layer flow and heat transfer over a stretching sheet has been analyzed. The governing differential equations are transformed to ordinary differential equations and solved numerically. A local similarity analysis identified governing dimensionless parameters: Biot number and internal heat generation parameter. Our result reveal that the thermal boundary layer thickness decrease with an increase in the local Biot number. An increase in internal heat generation prevents the rapid flow of heat from the lower surface to the upper surface of the stretching sheet and caused to increase temperature.

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