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Fuzzy Topsis Algorithm for Multi Criteria Group Decision Making Under A-Cuts

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ABSTRACT

The aim of this paper is to express the importance of a fuzzy TOPSIS (Technique for Ordering Preference by Similarity to Ideal Solution) model for Multi Criteria Group Decision Making (MCGDM) problem. In this paper the weight of each alternative and the weight of each criterion are described by linguistic in positive trapezoid fuzzy numbers. A α-cut method is proposed to calculate a closeness coefficient for the rating of all alternatives from fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). Finally, a numerical method demonstrates the possibility of the proposed method.

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INTRODUCTION

Decision making is a procedure for finding the alternative among a series of available alternatives. When we consider several criterions in decision making problems, they can be referred to as multi criteria decision making (MCDM). MCDM, has been one of the research areas in management operations and sciences, that due to the various applied need, has developed rapidly during the recent decade. Decisions in the public and private sector decision making often involve the assessment and ranking of available alternatives on multi-criteria(Anisseh, M. *et al*., 2012). Most crucial and significant decisions in organizations are made by group of managers or experts (Anisseh, M. *et al*., 2012). In such an environment, dissension always occurs in group decision making as members in a group generally do not come to the same decision (Anisseh, M. *et al*., 2012). Solving dissension is a significant subject in group decision making, which needs methods of aggregating preferences and settling differences (Anisseh, M. *et al*., 2012. Cheng, C.H., Y. Lin, 2002). To solve dissension for one decision maker that includes of multi criteria assessment and ranking problems, the multi criteria decision making (MCDM) has been developed (Yeh, C.H., Y.H. Chang, 2008. Ho, W., *et al.*, 2006. Corner, J.L., C.W. Kirkwood, 1991. Korhonen, P. *et al.*, 1984. Saaty, T.L., 1980). MCDM in the field is one of the most extensively used methods (Yeh, C.H., Y.H. Chang, 2008). The MCDM purpose is to choose the best alternative from several alternatives relating with various criteria decided by the decision maker (Anisseh, M. *et al*., 2012). A MCDM problem can be concisely expressed in matrix format as:

$$G = \begin{array}{c} A_1 \\ A_2 \\ . \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ . & . & & . \\ . & . & \cdots & . \\ . & . & & . \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \end{array}, \quad w = [w_1, w_2 .. w_n]$$

Where A_1, A_2, …, A_m are possible alternatives among which decision makers have to be chosen, C_1 , C_2,…,C_n are criteria with which alternative performance are measured, G_{ij} is the rating of alternative A_i with respect to criterion C_j, W_j is weight of criterion C_j. MCDM problems can be divided into two types of problems.

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One of them is classic MCDM problems, in which the rates and the weights of the criteria are measured precisely (Dyer, J.S., P.C. Fishburn,1992. Hwang, C.L., K. Yoon, 1981. Teghem, J., C. Jr. Delhaye, 1989). The other decision making is fuzzy multi criteria (FMCDM) in which rates and weights are appraised in uncertain and vague form and usually are stated in linguistic variables and fuzzy numbers (Bellman, R.E., L.A. Zadeh, 1970. Wang, Y.J., H.S. Lee, 2003). Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world (Anisseh, M. *et al*., 2012. Ertugrul, I., A. Tus, 2007). Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: nature and humanity; uncertain systems in industry; and facilitators for common-sense reasoning in decision making in the absence of complete and precise information (Anisseh, M. *et al*., 2012).TOPSIS is a useful technique in relating with multi criteria decision making problems(Hwang, C.L., K. Yoon, 1981). It helps decision maker(s) (DMs) organize the problems to be solved, and carry out analysis , comparisons and rankings of the alternatives . In many situations decision makers may provide imprecise information which comes from a variety of sources such as unquantifiable information about alternatives with related to criterions (Li, D.F. *et al*, 2009). The involved criteria or information are often uncertain, and most of these criteria can be evaluated by human judgment and human perception (Bashiri, M., H. Badri, 2010). In many cases, human judgmentare vague or ambiguous and thus may not be suitable to present them by precisely numerical values (Li, D.F., 2007. Park, J.H. *et al.*, 2010. Tan, C., 2010). TOPSIS technique is one of the known technique for classical MCDM which was first introduced by Yoon and Hwang (Hwang, C.L., K. Yoon, 1981). Jahanshahloo *et al*. extended the TOPSIS method by using the concept of α-Cuts (Jahanshahloo, G.R. *et al,* 2006). Wang and Lee generalized TOPSIS to fuzzy multiple criteria decision making in a fuzzy environment and proposed Up and Lo operations on fuzzy numbers to find the ideal solution and negative ideal solution (Wang, Y.J., H.S. Lee, 2007). Mahdavi *et al*. designed a model of TOPSIS for the fuzzy environment with the introduction of appropriate negations for obtaining ideal solution and applied a new measurement of fuzzy distance value a lower bound of alternatives (Mahdavi, I. *et al*., 2008). Chen and Tsao, Ashtiani *et al*, and Tan extended TOPSIS method for interval valued fuzzy data and a comprehensive experimental analysis to observe the interval valued fuzzy TOPSIS results yielded by different distance measures (Chen, T.Y., C.Y. Tsao, 2008. Ashtiani B *et al.*, 2009. Tan, C., 2010). Hosseinzadeh Lotfi *et al*. designed a new method for complex decision making based on TOPSIS for complex decision making problems with fuzzy data (Hosseinzadeh Lotfi *et al*., 2007). Mahmoodzadeh *et al*. designed project selection by using fuzzy AHP and TOPSIS technique (Mahmoodzadeh, S. *et al*., 2007). Salehi and Tavakkoli-Moghaddam designed project selection by using a fuzzy TOPSIS technique (Salehi, M., R. Tavakkoli-Moghaddam, 2008).Triantaphyllou and Lin developed a fuzzy version of TOPSIS method based on fuzzy arithmetic operations (Triantphyllou, E., C.T. Lin, 1996.). The main concept of TOPSIS algorithm is the definition of positive and negative ideal solution. The ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria. The optimal alternative is the one which has the shortest distance from the positive ideal solution (PIS) and the farthest from negative ideal solution (NIS).Fuzzy TOPSIS is to assign the importance of criteria and the performance of alternatives by using fuzzy number instead of crisp numbers. According to the concept of TOPSIS, we define the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS). Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives. The alternative is close to the (FPIS), and far from the (FNIS), and then this alternative will get a high ranking order. Finally, the paper will be finished by a conclusion.

Preliminaries:

Zadeh (1965) first introduced the fuzzy sets theory to resolve the vagueness, ambiguity of human judgment. In this section, we present some basic definitions related to the fuzzy sets theory.

A. Fuzzy set and fuzzy numbers:

Definition 1:

Let X be the universe of discourse. A fuzzy set $\tilde{Q}$ of X is characterized by a membership function $\mu_{\tilde{Q}}$(x): X → [0,1] which indicates the degree of $x \in X \text{ in } Q.$

Definition 2:

The fuzzy number Q is a triangular fuzzy number if its membership function f_Q is given by van Larrhoven & Pedrycz(Chu, T.C., C. Lin Yi, 2009. Van Larrhoven, P.J.M. & W. Pedrycz, 1983):

$$f_Q(x) = \begin{cases} 0, & x < a, \\ (x-a)/(b-a), & a \le x \le b, \\ (x-c)/(b-c), & b \le x \le c, \\ 0, & x > c, \end{cases} \tag{1}$$

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Where a, b and c are real numbers. For convenience, Q can be defined as a triplet (a, b, c).

Definition 3:

The membership function f_Q of the fuzzy number Q can also be expressed as:

$$f_Q(x) = \begin{cases} f_Q^L(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ f_Q^R(x), & c \le x \le d, \\ 0, & otherwise \end{cases} \tag{2}$$

Where $f_Q^L(x) = \dfrac{x-a}{b-a}$ and $f_Q^R(x) = \dfrac{x-d}{c-d}$ are the left and right membership functions of fuzzy number Q, respectively(Chu, T.C., C. Lin Yi, 2009).

It is assumed that Q is convex, normal and bounded, i.e. $-\infty < a, d < \infty$. For convenience, the fuzzy number in definition 3 can be denoted by Q=[a, b, c, d].

Definition 4:

The α-cut of fuzzy number Q can be defined as (Kauffman & Gupta (Kauffman, A., & M.M. Gupta, 1991):

$$Q^{\alpha} = \{x \mid f_Q(x) \ge \alpha,\}, \quad where\ x \in R, \alpha \in [0,1].$$

Q^{α} is a non-empty bounded closed interval contained in R and can be denoted by $Q^{\alpha} = [Q_L^{\alpha}, Q_u^{\alpha}]$, $where\ Q_L^{\alpha}\ and\ Q_u^{\alpha}$ are its lower and upper bounds, respectively. For example, if trapezoid fuzzy number Q=[a, b, c, d], then the α -cut of Q is expressed as:

$$Q^{\alpha} = [(b-a)\alpha + a, (c-d)\alpha + d] \tag{3}$$

B. Arithmetic operations of fuzzy numbers:

Given fuzzy numbers Q and K, Q, k$\in$R$^+$, the α -cuts of Q and K are $Q^{\alpha} = [Q_L^{\alpha}, Q_u^{\alpha}]\ and\ K^{\alpha} = [K_L^{\alpha}, K_u^{\alpha}]$, respectively. By the interval arithmetic, some operations of Q and K can be expressed as follows (Kauffman & Gupta,(Kauffman, A., & M.M. Gupta, 1991).

$$(Q \oplus K)^{\alpha} = [Q_L^{\alpha} + K_L^{\alpha}, Q_u^{\alpha} + K_u^{\alpha}], \tag{4}$$

$$(Q \Theta K)^{\alpha} = [Q_L^{\alpha} - K_u^{\alpha}, Q_u^{\alpha} - K_L^{\alpha}], \tag{5}$$

$$(Q \otimes K)^{\alpha} = [Q_L^{\alpha} . K_L^{\alpha}, Q_u^{\alpha} . K_u^{\alpha}], \tag{6}$$

$$(Q \phi K)^{\alpha} = \left[\frac{Q_L^{\alpha}}{K_u^{\alpha}}, \frac{Q_u^{\alpha}}{K_L^{\alpha}} \right], \tag{7}$$

$$(Q \otimes r)^{\alpha} = [Q_L^{\alpha} . r, Q_u^{\alpha} . r], \ \ r \in R^{+}. \tag{8}$$

3. Ranking fuzzy numbers:

There are various methods for ranking fuzzy numbers. The articles (Bortolan, G., R. Degani, 1985. Wang, X., E. Kerre, 2001) are a comprehensive overview of existing approaches.

3.1. Ranking fuzzy numbers by mean of removals:

The mean of removals, by Kauffman & Gupta (1991) is applied to consider a fuzzy number Q = [a, b, c, d] . The left removal of Q , denoted by Q_L, and the right removal of Q, denoted by Q_R, are defined as follows (Chu, T.C., C. Lin Yi, 2009).

$$Q_L = b - \int_a^b f_Q^L(x)dx, \tag{9}$$

$$Q_R = c + \int_c^d f_Q^R(x)dx, \tag{10}$$

The mean removal of the Q_L and Q_R is then defined as:

$$M(Q) = \frac{1}{2}(Q_L + Q_R) = \frac{1}{2}(b+c) + \frac{1}{2}\left(\int_c^d f_Q^R(x)\,dx - \int_a^d f_Q^L(x)\,dx\right) \tag{11}$$

Herein, M(Q) is used to compare fuzzy numbers. The larger the M(Q), the larger the fuzzy number Q. Therefore, for any two fuzzy numbers Q_i and Q_j, if $M(Q_i) > M(Q_j)$, then $Q_i > Q_j$. if $M(Q_i) = M(Q_j)$, then $Q_i = Q_j$. Finally, if $M(Q_i) < M(Q_j)$, then $Q_i < Q_j$.

4. The proposed fuzzy TOPSIS algorithm:

Thought our work is similar to Yao and Wu [35], we have applied the Generalized D_α as below:

$$D_\alpha(\tilde{A},\tilde{B}) = \frac{1}{2}\int_0^1 \left(\left[\tilde{A}\right]_\alpha^L + \left[\tilde{A}\right]_\alpha^u - \left[\tilde{B}\right]_\alpha^L - \left[\tilde{B}\right]_\alpha^u\right)\alpha^n\,d\alpha \qquad \alpha \in [0,1] \tag{12}$$

Where $[\tilde{A}]_\alpha^L, [\tilde{A}]_\alpha^u, [\tilde{B}]_\alpha^L, [\tilde{B}]_\alpha^u$ are lower and upper bounds of $\tilde{A}, \tilde{B}$, respectively and $\tilde{A}, \tilde{B} \in$ F (F is the family of the fuzzy numbers on R). From (12) let:

$D_\alpha(\tilde{A},\tilde{B}) > 0$ *iff* $D_\alpha(\tilde{A},O) > D_\alpha(\tilde{B},O)$ *iff* $\tilde{A} > \tilde{B}$

$D_\alpha(\tilde{A},\tilde{B}) < 0$ *iff* $D_\alpha(\tilde{A},O) < D_\alpha(\tilde{B},O)$ *iff* $\tilde{A} < \tilde{B}$

$D_\alpha(\tilde{A},\tilde{B}) = 0$ *iff* $D_\alpha(\tilde{A},O) = D_\alpha(\tilde{B},O)$ *iff* $\tilde{A} \approx \tilde{B}$

This method mentioned above is to preserve the property of ranking fuzzy numbers belong: $\alpha \in [0, 1]$.

The concept of linguistic variables is used for providing approximate characterization, when conventional quantitative terms are complex or ill defined. These linguistic variables can be expressed in positive trapezoid fuzzy numbers as table I and Table II.

Table I: Linguistic variables for the rating.

Linguistic variable	fuzzy number
Very Poor (VP)	(0,0,1,2)
Poor (P)	(1,2,2,3)
Medium Poor (MP)	(2,3,4,5)
Fair (F)	(4,5,5,6)
Medium Good (MG)	(5,6,7,8)
Good (G)	(7,8,8,9)
Very Good (VG)	(8,9,10,10)

Table II: Linguistic variables for importance weight of each criterion.

Linguistic variable	Corresponding trapezoid fuzzy number
Very Low (VL)	(0,0,1,2)
Low (L)	(1,2,2,3)
Medium Low (ML)	(2,3,4,5)
Medium (M)	(4,5,5,6)
Medium High (MH)	(5,6,7,8)
High (H)	(7,8,8,9)
Very High (VH)	(8,9,10,10)

Assume that x= $\{x_{ij} \mid i = 1,2,...,m\ ,\ j = 1,2,...,n\}$ is the rate alternatives with respect to various criteria (C_j, j=1…,n).

A decision maker D_K (k=1, 2,…, K) can express his membership function $\mu_{\tilde{R}_k}(x)$ with the positive trapezoid fuzzy numbers $\tilde{R}_k (k = 1,2,..., K)$.

If decision maker's fuzzy rates be positive trapezoid fuzzy numbers $\tilde{R}_k = (r_k^a, r_k^b, r_k^c, r_k^d)$ then the aggregation of decision makers' fuzzy rates can be denoted by $\tilde{R} = (r^a, r^b, r^c, r^d)$.

Where:

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$$r^{a} = \min_{k}\{r_{k}^{a}\},$$

$$r^{b} = \frac{1}{K}\sum_{k=1}^{K} r_{k}^{b},$$

$$r^{c} = \frac{1}{K}\sum_{k=1}^{K} r_{k}^{c},\ r^{d} = \max_{k}\{r_{k}^{d}\} \qquad (13)$$

Assume that rating of alternatives and the importance fuzzy weights of each criteria for the kth decision maker is:

$$\tilde{X}_{ijk} = (x_{ijk}^{a}, x_{ijk}^{b}, x_{ijk}^{c}, x_{ijk}^{d}),$$

$$\tilde{w}_{jk} = (w_{jk}^{a}, w_{jk}^{b}, w_{jk}^{c}, w_{jk}^{d})$$

(i=1, 2, …, m , j=1, 2,…, n)

As it is state above, the combination of alternatives' fuzzy raters $\tilde{X}_{ij}$ can be described as:

$$\tilde{X}_{ij} = (x_{ij}^{a}, x_{ij}^{b}, x_{ij}^{c}, x_{ij}^{d}).$$

Where:

$$x_{ij}^{a} = \min_{k}\{x_{ijk}^{a}\},$$

$$x_{ij}^{b} = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^{b},$$

$$x_{ij}^{c} = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^{c}, \qquad (14)$$

$$x_{ij}^{d} = \max_{k}\{x_{ijk}^{d}\}$$

For combining of each criteria of fuzzy weights $\tilde{w}_{j}$ can be denoted by:

$$\tilde{w}_{j} = (w_{j}^{a}, w_{j}^{b}, w_{j}^{c}, w_{j}^{d})$$

where:

$$w_{j}^{a} = \min_{k}\{w_{jk}^{a}\},\ \ w_{j}^{b} = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^{b} \qquad (15)$$

$$w_{j}^{c} = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^{c},\ \ w_{j}^{d} = \max_{k}\{w_{jk}^{d}\}$$

A fuzzy multi criteria decision making problem can be briefly expressed in matrix format as:

$$\tilde{W} = (\tilde{w}_{1}, \tilde{w}_{2}, \ldots, \tilde{w}_{n})$$

$$\tilde{D} = \begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} & \cdots & \tilde{X}_{1n} \\ \tilde{X}_{21} & \tilde{X}_{22} & \cdots & \tilde{X}_{2n} \\ \tilde{X}_{m1} & \tilde{X}_{m2} & \cdots & \tilde{X}_{mn} \end{bmatrix}$$

Therefore, we can obtain the normalized fuzzy decision matrix denoted by:

$$\tilde{R} = [\tilde{r}_{ij}]_{mxn} \qquad (16)$$

Where B and C are the set of benefit criteria and cost criteria, respectively, and

$$\tilde{r}_{ij} = x_{ij} / x_{ij}^{+}\ ,\ \ x_{ij}^{+} = \max_{i}\{x_{ij}^{d}\} \text{ for each } j \in B$$

$$\tilde{r}_{ij} = x_{ij}^{-} / x_{ij}\ ,\ \ x_{ij}^{-} = \min_{i}\{x_{ij}^{a}\} \text{ for each } j \in C \qquad (17)$$

The normalization method mentioned above is to preserve the property that the ranges of normalized trapezoid fuzzy numbers belong to [0,1].

The weighted normalized fuzzy decision matrix is denoted by:

$$\tilde{P} = [\tilde{p}_{ij}]_{m\times n} \quad (i = 1,2,\ldots,m,\ j = 1,2,\ldots,n)$$

where : (18)

$$\tilde{P}_{ij} = \tilde{r}_{ij}\ (.)\ \tilde{w}_j$$

So we can define the fuzzy positive ideal solution (FPIS,O^+) and fuzzy negative ideal solution(FNIS,O^-) as:

$$O^+ = (\tilde{F}_1^*, \tilde{F}_2^*, \ldots, \tilde{F}_n^*) \tag{19}$$

$$O^- = (\tilde{F}_1^-, \tilde{F}_2^-, \ldots, \tilde{F}_n^-)$$

Where:

$$\tilde{F}_j^* = (f_j^*, f_j^*, f_j^*, f_j^*),\quad \tilde{F}_j^- = (f_j^-, f_j^-, f_j^-, f_j^-) \qquad , \quad \forall i, j$$

$$f_j^* = \max_i\{p_{ij}^d\},\ f_j^- = \min_i\{p_{ij}^a)$$

Where: $\tilde{F}_j^* = (1,1,1,1)$, $\tilde{F}_j^- = (0,0,0,0)$

The distance of each alternative from O^+, O^- for $\alpha \in [0,1]$ can be calculated as:

$$d_i^*(\alpha) = \sum_{j=1}^{n} d(\tilde{F}_j^*, \tilde{p}_{ij}) = \sum_{j=1}^{n} (2f_j^* - p_{ij}^a - p_{ij}^d) + \left(\frac{\alpha}{2}\right)\sum_{j=1}^{n}(p_{ij}^a + p_{ij}^d - p_{ij}^b - p_{ij}^c)\ ,\ \forall i \tag{20}$$

$$d_i^-(\alpha) = \sum_{j=1}^{n} d(\tilde{p}_{ij}, \tilde{F}_j^-) = \sum_{j=1}^{n} (p_{ij}^a + p_{ij}^d - 2f_j^-) + \left(\frac{\alpha}{2}\right)\sum_{j=1}^{n}(p_{ij}^b + p_{ij}^c - p_{ij}^a - p_{ij}^d) \quad ,\ \forall i \tag{21}$$

A closeness coefficient is defined to determine the ranking order for all alternatives: Once the $d_i^*(\alpha)$ *and* $d_i^-(\alpha), \alpha \in [0,1]$ of each alternative has been calculated, the closeness coefficient of each alternative is calculated as:

$$CC_i(\alpha) = \frac{d_i^-(\alpha)}{d_i^*(\alpha) + d_i^-(\alpha)} \qquad i = 1, 2, \ldots, m \tag{22}$$

Obviously, a large value of index $CC_i(\alpha)$ demonstrates that the alternative is closer to the FPIS (O^+) and farther from FNIS (O^-) , and thus alternative will approach to rank 1.

5. Numerical illustrations:

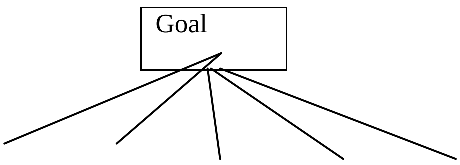
In this section, first we work out a numerical example, taken from(Chen, C.T., 2009), to illustrate the TOPSIS algorithm for multi criteria decision making with fuzzy data under α-cut method and then compare the performance of our method with the method of Chen (2009) and Mahdavi I. (2008).

5.1. A numerical example:

Suppose that a software company desires to hire a system analysis engineer. After preliminary screening, three candidates A_1, A_2 and A_3 remain for further evaluation. A committee of three decision-makers, D_1, D_2 and D_3 has been formed to conduct the interview and to select the most suitable candidate. Five benefit criteria are considered:

(1) Emotional steadiness (C_1),
(2) Oral communication skill (C_2),
(3) Personality (C_3),
(4) Past experience (C_4),
(5) Self-confidence (C_5).

The hierarchical structure of this decision problem is shown as Fig. 1. The importance weight of the criteria and the ratings of the three candidates by decision makers under all criteria can be expressed in positive trapezoid fuzzy numbers as table III and table IV.



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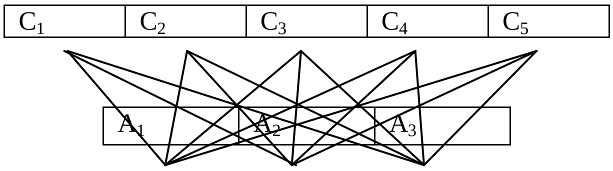

Fig. 1: The hierarchical structure.

Table III: The importance weight of the criteria.

Criteria	Decision-makers		
	D_1	D_2	D_3
C_1	H	VH	MH
C_2	VH	VH	VH
C_3	VH	H	H
C_4	VH	VH	VH
C_5	M	MH	MH

Table IV: The ratings of the three candidates by decision makers under all criteria.

Criteria	Candidates	Decision-makers		
		D_1	D_2	D_3
C_1	A_1	MG	G	MG
	A_2	G	G	MG
	A_3	VG	G	F
C_2	A_1	G	MG	F
	A_2	VG	VG	VG
	A_3	MG	G	VG
C_3	A_1	F	G	G
	A_2	VG	VG	G
	A_3	G	MG	VG
C_4	A_1	VG	G	VG
	A_2	VG	VG	VG
	A_3	G	VG	MG
C_5	A_1	F	F	F
	A_2	VG	MG	G
	A_3	G	G	MG

5.2. Comparison result:

The example shows that preference of the alternatives is A_2, A_3 and A_1. The results, shown in Table X, indicate that the correct solution is found only by our method, and the other two methods obtain a correct result.

Table V: Comparison results.

A	Method	di*	di^-	CC_i	Rank
A_1	Chen [3]	2.10	3.45	0.62	**3**
A_2		1.24	4.13	0.77	**1**
A_3		1.59	3.85	0.71	**2**

Table VI:

A	Method	di*	di^-	CC_i	Rank
A_1	Mahdavi [10]	4.3673	4.9326	0.5304	**3**
A_2		5.0000	4.3044	0.4626	**1**
A_3		4.6722	4.5614	0.4940	**2**

5.3. Our method's comparison results:

Table VII: The Closeness Coefficient for n= 0, 1, 2 from (12), (A=Alternative).

A	for n=0				
	α =0	α =0.25	α =0.5	α =0.75	α =1
A_1	0.5069	0.5186	0.5186	0.5186	0.5186
A_2	0.6138	0.6440	0.6440	0.6440	0.6440
A_3	0.5216	0.5425	0.5425	0.5425	0.5425

Table VIII:

A	for n=1				
	α =0	α =0.25	α =0.5	α =0.75	α =1
A_1	0.5069	0.5098	0.5128	0.5157	0.5186
A_2	0.6138	0.6213	0.6289	0.6365	0.6440
A_3	0.5216	0.5268	0.5320	0.5372	0.5425

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Table IX:

A	for n=2				
	α =0	α =0.25	α =0.5	α =0.75	α =1
A_1	0.5069	0.5076	0.5098	0.5135	0.5186
A_2	0.6138	0.6156	0.6213	0.6308	0.6440
A_3	0.5216	0.5229	0.5268	0.5333	0.5425

As TABLE X the ranking order of all alternatives can be calculated as:

Table X: TOTAL.

n / A	$n=0$ (*)	RANK	$n=1$ (**)	RANK	$n=2$ (***)	RANK
A_1	0.5162	3	0.5156	3	0.5155	3
A_2	0.6379	1	0.6364	1	0.6360	1
A_3	0.5383	2	0.5372	2	0.5369	2

From (*) for A_1 the TOTAL is calculated as:

$$\frac{\sum_{\alpha} CC_i\big|_{\alpha}}{5} \qquad \alpha = (0, 0.25, 0.5, 0.75, 1), (i = 1, ..., m)$$

From (**) for A_1 the TOTAL is calculated as:

$$\frac{1}{\sum_{\alpha}}[CC_i\big|_{\alpha=0} * (0) + CC_i\big|_{\alpha=0.25} * (0.25) + CC_i\big|_{\alpha=0.5} * (0.5) + CC_i\big|_{\alpha=0.75} * (0.75) + CC_i\big|_{\alpha=1} * (1)$$

]

From (***) for A_1 the TOTAL is calculated as:

$$\frac{1}{\sum_{\alpha^2}}[CC_i\big|_{\alpha=0} * (0)^2 + CC_i\big|_{\alpha=0.25} * (0.25)^2 + CC_i\big|_{\alpha=0.5} * (0.5)^2 + CC_i\big|_{\alpha=0.75} * (0.75)^2 +$$

$$+ CC_i\big|_{\alpha=1} * (1)^2]$$

Conclusion:

In general, this paper has two purposes: the first one is to propose fuzzy TOPSIS algorithm for multi criteria decision making, and the second one is to examine the proposed algorithm by a case study of an example. We proposed TOPSIS for multi criteria decision making (MCDM) problems with fuzzy data and developed an algorithm to determine the ranking order of all alternatives and select the best one from among a set of possible alternatives.

Here, the linguistic variables are applied instead of numerical values to solve the MCDM problem under α-cuts method. The distances of an alternative from the FPIS and the FNIS are considered. The alternative is closer to the FPIS and farther from FNIS, will get a high ranking order. A numerical example was illustrated to examine the applicability of the proposed approach. Comparative results were shown to illustrate the advantages of the proposed model.

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