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Point To Point Interval Type-2 Fuzzy Sets Is More Flexible and Applied In Mcdm Problems with Fuzzy Topsis Method

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ABSTRACT
It has been widely used in ranking of multi alternatives with respect to multi criteria with the superiority of fuzzy set type-1. In this paper based on the interval type-2 fuzzy sets, we introduce a new method of fuzzy TOPSIS for handling fuzzy ranking multi criteria group/individual decision making (MCGDM/MCDM) problems. In the proposed method we obtain type-1 fuzzy set for each one of the membership function in the part of positive linguistic variable for trapezoidal and triangular (in the shape of trapezoidal) fuzzy numbers for the rating of alternatives and the importance weight of each criterion in the form of point to point interval type-2 fuzzy sets (PTP IT2 FS). To illustrate the performance of the proposed method, a numerical method demonstrates the possibility of the proposed method. At the end of paper results demonstrate that this method provides us with a useful way to handle the fuzzy multi criteria decision making problems.

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INTRODUCTION

Decision making is a procedure for finding the alternative among a series of available alternatives. One of the well known methods in multi criteria decision making (MCDM) is Technique for Order Preference by Similarly to Ideal Solution (TOPSIS). The technique was developed by Hwang and Yoon (Hwang, C.L. and K. Yoon, 1981). Decisions in the public and private sector decision making often involve the assessment and ranking of available alternatives on multi-criteria (Anisseh, M., *et al*., 2012). There are a bunch of research articles on fuzzy TOPSIS for MCMD, see references (Saeednamaghi, S.H., A. Zare, 2014). MCDM in the field is one of the most extensively used methods (Yeh, C.H., Y.H. Chang, 2008). Chen (2009) introduced an extension of TOPSIS under fuzzy environment. One of them is classic MCDM problems, in which the rates and the weights of the criteria are measured precisely (Dyer, J.S., P.C. Fishburn, 1992; Hwang C.L., K. Yoon, 1981; Teghem, J., Jr, 1989). The other decision making is fuzzy multi criteria (FMCDM) in which rates and weights are appraised in uncertain and vague form and usually are stated in linguistic variables and fuzzy numbers (Bellman R.E., L.A. Zadeh, 1970; Wang, Y.J., H.S. Lee, 2003). Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world (Anisseh, M., *et al*., 2012; Ertugrul, I., A. Tus, 2007). TOPSIS is a useful technique in relating with multi criteria decision making problems (Hwang C.L., K. Yoon, 1981). It helps decision maker(s) (DMs) organize the problems to be solved, and carry out analysis , comparisons and rankings of the alternatives. Jahanshahloo *et al*. extended the TOPSIS method by using the concept of α-Cuts (Ertugrul, I., A. Tus, 2007). Mahdavi *et al*. designed a model of TOPSIS for the fuzzy environment with the introduction of appropriate negations for obtaining ideal solution and applied a new measurement of fuzzy distance value a lower bound of alternatives (Mahdavi, I., *et al*., 2008). Chen and Tsao, Ashtiani *et al*, and Tan extended TOPSIS method for interval valued fuzzy data and a comprehensive experimental analysis to observe the interval valued fuzzy TOPSIS results yielded by different distance measures(Chen, T.Y., C.Y. Tsao, 2008; Ashtiani, B., *et al*., 2009; Tan, C., 2010). Hosseinzadeh Lotfi *et al*. designed a new method for complex decision making based on TOPSIS for complex decision making problems with fuzzy data (Hosseinzadeh, Lotfi *et al*., 2007). Salehi and Tavakkoli-Moghaddam designed project selection by using a fuzzy TOPSIS technique (Salehi, M., R. Tavakkoli-Moghaddam, 2008). The main concept of TOPSIS algorithm is the definition of positive and negative ideal solution. The ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria. The optimal alternative is the one which has the

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shortest distance from the positive ideal solution (PIS) and the farthest from negative ideal solution (NIS).Fuzzy TOPSIS is to assign the importance of criteria and the performance of alternatives by using fuzzy number instead of crisp numbers. According to the concept of TOPSIS, we define the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS). Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives. The alternative is close to the (FPIS), and far from the (FNIS), and then this alternative will get a high ranking order. However, fuzzy TOPSIS is still believed not suitable to represent the uncertainties. Zade (1975) was developed a new fuzzy set method known as type-2 fuzzy set. Then Mendel *et al*. (2006) improved this type-2 fuzzy set into an interval type-2 fuzzy set. In (Wu, D., J.M. Mendel, 2007), Wu and Mendel pointed out that interval type-2 fuzzy sets have been applied in many areas, such as: decision making (Mendel, J.M., 2002; Ozen, T., J.M. Garibaldi, 2004; Sevastjanov, P., P. Figat, 2007; Wu, D., J.M. Mendel, 2007; Wu, H., J.M. Mendel, 2004), time series forecasting (Baguley, P., T. Page, 2006; Karnik, N.N., J.M. Mendel, 1999; Mendel, J.M., 2001), survey processing (Auephanwiriyakul, S., A. Adrian, 2002; Liu F., J.M. Mendel, 2007 Mendel, J.M., 2001), document retrieval (Castillo, O., N. Cazarez, 2006), speech recognition (Melin, P., J. Urias; Zeng, J., Z.O. Liu, 2006), noise cancellation (Castillo O., P. Melin, 2004; Castillo O., P. Melin, 2008), word modeling (Liu F., J.M. Mendel, 2007; Mendel, J.M., H. Wu, 2004; Wu, H., J.M. Mendel, 2004), clustering (Rhee, F.C.H., 2007), control (Astudillo, L., O. Castillo, 2007; Castillo, O., N. Cazarez, 2006;Castillo O., P. Melin, 2008; Figueroa, J., J. Posada, 2005; Hagras, H., 2007; Lee C.H., Y.C. Lin, 2004; Lin P.Z., C.F. Hsu, 2005; Lynch C., H. Hagras, 2006; Melin P., O. Castillo, 2007; Sepulveda R., O. Castillo, 2007; Wu, D., W.W. Tan, 2006; Wu, D., W.W. Tan, 2006), wireless communication (Liang Q., L. Wang, 2005; Shu, H., Q. Liang, 2005), web-shopping (Gu, L., Y.Q. Zhang, 2007) and linguistic summarization of databases(Niewiadomski, A., M. Bartyzel, 2006; Niewiadomski, A., P.S. Szczepaniak, 2006). Zhang (2012) introduced interval type-2 rough fuzzy set as a combination of IT2FS and rough set theory. He also manage to get the relationship between IT2FS and interval type 2 fuzzy topology operators. Until now, there are a lot of studies discussed on the improvement of ranking type-2 fuzzy numbers. For example, Dongrui Wu, Jerry M. Mendel (2009) proposed a new ranking method and a new similarity measure for IT2 FSs. Chen Shyi-Ming, Wang Cheng-Yi (2013) present a new method for fuzzy multiple attributes decision making based on interval type-2 fuzzy sets. Their proposed method can overcome the drawbacks of Liu and Su's method (2005) and Wang and Luo's method (2009) due to the fact that it can deal with the ranking of interval type-2 fuzzy sets to distinguish the preference order of the alternatives. Chen and Lee (2010) presented a method for fuzzy multiple attributes group decision making based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets. Huynh *et al*. (2008) presented a method for ranking fuzzy numbers based on the probability-based comparison relation of fuzzy numbers and then applied the fuzzy ranking method for target-based fuzzy decision making. Yager (1999) presented a method for fuzzy multiple attributes decision making based on fuzzy probability assessments. Yager (1999), presented a method for fuzzy multiple attributes decision making by including the decision-maker's attitude based on comparing fuzzy sets. Yager (2000) presented a method for fuzzy multiple attributes decision making based on evaluating fuzzy sets and the attitude of the decision-maker. The rest of this paper is organized as follows: In Section 2, we briefly review some basic concepts of interval type-2 fuzzy sets. In Section 3, we propose a new fuzzy ranking method based on point to point interval type-2 fuzzy sets (PTP IT2 FS) with considering of new decision-maker's alternatives and weights. Then, based on the proposed method, we design type-1 fuzzy sets for an example, see (Salehi, M., R. Tavakkoli-Moghaddam, 2008). In Section 4, we compare our results method with type-1 fuzzy for an example was mentioned in section 3 .Finally, the conclusions are discussed in Section 5.

2. Preliminaries:

For convenience in defining the Upper and Lower MFs (Membership Functions) of a type-2 MF, we first give the definition of footprint of uncertainty (FOU) of a type-2 MF.

Definition 2.1 (Footprint of Uncertainty of a Type-2MF):

Uncertainty in the primary membership grades of a type-2 MF consists of a bounded region that we call the footprint of uncertainty of a type-2 MF (e.g., see Fig. 1). It is the union of all primary membership grades.

Definition 2.2 (Upper and Lower MFs):

An Upper MF (UMF) and a Lower MF (LMF) are two type-1 MFs that are bounds for the footprint of uncertainty of an interval type-2 MF. The Upper MF is a subset that has the maximum membership grade of the footprint of uncertainty; and the Lower MF is a subset that has the minimum membership grade of the footprint of uncertainty.

The propose a new fuzzy ranking method based on point to point interval type-2 fuzzy sets:

In this Section, we propose a new fuzzy ranking method based on point to point interval type-2 fuzzy sets with the consideration of the decision maker's (DMs) alternatives and weights. Chen (2000) proposed extensions of the TOPSIS for group decision-making under fuzzy environment. Now, we present a new fuzzy ranking method based on point to point interval type-2 fuzzy sets with the new of alternatives and weights for trapezoidal and triangular (in the shape of trapezoidal) fuzzy numbers. TABLE I, TABLE II, TABLE III and TABLE IV are used for new linguistic variables for the rating of alternatives and linguistic variables for importance weight of each criterion.

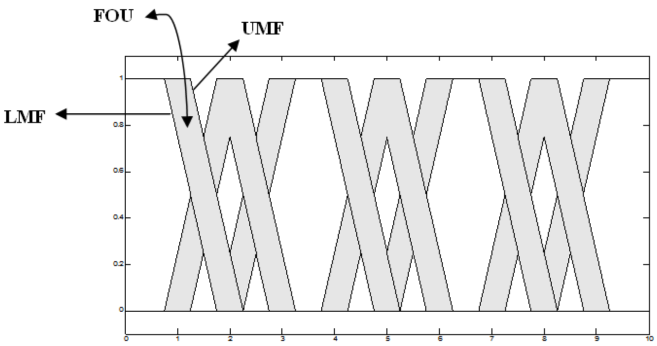

Fig. 1: Interval type-2 fuzzy set (IT2 FS) and associated quantities for triangular and trapezoidal MFs.

Table I: Linguistic variables for the rating of alternatives.

Linguistic variable	fuzzy number
Very Poor (VP)	(1,2,2,3)
Poor (P)	(2,3,3,4)
Medium Poor (MP)	(3,4,4,5)
Fair (F)	(4,5,5,6)
Medium Good (MG)	(5,6,6,7)
Good (G)	(6,7,7,8)
Very Good (VG)	(7,8,8,9)

Table II: Linguistic variables for importance weight of each criterion.

Linguistic variable	Corresponding triangular (in the shape of trapezoidal) fuzzy number
Very Low (VL)	(1,2,2,3)
Low (L)	(2,3,3,4)
Medium Low (ML)	(3,4,4,5)
Medium (M)	(4,5,5,6)
Medium High (MH)	(5,6,6,7)
High (H)	(6,7,7,8)
Very High (VH)	(7,8,8,9)

Interval type-2 fuzzy sets for the rating of alternatives and importance weight of each criterion with fuzzy numbers, can be used as Fig. 2., Fig. 3., Fig. 4. and Fig. 5. The blue lines denote upper MFs and the black lines denote lower MFs and the dashed lines denote a specific MFs in FOU. There are numerous random type-1 fuzzy TOPSIS ranking in FOU between lower and upper MFs (for example it has been shown for VP as below). We consider a distinguished distance (Alfa=0.5) from specific MFs to upper and lower MFs, because the ranking of fuzzy numbers be preserved. For example:

Very Poor (VP) ⟶ (1,2,2,3) ⟶ VP_{UPPER} ⟶ (0.5,2,2,3.5)

(1,2,2,3) ⟶ VP_{LOWER} ⟶ (1.5,2,2,2.5) ⟶ $VP_{LOWER} \leq VP_K \leq VP_{UPPER}$

K=1,…,N

3. 1. For triangular (In the shape of trapezoidal) fuzzy numbers.

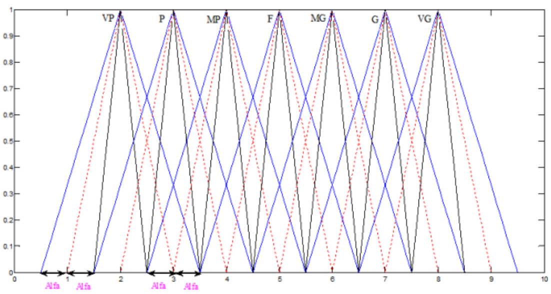

Fig. 2: Linguistic variables for the rating of alternatives.

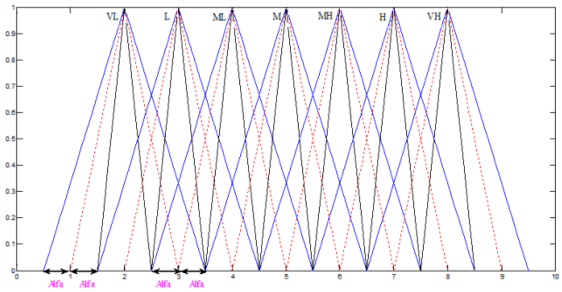

Fig. 3: Linguistic variables for the importance weight of each criterion.

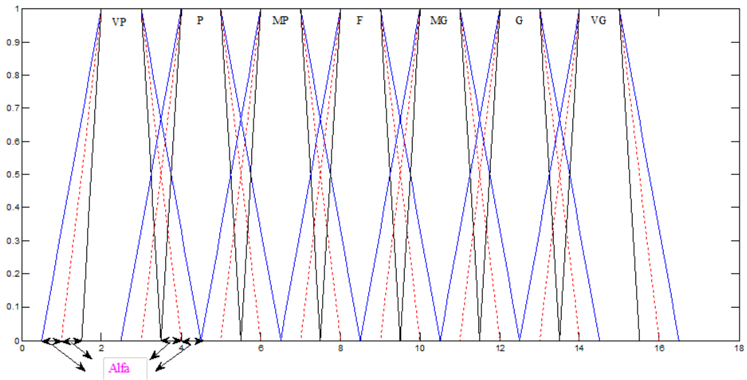

Fig. 4: Linguistic variables for the rating of alternatives.

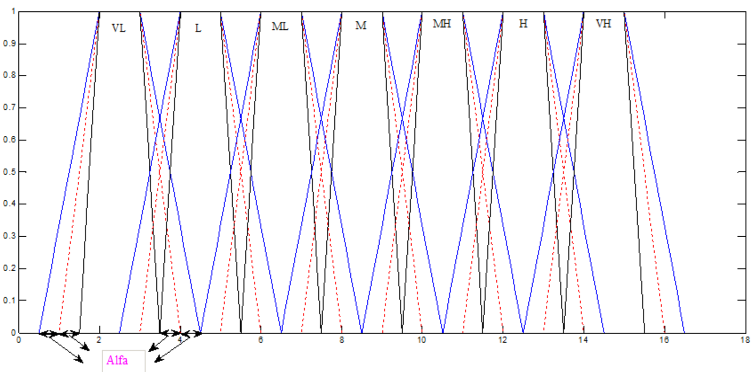

Fig. 5: Linguistic variables for the importance weight of each criterion.

In this paper the proposed method is the same as (Saeednamaghi, S.H., A. Zare, 2014) with considering of point to point interval type-2 fuzzy sets (PTP IT2 FS) for the new rating of alternatives and importance weight of each criterion with fuzzy numbers was mentioned above and solve an example [11] with this method. Finally, been compared the comparison result (Closeness Coefficient (CC)) between random type-1 fuzzy TOPSIS ranking in FOU and Point to Point interval type-2 fuzzy TOPSIS ranking (for meaning **50** times solving in the case of type-1 fuzzy), that the obtained result will be compared with type-1 fuzzy TOPSIS ranking in [11] for each definitions 3. 1. and 3. 2. in the following TABLE V and TABLE VI.

3. 2. For trapezoidal fuzzy numbers:

*Note: All things was mentioned about Alfa for triangular (In the shape of trapezoidal) fuzzy numbers can be mentioned for trapezoidal fuzzy numbers.

Journal of Applied Science and Agriculture, 9(6) May 2014, Pages: 2307-2316

Table III: Linguistic variables for the rating of alternatives.

Linguistic variable	fuzzy number
Very Poor (VP)	(1,2,3,4)
Poor (P)	(3,4,5,6)
Medium Poor (MP)	(5,6,7,8)
Fair (F)	(7,8,9,10)
Medium Good (MG)	(9,10,11,12)
Good (G)	(11,12,13,14)
Very Good (VG)	(13,14,15,16)

Table IV: Linguistic variables for importance weight of each criterion.

Linguistic variable	Corresponding trapezoidal fuzzy number
Very Low (VL)	(1,2,3,4)
Low (L)	(3,4,5,6)
Medium Low (ML)	(5,6,7,8)
Medium (M)	(7,8,9,10)
Medium High (MH)	(9,10,11,12)
High (H)	(11,12,13,14)
Very High (VH)	(13,14,15,16)

Table V: Comparison results for triangular (In the shape of trapezoidal) fuzzy numbers

Alternatives	CC random T1 fuzzy in FOU	Ranking randomT1 fuzzy in FOU	CC PTP IT2 FS	Ranking PTP IT2 FS	CC T1fuzzy mentioned in [11]	Ranking T1fuzzy mentioned in [11]
1	0.5358	3	0.5421	3	0.62	3
2	0.6376	1	0.6428	1	0.77	1
3	0.5575	2	0.5622	2	0.71	2

(CC=Closeness Coefficient)

Table VI: Comparison results for trapezoidal fuzzy numbers.

Alternatives	CC random T1 fuzzy in FOU	Ranking randomT1 fuzzy in FOU	CC PTP IT2 FS	Ranking PTP IT2 FS	CC T1fuzzy mentioned in [11]	Ranking T1fuzzy mentioned in [11]
1	0.3407	3	0.3479	3	0.62	3
2	0.5190	1	0.5249	1	0.77	1
3	0.3805	2	0.3857	2	0.71	2

(CC=Closeness Coefficient)

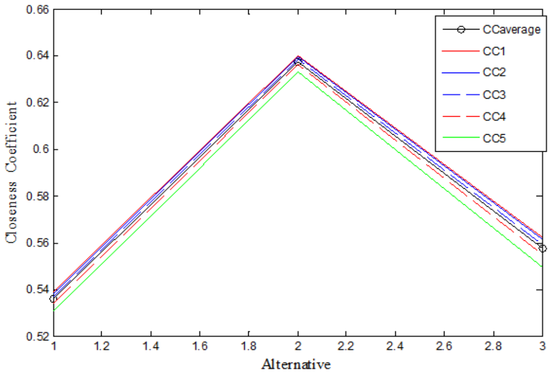

Fig. 6: CC for random T1 fuzzy in FOU for triangular (In the shape of trapezoidal) fuzzy numbers.

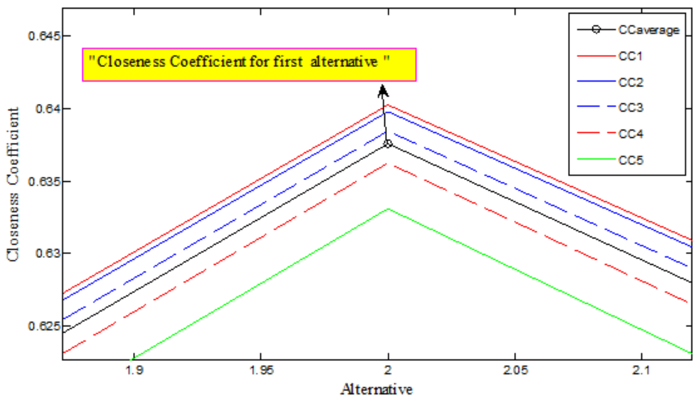

Fig. 7: CC for first alternative.

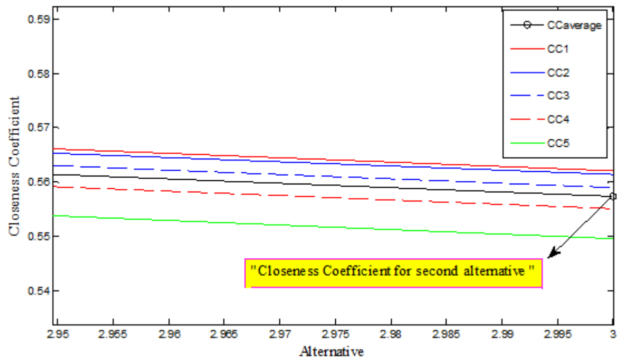

Fig. 8: CC for second alternative.

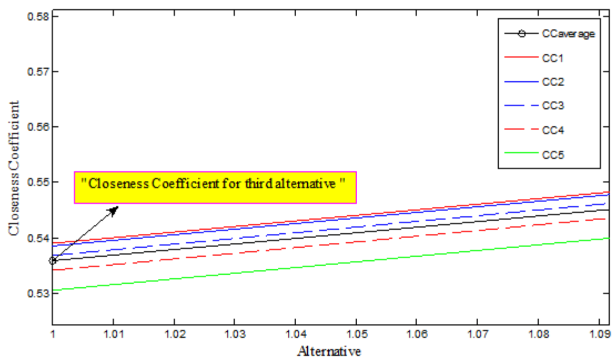

Fig. 9: CC for third alternative.

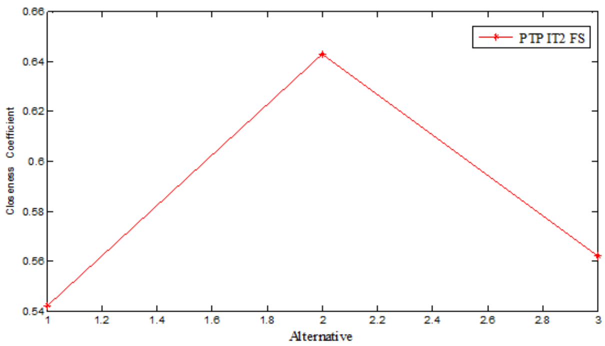

Fig. 10: CC for PTP IT2 FS for triangular (In the shape of trapezoidal) fuzzy numbers.

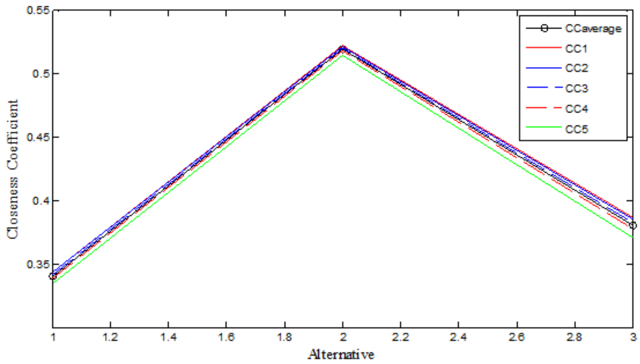

Fig. 11: CC for random T1 fuzzy in FOU for trapezoidal fuzzy numbers.

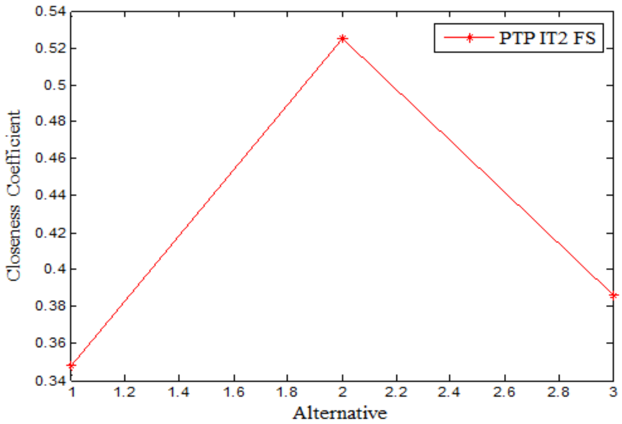

Fig. 12: CC for PTP IT2 FS for trapezoidal fuzzy numbers.

Conclusion:

Fuzzy Technique for Order Preference by Similarly to Ideal Solution (TOPSIS) is one of the most commonly used approaches in solving numerous multi criteria decision making problems. It has been widely used in ranking of multi alternatives with respect to multi criteria with the superiority of fuzzy set type-1. In this paper, we have presented a new method for ranking fuzzy multi criteria decision making based on point to point interval type-2 fuzzy sets (PTP IT2 FS). This method, which can easily be re-done for new data sets that a reader collects, should help people better understand the uncertainties associated with linguistic terms and hence how to use the uncertainties effectively in survey design and linguistic information processing. The proposed method is more flexible and more applied in the ranking of alternatives. A numerical example was illustrated to examine the applicability of the proposed approach. Comparative results were shown to illustrate the advantages of the proposed model.

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Journal of Applied Science and Agriculture, 9(6) May 2014, Pages: 2307-2316

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