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Rating and Comparing of Multiple Aspect Alternatives Based on Centroid Index

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ABSTRACT

In this study, a simple and a new centroid-indexranking method of fuzzy numbers is proposed. Our method isbased on calculating the centroid point, where the distance meansfrom minimum point to the centroid $(\bar{x}_0, \bar{y}_0)$, and the $\bar{x}_0$ and $\bar{y}_0$ indexes are the same as Wang *et al.*'s (Wang *et al.*, 2006) $\bar{x}_0$ and $\bar{y}_0$. Thus, weuse a ranking function (distance index) as the order quantities invague environment. By utilizing this index, a method is presentedfor effectively ranking various fuzzy numbers and their imagesto overcome the deficiencies of the previous techniques. Finally,several numerical examples following the procedure indicate theranking results to be valid.

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INTRODUCTION

Ranking of fuzzy numbers has been a concern in fuzzymultiple attribute decision-making because of its inception. Morethan 20 fuzzy ranking indices have been proposed since 1976.Various techniques are applied to compare the fuzzy numbers. Centroid-index methods among the existing ranking methods, have been extensively studied and applied to solve many decision makingproblems. However, there are some drawbacks in the existingcentroid ranking methods, i.e., they can't correctly rankfuzzy numbers in some situations. Hence, in this article, theauthors present a novel technique for ordering fuzzy numbers (normal/non-normal/trapezoidal/general). The proposedmethod considers the centroid points and the minimum crispvalue of fuzzy numbers to deal with fuzzy number rankingproblems. The proposed method can overcome the drawbacksof the existing centroid index ranking methods. In this articlealso there are some examples, comparing the proposed method withother ranking techniques.

The Centroid Formulae For Fuzzy Numbers:

A fuzzy number is a convex fuzzy subset of the real line $\Re$ and is completely defined by itsmembership function. If A be a fuzzy number, its membership function $f_A(x)$ can generallybe defined as (Dubois *et al.*, 1978;Saneifard *et al.*, 2010b;Saneifard, 2010b;Saneifard *et al.*, 2011b),

$$f_A(x)=\begin{cases} f_A^L(x), & when \ a\le x\le b, \\ \omega, & when \ b\le x\le c, \\ f_A^R(x), & when \ c\le x\le d, \\ 0, & otherwise. \end{cases} \tag{1}$$

Where $0\le\omega\le 1$is a constant, $f_A^L:[a,b]\to[0,\omega]$ and $f_A^R:[c,d]\to[0,\omega]$are two strictlymonotonically and continuous mappings from $\Re$ to closed interval $[0,\omega]$. If $\omega=1$, then A is anormal fuzzy number; otherwise it is said to be a non-normal fuzzy number. If the membershipfunction $f_A(x)$ is piecewise linear, then A is consider as a trapezoidal fuzzy number and isusually denoted by $A=(a,b,c,d;\omega)$. In particular, when $b=c$, the trapezoidal fuzzy numberis reduced to a triangular fuzzy number.Since $f_A^L(x)$ and $f_A^R(x)$ are both strictly monotonically and continuous functions, their

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inversefunctions exist and should also be continuous and strictly monotonically. Let $g_A^L:[0,\omega]\to[a,b]$ and $g_A^R:[0,\omega]\to[c,d]$ be the inverse functions of f_A^L and f_A^R, respectively. Then $g_A^L(y)$ and $g_A^R(y)$ should be integrable on the closed interval $[0,\omega]$. In other words, both $\int_0^{\omega} g_A^L(y)dy$ and $\int_0^{\omega} g_A^R(y)dy$ should exist. In the case of trapezoidal fuzzy number, the inverse function $g_A^L(y)$ and $g_A^R(y)$ can be analytically expressed as:

$$g_A^L(y)=a+\frac{(b-a)y}{\omega},\ 0\le y\le\omega, \tag{2}$$

$$g_A^R(y)=d-\frac{(d-c)y}{\omega},\ 0\le y\le\omega. \tag{3}$$

In order to determine the centroid point $(\bar{x}_0,\bar{y}_0)$ of a fuzzy number A, Wang et. al. (Wang *et al.*, 2006) providedthe following centroid formulae:

$$\bar{x}_0(A)=\frac{\int_a^b x\,f_A^L(x)dx+\int_b^c (x\omega)dx+\int_c^d x\,f_A^R(x)dx}{\int_a^b f_A^L(x)dx+\int_b^c(\omega)dx+\int_c^d f_A^R(x)dx}, \tag{4}$$

$$\bar{y}_0(A)=\frac{\int_0^{\omega} y(g_A^R(y)-g_A^L(y))dy}{\int_0^{\omega}(g_A^R(y)-g_A^L(y))dy}. \tag{5}$$

Consider a general trapezoidal fuzzy number $A=(a,b,c,d;\omega)$, which its membership function isdefined as

$$f_A(x)=\begin{cases}\dfrac{\omega(x-a)}{b-a}, & when\ \ a\le x\le b,\\ \omega, & when\ \ b\le x\le c,\\ \dfrac{\omega(d-x)}{d-c}, & when\ \ c\le x\le d,\\ 0, & otherwise.\end{cases} \tag{6}$$

For this trapezoidal fuzzy number, the following results have been derived from (4)and (5)

$$\bar{x}_0(A)=\frac{1}{3}[a+b+c+d-\frac{dc-ab}{(d+c)-(a+b)}], \tag{7}$$

$$\bar{y}_0(A)=\omega\frac{1}{3}[1+\frac{c-b}{(d+c)-(a+b)}]. \tag{8}$$

The ranking value $R(A)$ of the fuzzy number A is defined as follows (Cheng, 1998):

$$R(A)=\sqrt{\bar{x}_0^2(A)+\bar{y}_0^2(A)}. \tag{9}$$

The larger value of $R(A)$, the better ranking of A.In (Chu, 2002), the authors presented a centroid-index ranking method order fuzzy numbers. Thecentroid point of a fuzzy number A is $(\bar{x}_A,\bar{y}_A)$, where $\bar{x}_A$ and $\bar{y}_A$ are the same as (2)and (3) in (Chu, 2002). The ranking value $S(A)$ of the fuzzy number A is defined as follow:

$$S(A)=\bar{x}_A\times\bar{y}_A. \tag{10}$$

The larger value of $S(A)$, the better ranking of A.

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A New Ranking Method For Fuzzy Numbers:

In this section, we present a new approach for ranking fuzzy numbers based on the distancemethod. This method not only considers the centroid point of a fuzzy number, but also considersthe minimum crisp value of fuzzy numbers.

For ranking fuzzy numbers, this study firstly defines a minimum crisp value $\tau_{\min}$ to be thebenchmark and its characteristic function $\mu_{\tau_{\min}}(x)$ is as follow:

$$\mu_{\tau_{\min}}(x)=\begin{cases}1, & when\ \ x=\tau_{\min},\\ 0, & when\ \ x\neq\tau_{\min}.\end{cases} \tag{11}$$

By ranking n fuzzy numbers $A_1, A_2, \ldots, A_n$ the minimum crisp value $\tau_{\min}$ is defined as:

$$\tau_{\min}=\min\{x \mid x\in Domain(A_1, A_2, \cdots, A_n)\}. \tag{12}$$

The advantages of the definition of minimum crisp value are two-fold: first, the minimumcrisp values will be obtained by themselves, and second, it is easy to compute.

Example 1:

Three fuzzy numbers A, B and C that had been illustrated by Chen (Chen, 1985) and theirmembership functions are shown in Table (1). The inverse functions calculated by (2)and (3) are also shown in this Table. By the (2), (3) and (12), this study obtains $\tau_{\min}$ and inverse functions as follows:

$$\tau_{\min}=\min\{x \mid x\in Domain(A,B,C)\}=\min\{0.01,0.1,0.2,0.4,0.5,0.6,0.7,0.8,0.9\}=0.01,$$

and

$$g_{\min}(x)=\begin{cases}g^{L}_{\min}(x)=0.01,\\ g^{L}_{\max}(x)=0.01.\end{cases}$$

Table 1: Fuzzy numbers A, B and C.

Fuzzy number	Membership function	Inverse functions
A	$\mu_A(x)=\begin{cases}2.546x+0.256, & 0.01\le x\le 0.4,\\ 1, & 0.40\le x\le 0.7,\\ 2.6-3.3x, & 0.70\le x\le 0.8.\end{cases}$	$g_A(x)=\begin{cases}\tilde{A}_L(x)=0.39x+0.01,\\ \tilde{A}_R(x)=-0.1x+0.8.\end{cases}$
B	$\mu_B(x)=\begin{cases}3.33x-0.66, & 0.2\le x\le 0.5,\\ 1, & x=0.5,\\ 7-10x, & 0.5\le x\le 0.9.\end{cases}$	$g_B(x)=\begin{cases}L_B(x)=0.3x+0.2,\\ R_B(x)=0.9-0.4x.\end{cases}$
C	$\mu_C(x)=\begin{cases}2x-0.2, & 0.1\le x\le 0.6,\\ 1, & x=0.6,\\ 4-5x, & 0.6\le x\le 0.8.\end{cases}$	$g_C(x)=\begin{cases}L_C(x)=0.5x+0.1,\\ R_C(x)=0.8-0.2x.\end{cases}$

Assume that there are n fuzzy numbers $A_1, A_2, \ldots, A_n$. The proposed method for rankingfuzzy numbers $A_1, A_2, \ldots, A_n$ could be presented in 3 steps as follow:

Step 1. Use (4) and (5) to calculate the centroid point $(\bar{x}_{0_{A_j}}, \bar{y}_{0_{A_j}})$ of each fuzzynumbers A_j, where $1\le j\le n$.

Step 2. Calculate the minimum crisp value $\tau_{\min}$ of all fuzzy numbers A_j, where $1\le j\le n$.

Step 3. Use the point $(\bar{x}_{0_{A_j}}, \bar{y}_{0_{A_j}})$ to calculate the ranking value $Dist(A_j)$ of the fuzzy numbers A_j, where $1\le j\le n$, as follow:

$$Dist(A_j)=\sqrt{(\bar{x}_{0_{A_j}}-\tau_{\min})^2+(\bar{y}_{0_{A_j}}-0)^2}=\sqrt{(\bar{x}_{0_{A_j}}-\tau_{\min})^2+(\bar{y}_{0_{A_j}})^2}\ . \tag{13}$$

In (13), we get that $Dist(A_j)$ can be considered as the Euclidean distancebetween the points $(\bar{x}_{0_{A_j}}, \bar{y}_{0_{A_j}})$ and $(\tau_{\min}, 0)$, as shown in Figure 1.It could be obtain that the larger value of $Dist(A_j)$, the better ranking of A_j, where $1 \le j \le n$.

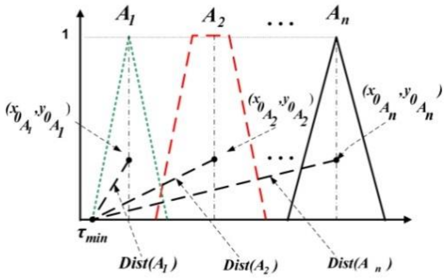

Fig. 1: The distance between $(\bar{x}_{0_{A_j}}, \bar{y}_{0_{A_j}})$ and the point $(\tau_{\min}, 0)$.

A is a fuzzy number characterized by (1) and $Dist(A)$ is the Euclidean distance betweenthe points $(\bar{x}_{0_{A_j}}, \bar{y}_{0_{A_j}})$ and $(\tau_{\min}, 0)$ of it.

In order to calculate a fuzzy number approximately by a scalar value, the researchershave to use an operator $Dist : F \to \Re$ (A space of all fuzzy numbers denoted by F) whichtransforms fuzzy numbers into a family of real line. Operator $Dist$ is a crisp approximation operator. Since above defuzzification can be used as a crisp approximation of a fuzzynumber, therefore the resultant value is used to rank the fuzzy numbers. Thus, $Dist$ is used torank fuzzy numbers. The larger $Dist$, the larger fuzzy number.

Let $A_1, A_2 \in F$ be two arbitrary fuzzy numbers. Define the ranking of A_1 and A_2 by $Dist$ on F as follow:

(1) $Dist(A_1) > Dist(A_2)$ if only if $A_1 \succ A_2$,

(2) $Dist(A_1) < Dist(A_2)$ if only if $A_1 \prec A_2$,

(3) $Dist(A_1) = Dist(A_2)$ if only if $A_1 \sim A_2$.

However, this article formulates the orders $\succeq$ and $\preceq$ as $A_1 \succeq A_2$ if and only if $A_1 \succ A_2$ or $A_1 \sim A_2$, $A_1 \preceq A_2$ if and only if $A_1 \prec A_2$ or $A_1 \sim A_2$.

Remark 1. If $\inf Supp(A) \ge 0$, then $Dist(A) \ge 0$.

Remark2. If $\inf Supp(A) \le 0$, then $Dist(A) \ge 0$.

Here are some examples to illustrate the ranking of fuzzy numbers.

Examples:

In this section, we want to compare the proposed method with (Saneifard *et al*., 2011a;Cheng, 1999;Yager *et al*., 1993;Bass *et al*., 1977;Chang, 1981;Saneifard *et al*., 2007;Saneifard, 2009;Saneifard, 2010a;Saneifard *et al*., 2010a;Saneifard *et al*., 2010b).

Example 2:

Consider the data used in (Saneifard *et al*., 2011), i.e. the three fuzzy numbers, $A = (5,6,7)$, $B = (5.9,6,7)$ and $C = (6,6,7)$, as shown in Fig. 2.According to (13), the ranking index values are obtained i.e. $Dist(A) = 1.05$, $Dist(B) = 1.34$ and $Dist(C) = 1.37$. Accordingly, the ranking order of fuzzy numbers is $A \prec B \prec C$.However, by Chu and Tsao's approach (Chu *et al*., 2002), the ranking order is $A \prec C \prec B$. Meanwhile, usingCV index proposed (Cheng, 1998), the ranking order is $C \prec B \prec A$. From Figure 2, it is easy to seethat the ranking results obtained by the existing approaches in (Chu *et al*., 2002)and (Cheng, 1998) are unreasonable and are notconsistent with human intuition. On the other hand, in (Saneifard *et al*., 2011), the ranking result is $A \prec B \prec C$,which is the same as the one obtained by the writers approach. However, their approach issimpler in the computation procedure.

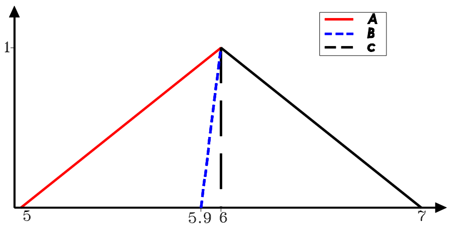

Fig. 2: Fuzzy numbers A, B, C

Example 3:

Consider the following sets: $A=(1,2,5)$, $B=(0,3,4)$ and $C=(2,2.5,3)$.By using this new approach, $Dist(A)=1.69$, $Dist(B)=1.37$ and $Dist(C)=1.53$. Hence, theranking order is $B \prec C \prec A$ too. It seems that, the result obtained by "Distance Minimization"method is unreasonable. To compare with some of the other methods as in (Chu *et al.*, 2002), the readers can referto Table 2.Furthermore, in the mentioned example, $Dist(-A)=2.34$, $Dist(-B)=3.48$ and $Dist(-C)=2.85$, consequently the ranking order of the images of the three fuzzy numbers is $-A \prec -C \prec -B$.Clearly, this proposed method has consistency in ranking fuzzy numbers and their images, whichcould not be supported by CV-index method.

Table 2: Comparative results of Example 3.

Fuzzynumber	New approach	Sign Distance With p=2	Distance Minimization	Chu-Tsao	CV Index	Magnitude method
A	1.69	3.91	2.5	0.74	0.32	2.16
B	1.37	3.91	2.5	0.74	0.36	2.83
C	1.53	3.55	2.5	0.75	0.08	2.50
Results	$B \prec C \prec A$	$C \prec A \sim B$	$C \sim A \sim B$	$A \sim B \prec C$	$B \prec A \prec C$	$A \prec C \prec B$

Table 3: The comparison with different ranking approaches.

Proposed method	Yager	Kerre	Chang	Bass and Kwakernaak
$\tilde{A}_1 \prec \tilde{A}_2$	$\tilde{A}_1 \prec \tilde{A}_2$	$\tilde{A}_1 \prec \tilde{A}_2$	$\tilde{A}_1 \prec \tilde{A}_2$	$\tilde{A}_1 \prec \tilde{A}_2$
$\tilde{B}_1 \prec \tilde{B}_2$	$\tilde{B}_1 \sim \tilde{B}_2$	$\tilde{B}_1 \prec \tilde{B}_2$	$\tilde{B}_1 \succ \tilde{B}_2$	$\tilde{B}_1 \sim \tilde{B}_2$
$\tilde{C}_1 \prec \tilde{C}_2 \prec \tilde{C}_3$	$\tilde{C}_1 \prec \tilde{C}_2 \prec \tilde{C}_3$	$\tilde{C}_1 \sim \tilde{C}_2 \prec \tilde{C}_3$	$\tilde{C}_1 \prec \tilde{C}_2 \prec \tilde{C}_3$	$\tilde{C}_1 \sim \tilde{C}_2 \prec \tilde{C}_3$
$\tilde{D}_1 \prec \tilde{D}_2 \prec \tilde{D}_3$	$\tilde{D}_1 \prec \tilde{D}_2 \prec \tilde{D}_3$	$\tilde{D}_1 \prec \tilde{D}_2 \prec \tilde{D}_3$	$\tilde{D}_1 \prec \tilde{D}_2 \prec \tilde{D}_3$	$\tilde{D}_1 \sim \tilde{D}_2 \prec \tilde{D}_3$
$\ddot{E}_1 \succ \tilde{E}_2$	$\ddot{E}_1 \succ \tilde{E}_2$	$\ddot{E}_1 \succ \tilde{E}_2$	$\ddot{E}_1 \succ \tilde{E}_2$	$\ddot{E}_1 \prec \tilde{E}_2$
$\tilde{F}_1 \succ \tilde{F}_2$	$\tilde{F}_1 \prec \tilde{F}_2$	$\tilde{F}_1 \prec \tilde{F}_2$	$\tilde{F}_1 \succ \tilde{F}_2$	$\tilde{F}_1 \prec \tilde{F}_2$
$\tilde{G}_1 \prec \tilde{G}_2 \prec \tilde{G}_3$	$\tilde{G}_1 \prec \tilde{G}_2 \prec \tilde{G}_3$	$\tilde{G}_1 \prec \tilde{G}_2 \prec \tilde{G}_3$	$\tilde{G}_1 \prec \tilde{G}_2 \prec \tilde{G}_3$	$\tilde{G}_1 \prec \tilde{G}_2 \prec \tilde{G}_3$
$\tilde{H}_1 \prec \tilde{H}_2 \prec \tilde{H}_3$	$\tilde{H}_1 \prec \tilde{H}_2 \prec \tilde{H}_3$	$\tilde{H}_1 \prec \tilde{H}_2 \prec \tilde{H}_3$	$\tilde{H}_1 \prec \tilde{H}_2 \prec \tilde{H}_3$	$\tilde{H}_1 \prec \tilde{H}_2 \prec \tilde{H}_3$
$\tilde{I}_1 \succ \tilde{I}_2$	$\tilde{I}_1 \sim \tilde{I}_2$	$\tilde{I}_1 \sim \tilde{I}_2$	$\tilde{I}_1 \succ \tilde{I}_2$	$\tilde{I}_1 \sim \tilde{I}_2$
$\tilde{j}_1 \prec \tilde{j}_2 \prec \tilde{j}_3$	$\tilde{j}_1 \prec \tilde{j}_2 \prec \tilde{j}_3$	$\tilde{j}_1 \prec \tilde{j}_2 \prec \tilde{j}_3$	$\tilde{j}_1 \prec \tilde{j}_2 \prec \tilde{j}_3$	$\tilde{j}_1 \prec \tilde{j}_2 \prec \tilde{j}_3$
$\tilde{K}_1 \prec \tilde{K}_2$	$\tilde{K}_1 \prec \tilde{K}_2$	$\tilde{K}_1 \prec \tilde{K}_2$	$\tilde{K}_1 \prec \tilde{K}_2$	$\tilde{K}_1 \prec \tilde{K}_2$
$\tilde{L}_1 \prec \tilde{L}_2$	$\tilde{L}_1 \prec \tilde{L}_2$	$\tilde{L}_1 \prec \tilde{L}_2$	$\tilde{L}_1 \succ \tilde{L}_2$	$\tilde{L}_1 \prec \tilde{L}_2$

Example 4:

The $Dist$ values of 12 examples are shown asfollows. Also the ranking results are shown in Table 3. In this Table, the main findings and *Dist* with some advantages are as follow:

1. In example L and K, some methods use complicated and normalized process to rank andthey can't obtain consistent results. However, the proposed method is more suitable for rankingany kind of fuzzy number without normalization process.

2.About fuzzy numbers with the same mean (Examples B, I), Yager(Yager *et al.*, 1993), Kerre(Wang *et al.*, 2001), Bass (Bass *et al.*, 1977) have not been able to obtain their orderings. Chang's method (Chang, 1981) has been ableto rank fuzzy number orderings, but Chang's results to eclipsedthe smaller spread, the higher ranking order.In Example B and I, shows that the proposed method can rank instantlyand their results comply with intuition of human being.

3. In Examples C, D and L, we can see that the methods of Kerre(Wang *et al*., 2001), Bass (Bass *et al*., 1977) have many limitations in triangle, trapezoid, non-normalized fuzzy numbers and so on.

4. The proposed method can be used for ranking fuzzy numbers and crisp values,whileYager hasnot been able to handle the crisp value problem (Yager *et al*., 1993).

5.Kerre's method would support a fuzzy number with smaller area measurement, regardless ofits relative location on the X -axis (Wang *et al*., 2001). The results are against their intuition in examples C and D. According to Table 3, one can get that the proposed method is able to solve the problem.

All above examples show that this method is more consistent with institution than theprevious ranking methods.

Conclusion:

In this article, authors presented a new method for ranking fuzzy numbers. The proposedmethod considers the centroid points and maximum crisp value of fuzzy numbers to rankingfuzzy numbers. Centroid method can overcome the drawback previous centroid methods.

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