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An Application of Fuzzy Risk Analysis In Fuzzy Logic Controller

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ABSTRACT

Fuzzy systems have gained more attention from researchers and practitioners of various fields. In such systems, the output represented by a fuzzy set sometimes needs to be transformed into a scalar value, and this task is known as the defuzzification process. Several analytic methods have been proposed for this problem, but in this paper, we suggest a new approach to the problem of defuzzification using the weighted metric between fuzzy numbers. In this study some preliminary results on properties of such defuzzification will be reported.

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INTRODUCTION

As most modeling and control applications require crisp outputs, when applying fuzzy inference systems, the fuzzy system output $A(y)$ usually has to be converted into a crisp output y^* . This operation is called defuzzification. It is essentially a process guided by the output fuzzy subset of the model. One selects a single crisp value as the system output. In fuzzy logic controller applications the typical methods used for this process are the center of area (COA) method, and the mean of maximal (MOM) method (Allahviranloo *et al*, 2012). Both of these methods can be seen to be based on a weighted type aggregation which can be seen as a blending or mixing of different solutions. In (Filev *et al*, 1991; Yager *et al*, 1993), the researchers introduced a general approach to defuzzification based upon the BADD transformator. The output for the fuzzy controller F is a fuzzy subset of the real line; for simplicity the researchers shall assume the support set Y is finite, $Y = \{y_1, y_2, \dots, y_n\}$. For $y_i \in Y$, $F(y_i) = w_i$ indicates the degree to which each y_i is suggested as a good output value by the rule base under the current input. Two commonly used methods for defuzzification are the center of area (COA) method and the mean of maximum (MOM) method (Dubois *et al*, 1987; Diamond *et al*, 1990). In the (COA) method one calculates the output of the defuzzifier, y_{COA} , as: $y_{COA} = \frac{\sum_i y_i w_i}{\sum w_i}$. In the MOM method one calculates the output of the controller $y_{MOM} = \frac{1}{m} \sum_{y_i \in A} y_i$, where A is the set of elements in Y which provide the maximum value of $F(y)$ and m is the cardinality of A . Based upon these observations one can view the defuzzification process under the (COA) and MOM methods as first converting the fuzzy subset F of Y into a probability distribution on Y , in the spirit described in the earlier section and then taking the expected value as our output. Keeping with this probabilistic interpretation the researchers shall in the following use P_i instead of q_i to denote the transformation values. Moreover, in (Saneifard, 2009; Ming *et al*, 2000), authors used the concept of the symmetric triangular fuzzy number, and introduced an approach to defuzzify a fuzzy number based L_2 -distance. In this paper, the researchers suggest a new approach to the problem of defuzzification using the weighted metric between two fuzzy numbers. In this study some preliminary results on properties of such defuzzification are to be reported. Therefore, by the means of this defuzzification, this article aims to use the concept of symmetric triangular fuzzy number, and introduces a new approach to defuzzify a fuzzy quantity. The basic idea of the new method is to obtain the nearest symmetric triangular fuzzy number which a fuzzy quantity is related to. Unlike other methods, this study defuzzifies the fuzzy number, and at the same time obtains the fuzziness of the original quantity. The paper is organized as follows: In Section 2, this article recalls some fundamental results on fuzzy numbers. In Section 3, the new defuzzification method is proposed. In this Section some theorems and remarks are proposed and illustrated. Examples and applications of this study are carried out in Section 4.

Preliminary Notes:

The basic definitions of a fuzzy number are given in (Heilpern, 1992; Kauffman *et al*, 1991) as follows:

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Definition 2.1:

A fuzzy number $\tilde{A}$ is a mapping $A(x) : \mathfrak{R} \rightarrow [0, 1]$ with the following properties:

1. A is an upper semi-continuous function on $\mathfrak{R}$,
2. $A(x) = 0$ outside of some interval $[a_1, b_2] \subset \mathfrak{R}$,
3. There are real numbers a_2, b_1 such as $a_1 \leq a_2 \leq b_1 \leq b_2$ and
 - $A(x)$ is a monotonic increasing function on $[a_1, b_2]$,
 - $A(x)$ is a monotonic decreasing function on $[b_1, b_2]$,
 - $A(x) = 1$ for all x in $[a_2, b_1]$.

The trapezoidal fuzzy number $\tilde{A} = (x_0, y_0, \sigma, \beta)$, with two defuzzifier x_0, y_0 and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$ is a fuzzy set where the membership function is as

$$A(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma) & x_0 - \sigma \leq x \leq x_0, \\ 1 & x_0 \leq x \leq y_0, \\ \frac{1}{\beta}(y_0 - x + \beta) & y_0 \leq x \leq y_0 + \beta, \\ 0 & \text{otherwise.} \end{cases}$$

If $x_0 = y_0$ and $\sigma = \beta$, a popular fuzzy number is obtain. It is the symmetric triangular fuzzy number $S[x_0, \sigma]$ centered at x_0 with basis 2σ by following form

$$A(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma) & x_0 - \sigma \leq x \leq x_0, \\ 1 & x = x_0, \\ \frac{1}{\sigma}(x_0 - x + \sigma) & x_0 \leq x \leq x_0 + \sigma, \\ 0 & \text{otherwise.} \end{cases}$$

The parametric form of symmetric triangular fuzzy number is

$$\underline{A}(r) = x_0 - \sigma(1 - r), \quad (1)$$

$$\overline{A}(r) = x_0 + \sigma(1 - r) \quad (2)$$

Definition 2.2:

Let $\tilde{A}$ be a fuzzy number and $(\underline{A}(r), \overline{A}(r))$ be its parametric form. The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number $\tilde{A}$:

$$I(\tilde{A}) = \frac{1}{2} \int_0^1 (\underline{A}(r) + \overline{A}(r)) dr,$$

$$D(\tilde{A}) = \int_0^1 (\overline{A}(r) - \underline{A}(r)) f(r) dr.$$

The function $f(r)$ is also called weighting function.

Definition 2.3:

For arbitrary fuzzy numbers $\tilde{A}$ and $\tilde{B}$ the quantity

$$d_p = \sqrt{[I(\tilde{A}) - I(\tilde{B})]^2 + [D(\tilde{A}) - D(\tilde{B})]^2}, \quad (3)$$

is called the parametric distance between the fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

Nearest Parametric Symmetric Triangular Defuzzification:

In this Section, we will propose the nearest weighted symmetric triangular defuzzification approach associated with the weighted metric d_p in F . Let $\tilde{A}$ be a general fuzzy number and $(\underline{A}(r), \overline{A}(r))$ be its parametric form (Saneifard et al, 2011a; Saneifard et al, 2011b; Saneifard et al, 2011). To obtain a parametric symmetric

triangular fuzzy number $S[x_{0p}, \sigma_p]$, which is the nearest to $\tilde{A}$, the researchers use the weighted distance (3) and minimize

$$d_p(\tilde{A}, S[x_{0p}, \sigma_p]) = ([I(\tilde{A}) - I(S[x_{0p}, \sigma_p])]^2 + [D(\tilde{A}) - D(S[x_{0p}, \sigma_p])]^2)^{\frac{1}{2}}, \quad (4)$$

with respect to x_{0p} and σ_p , where

$$I(S[x_{0p}, \sigma_p]) = \frac{1}{2} \int_0^1 [x_{0p} - \sigma_p(1-r) + x_{0p} + \sigma_p(1-r)] dr = x_{0p},$$

$$D(S[x_{0p}, \sigma_p]) = \int_0^1 [x_{0p} + \sigma_p(1-r) - x_{0p} + \sigma_p(1-r)] f(r) dr = \int_0^1 2\sigma_p(1-r)f(r) dr.$$

In order to minimize it suffices to minimize

$$\bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p]) = d_p^2(\tilde{A}, S[x_{0p}, \sigma_p]) = \left[\frac{1}{2} \int_0^1 (\bar{A}(r) + \underline{A}(r) - 2x_{0p}) dr \right]^2 + \left[\int_0^1 (\bar{A}(r) - \underline{A}(r) - 2\sigma_p(1-r)) f(r) dr \right]^2.$$

If $S[x_{0p}, \sigma_p]$ minimizes $\bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p])$, then $S[x_{0p}, \sigma_p]$ provides a defuzzification of $\tilde{A}$ with a defuzzifier x_{0p} and fuzziness σ_p . So that to minimize $\bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p])$, this article has,

$$\frac{\partial \bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p])}{\partial \sigma_p} = -4 \int_0^1 (\bar{A}(r) - \underline{A}(r)) f(r)(1-r) dr + 8\sigma_p \int_0^1 f(r)(1-r)^2 dr,$$

and

$$\frac{\partial \bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p])}{\partial x_{0p}} = -2 \int_0^1 (\bar{A}(r) + \underline{A}(r) - 2x_{0p}) dr.$$

By solve system of equation as follows,

$$\frac{\partial \bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p])}{\partial \sigma_p} = 0, \quad \frac{\partial \bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p])}{\partial x_{0p}} = 0.$$

The solution is

$$\sigma_p = \frac{\int_0^1 (\bar{A}(r) - \underline{A}(r)) f(r)(1-r) dr}{2 \int_0^1 f(r)(1-r)^2 dr}, \quad x_{0p} = \frac{1}{2} \int_0^1 (\bar{A}(r) + \underline{A}(r)) dr.$$

Remark 3.1:

If we consider $f(r) = r$, the nearest parametric symmetric triangular defuzzification of $\tilde{A}$ is given by the defuzzifier $x_{0p} = \frac{1}{2} \int_0^1 (\bar{A}(r) + \underline{A}(r)) dr$, and fuzziness $\sigma_p = 6 \int_0^1 [\bar{A}(r) - \underline{A}(r)] r(1-r) dr$.

The above defuzzification approach can be applied to two fuzzy numbers whenever a single fuzzy quantity is desirable. Let $\tilde{A}$ and $\tilde{B}$ be a fuzzy numbers with parametric forms $\tilde{A} = (\underline{A}(r), \bar{A}(r))$ and $\tilde{B} = (\underline{B}(r), \bar{B}(r))$. To find a parametric symmetric triangular fuzzy number $S[x_{0p}, \sigma_p]$ near $\tilde{A}$ both and $\tilde{B}$, this article minimizes

$$\begin{aligned} \bar{D}(x_{0p}, \sigma_p) &= \bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p]) + \bar{D}_p(\tilde{B}, S[x_{0p}, \sigma_p]) \\ &= \left(\frac{1}{2} \int_0^1 [(\underline{A}(r) - \underline{S}[x_{0p}, \sigma_p](r)) + (\bar{A}(r) - \bar{S}[x_{0p}, \sigma_p](r))] dr \right)^2 \\ &= \left(\int_0^1 f(r) [(\bar{A}(r) - \bar{S}[x_{0p}, \sigma_p](r)) - (\underline{A}(r) - \underline{S}[x_{0p}, \sigma_p](r))] dr \right)^2 \\ &+ \left(\frac{1}{2} \int_0^1 [(\underline{B}(r) - \underline{S}[x_{0p}, \sigma_p](r)) + (\bar{B}(r) - \bar{S}[x_{0p}, \sigma_p](r))] dr \right)^2 \\ &+ \left(\int_0^1 f(r) [(\bar{B}(r) - \bar{S}[x_{0p}, \sigma_p](r)) - (\underline{B}(r) - \underline{S}[x_{0p}, \sigma_p](r))] dr \right)^2. \end{aligned}$$

Thus, this study must find a lodger point, (x_{0p}, σ_p) for which $\frac{\partial \bar{D}(x_{0p}, \sigma_p)}{\partial x_{0p}} = 0$, $\frac{\partial \bar{D}(x_{0p}, \sigma_p)}{\partial \sigma_p} = 0$. (*)

$$\begin{aligned} \text{Then, } \frac{\partial \bar{D}(x_{0p}, \sigma_p)}{\partial \sigma_p} &= \frac{\partial \bar{D}_p(\tilde{A}, S[x_{0p}, \sigma_p])}{\partial \sigma_p} + \frac{\partial \bar{D}_p(\tilde{B}, S[x_{0p}, \sigma_p])}{\partial \sigma_p} \\ &= -4 \int_0^1 [\bar{A}(r) - \underline{A}(r)] (1-r) f(r) dr + 8\sigma_p \int_0^1 f(r)(1-r)^2 dr \\ &- 4 \int_0^1 [\bar{B}(r) - \underline{B}(r)] (1-r) f(r) dr + 8\sigma_p \int_0^1 f(r)(1-r)^2 dr = 0, \end{aligned}$$

and

$$\frac{\partial \bar{D}(x_{0p}, \sigma_p)}{\partial x_{0p}} = \frac{\partial \bar{D}_p(\bar{A}, S[x_{0p}, \sigma_p])}{\partial x_{0p}} + \frac{\partial \bar{D}_p(\bar{B}, S[x_{0p}, \sigma_p])}{\partial x_{0p}}$$

$$= -2 \int_0^1 [\underline{A}(r) + \bar{A}(r) - 2x_{0p}] dr - 2 \int_0^1 [\underline{B}(r) + \bar{B}(r) - 2x_{0p}] dr = 0$$

Hence, by the solve system of equation(*), there is

$$\sigma_p = \frac{\int_0^1 (\bar{A}(r) + \bar{B}(r) - \underline{A}(r) - \underline{B}(r)) f(r)(1-r) dr}{4 \int_0^1 f(r)(1-r)^2 dr},$$

$$x_{0p} = \frac{1}{4} \int_0^1 (\bar{A}(r) + \underline{A}(r) + \bar{B}(r) + \underline{B}(r)) dr.$$

If this article assumesthat $f(r) = r$, then

$$\sigma_p = 3 \int_0^1 (\bar{A}(r) + \bar{B}(r) - \underline{A}(r) - \underline{B}(r)) r(1-r) dr, \quad x_{0p} = \frac{1}{4} \int_0^1 (\bar{A}(r) + \underline{A}(r) + \bar{B}(r) + \underline{B}(r)) dr.$$

Numerical Examples and Applications:

In this section the researchers present numerical examples to illustrate the differencebetween the introduced method in this paper and the given method in (Ming *et al*, 2000; Larkin, 1985). Throughoutthis section the researchers assumed that $f(r) = r$.

Example 4.1:

Consider a plateau $\underline{A}(r) = a + (b-a)r$, $\bar{A}(r) = d - (d-c)r$, where $a \leq b \leq c \leq d$. The nearest parametric triangular defuzzification procedure yields

$$x_{0p} = \frac{1}{2} \int_0^1 (\bar{A}(r) + \underline{A}(r)) dr = \frac{1}{2} \int_0^1 [(a+d) + (b-a)r - (d-c)r] dr = \frac{a+b+c+d}{4},$$

and

$$\sigma_p = \frac{\int_0^1 (\bar{A}(r) - \underline{A}(r)) f(r)(1-r) dr}{2 \int_0^1 f(r)(1-r)^2 dr} = \frac{1}{24} \int_0^1 [d - (d-c)r - a - (b-a)r] r(1-r) dr$$

$$= \frac{c+d-a-b}{2}.$$

In specific case $b = c$ there is $x_{0p} = \frac{1}{4}(a+2b+d)$, $\sigma_p = \frac{1}{2}(d-a)$.

Example 4.2:

Consider the Gaussian membership function $\tilde{A}(x) = e^{\frac{-(x-\mu_0)^2}{\sigma_0^2}}$ which its parametric form is $\underline{A}(r) = \mu_0 + \sigma_0 \sqrt{-\ln r}$, $\bar{A}(r) = \mu_0 - \sigma_0 \sqrt{-\ln r}$, then

$$x_{0p} = \frac{1}{2} \int_0^1 (\bar{A}(r) + \underline{A}(r)) dr = \mu_0 \text{ and } \sigma_p = \frac{\int_0^1 (\bar{A}(r) - \underline{A}(r)) f(r)(1-r) dr}{2 \int_0^1 f(r)(1-r)^2 dr} = \frac{\sigma_0 \sqrt{\pi}}{6} (9\sqrt{2} - 4\sqrt{3}).$$

Furthermore, $\sigma = \frac{3}{8} \sigma_0 \sqrt{\pi} (4 - \sqrt{2})$ (Ming *et al*, 2000; Larkin, 1985), it is clear that $\sigma_p \leq \sigma$.

Example 4.3:

Let $\tilde{A}$ be a plateau and $\tilde{B}$ a triangular fuzzy number given by

$$\underline{A}(r) = r, \quad \bar{A}(r) = 3 - r; \quad \underline{B}(r) = 2 + r, \quad \bar{B}(r) = 4 - r.$$

The defuzzification procedure yields

$$x_{0p} = \frac{1}{4} \int_0^1 (\bar{A}(r) + \underline{A}(r) + \bar{B}(r) + \underline{B}(r)) dr = \frac{1}{4} \int_0^1 9r dr = \frac{9}{4},$$

and

$$\sigma_p = \frac{\int_0^1 (\bar{A}(r) + \bar{B}(r) - \underline{A}(r) - \underline{B}(r)) f(r)(1-r) dr}{4 \int_0^1 f(r)(1-r)^2 dr} = 3 \int_0^1 (5 - 4r)(1-r) r dr = \frac{3}{2}.$$

Example 4.3:

In other segment, this article applies the parametric symmetric triangular defuzzification procedure to obtain a fuzzy partition from two extreme values. So, given the extreme values 0 and 1, we define the fuzzy number

"medium" $\tilde{A}^{(1)}$ as $\underline{A}^{(1)}(r) = \frac{r}{2}$, $\overline{A}^{(1)}(r) = 1 - \frac{r}{2}$. Defuzzify 0 and $\tilde{A}^{(1)}$, to obtain "lower medium" $\tilde{A}^{(2,1)}$ for which $x_{0p} = \frac{1}{4} \int_0^1 (\overline{A}^{(1)}(r) + 0 + \underline{A}^{(1)}(r) + 0) dr = \frac{1}{4}$ and $\sigma_p = 3 \int_0^1 (\overline{A}^{(1)}(r) + 0 - \underline{A}^{(1)}(r) - 0) r(1-r) dr = \frac{1}{4}$. Thus $\underline{A}^{(2,1)}(r) = 0 + \frac{1}{4}r$ and $\overline{A}^{(2,1)}(r) = \frac{1}{2} - \frac{1}{4}r$. This study now defuzzifies $\tilde{A}^{(1)}$ and 1, to get the "upper medium" $\tilde{A}^{(2,3)}$ with $x_{0p} = \frac{3}{4}$ and $\sigma_p = \frac{1}{4}$, i.e. $\underline{A}^{(2,3)}(r) = \frac{1}{2} + \frac{1}{4}r$, $\overline{A}^{(2,3)}(r) = 1 - \frac{1}{4}r$. Update the "medium" by defuzzifying the "lower medium" $\tilde{A}^{(2,1)}(r)$ and "upper medium" $\tilde{A}^{(2,3)}(r)$. The result is the "medium" $\tilde{A}^{(2,2)}(r)$ centered at

$$x_{0p} = \frac{1}{4} \int_0^1 \left(\frac{r}{4} + \frac{1}{2} - \frac{r}{4} + \frac{1}{2} + \frac{r}{4} + 1 - \frac{r}{4} \right) dr = \frac{1}{2} \quad \text{with} \quad \sigma_p = 3 \int_0^1 r(1-r)^2 dr = \frac{1}{4},$$

i.e. $\underline{A}^{(2,2)}(r) = 0.25 + \frac{1}{4}r$, and $\overline{A}^{(2,2)}(r) = \frac{3}{4} - \frac{1}{4}r$. Thus, we obtain a fuzzy partition with five elements $P = \{0, \tilde{A}^{(2,1)}, \tilde{A}^{(2,2)}, \tilde{A}^{(2,3)}, 1\}$.

As the fuzzy partition becomes finer, the fuzziness of its elements obviously decreases.

Example 4.4:

Consider the Gaussian membership function in Example (4.2) with $\mu_0 = 1$ and $\sigma_0 = \frac{1}{2}$ (Liu, 2001). We apply the parametric symmetric triangular defuzzification procedure to obtain a fuzzy partition from two extreme values 0 and 2. Then $\underline{A}^{(1)}(r) = 1 + \frac{\sqrt{-\ln r}}{2}$, $\overline{A}^{(1)}(r) = 1 - \frac{\sqrt{-\ln r}}{2}$. Defuzzify 0 and $\tilde{A}^{(1)}$, to obtain $\tilde{A}^{(2,1)}$ for which $x_{0p} = \frac{1}{4} \int_0^1 (\overline{A}^{(1)}(r) + 0 + \underline{A}^{(1)}(r) + 0) dr = \frac{1}{2}$ and $\sigma_p = 3 \int_0^1 (\overline{A}^{(1)}(r) + 0 - \underline{A}^{(1)}(r) - 0) r(1-r) dr = \frac{1}{2}$. Thus $\underline{A}^{(2,1)}(r) = 0 + \frac{1}{2}r$ and $\overline{A}^{(2,1)}(r) = 1 - \frac{1}{2}r$. We now defuzzifies $\tilde{A}^{(1)}$ and 2, to get the $\tilde{A}^{(2,3)}$ with $x_{0p} = \frac{3}{2}$ and $\sigma_p = \frac{1}{2}$, i.e. $\underline{A}^{(2,3)}(r) = 1 + \frac{1}{2}r$, $\overline{A}^{(2,3)}(r) = 2 - \frac{1}{2}r$. Update the "medium" by defuzzifying the "lower medium" $\tilde{A}^{(2,1)}$ and "upper medium" $\tilde{A}^{(2,3)}$. The result is the "medium" $\tilde{A}^{(2,2)}$ centered at $x_{0p} = 1$ with $\sigma_p = \frac{1}{2}$, i.e. $\underline{A}^{(2,2)}(r) = \frac{1}{2} + \frac{1}{2}r$ and $\overline{A}^{(2,2)}(r) = \frac{3}{2} - \frac{1}{2}r$. Thus, this article obtains a fuzzy partition with five elements $P = \{0, \tilde{A}^{(2,1)}, \tilde{A}^{(2,2)}, \tilde{A}^{(2,3)}, 1\}$.

As the fuzzy partition becomes finer, the fuzziness of its elements obviously decreases.

Conclusion:

In this study, we suggest a new approach to the problem of defuzzification using the parametric metric between two fuzzy numbers. In this paper some preliminary results on properties of such defuzzification are to be reported. The basic idea of the new method is to obtain the nearest parametric symmetric triangular fuzzy number which a fuzzy quantity is related to.

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