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### An Analysis of the Dynamic Stability of the Turning Process against Self-Excited Vibrations and Determining the Stability Range

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#### ABSTRACT

Chatter is a self-excited vibration which depends on several parameters such as the dynamic characteristics of the machine tool structure, the material of the workpiece, the material removal rate, and the geometry of tools. Chatter has an undesirable effect on dimensional accuracy, smoothness of the workpiece surface, and the lifetime of tools and the machine tool. Thus, it is useful to understand this phenomenon in order to improve the economic aspect of machining. In the present article, first the theoretical study and mathematical modeling of chatter in the cutting process were carried out, and then by performing modal testing on a milling machine and drawing chatter stability diagrams, we determined the stability regions of the machine tool operation.

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#### INTRODUCTION

Chatter is a self-excited vibration which occurs if the chip width is too large with respect to the dynamic stiffness of the system. Variable forces generated during the machining operation lead to vibrational excitation of the tools and workpiece and as a result, the surface of the workpiece becomes wavy. In the next machining stage, if the generated wavy surface is not in phase with the wavy surface of the previous stage, the thickness of the chip becomes variable and it will lead to a change in the shear force. The change in the shear force will change the energy entering the system – including tools and workpiece and if the system does not have the capacity for the extra energy input, it will start vibrating which is referred to as chatter. Chatter vibration has an exponential growth in amplitude over time, and this continues until the tool is separated from the workpiece or that it breaks. This phenomenon can be distinguished by its sound or its marks on the workpiece surface.

Machining with chatter is in most cases an unwanted phenomenon, since it has an adverse effect on the surface of the workpiece and besides, it may break the tools or other parts of the machine tool. Therefore, the cutting width and material removal rate must be below the limit at which chatter occurs. From this perspective, chatter is a factor decreasing material removal rate before it is limited by the power and moment of the machine tool.

The simplest theory for calculating the cutting force which has been used in studying the stability of machining processes was founded by Tlustý (Tlustý and Poláček 1963). (Rao and Shin 1999) presented a cutting force model of oblique turning process and used it for drawing chatter stability diagrams. Baker and Rouch (Baker and Rouch 2002) used a finite element model for analyzing the stability of the turning process. In references [4-7] the effect of tool wear parameter on chatter has been examined. Targ (Tang, Kao and Lee 2000) controlled chatter in the turning process by placing piezoelectric actuators on the tool. Liao (Chen 1972) controlled chatter using a system for automatic spindle speed regulation. Pan *et al.* (Galewski and Kalinski, 2009) developed an intelligent system for controlling chatter in the turning process using neural networks.

The width of the chip is the most important factor which plays a crucial role in generating chatter vibration. If chip width is small enough, machining will be stable. With the increase of material removal width, chatter starts at a limit and will be very severe for chip widths larger than this. The critical chip width depends on some factors such as the dynamic characteristics of the machine tool structure, the material of the workpiece, material removal rate, and the geometry of tools (Schmitz 2004). A negative cutting angle will increase the possibility of chatter, since the direction of cutting force with the negative cutting angle of tools will be mostly along the direction of vibration. The effect of the relief angle of the tool on the occurrence of chatter is displayed

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in the figure. Since the tool moves on a wavy surface, the relief angle (the angle in the above figure) is always changing and in location B where the relief angle is negative, much energy is lost from the system due to the contact between the tool and the workpiece. This phenomenon has been discussed as process damping and is more noticeable at lower cutting speeds, since at higher cutting speeds the wavelength of oscillations increases and assuming the constancy of the vibration amplitude, the slope of the oscillations decreases and thus there will be less change in the relief angle (Schmitz 2004).

*Modeling a Fixed-Hinged Beam with Cutting Tool in the Middle of the Beam:*

The workpiece studied in this article is a fixed-hinged beam; thus considering the new conditions, a support is obtained for the beam using the following equations:

$$\begin{aligned}
 \frac{\partial u_2}{\partial x} &= \frac{T}{GJ} U_3 + \theta_3 \\
 \frac{\partial u_3}{\partial x} &= \frac{-T}{GJ} U_2 - \theta_2 \\
 \frac{\partial \theta_2}{\partial x} &= \frac{M_2}{EI_{22}} + \frac{T}{GJ} \theta_3 \\
 \frac{\partial \theta_3}{\partial x} &= \frac{M_3}{EI_{33}} - \frac{T}{GJ} \theta_2 \\
 \frac{\partial M_2}{\partial X} &= J_{22} \frac{\partial^2 \theta_2}{\partial t^2} - \left[ \frac{T}{EI_{33}} - \frac{T}{GJ} \right] M_3 + V_3 \\
 \frac{\partial M_3}{\partial X} &= J_{33} \frac{\partial^2 \theta_3}{\partial t^2} - \left[ \frac{T}{EI_{22}} - \frac{T}{GJ} \right] M_2 - V_2 \\
 \frac{\partial V_2}{\partial X} &= n_o \frac{\partial^2 U_2}{\partial t^2} - \frac{PM_3}{EI_{33}} + \frac{T}{GJ} V_3 - F_1 \\
 \frac{\partial V_3}{\partial X} &= n_o \frac{\partial^2 U_3}{\partial t^2} + \frac{PM_2}{EI_{22}} - \frac{T}{GJ} V_2 - F_2
 \end{aligned} \tag{1}$$

where  $\theta_2$  and  $\theta_3$  are the rotations of the beam in the 2 and 3 directions,  $U_3$  and  $U_2$  are displacements of the beam in the 2 and 3 directions,  $M_2$  and  $M_3$  are the bending moments in these two directions, and  $V_2$  and  $V_3$  are the shear forces in these two directions.

*Solving the Equations Governing the Beam:*

The studied beam is a steel beam with 1m length and 1cm×2cm rectangular cross-section. We take the vector of unknowns as  $[U_2 \ U_3 \ \theta_2 \ \theta_3 \ M_2 \ M_3 \ V_2 \ V_3]$ , and use the method of separation of variables in order to solve the equations. That is,

$$Z_{(x,t)} = Z_{(x)} e^{i\omega\tau} \tag{2}$$

The dynamic equations of the system consisting of 8 equations varying with time and space are solved using the method of separation of variables with respect to time and space. The dynamic effect of the temporal term appears in these equations in the form of a frequency term. By eliminating the temporal dependence from dynamic equations governing the behavior of the system, we arrive at 8 spatial equations for positions along the beam which can be solved by applying boundary constraints (conditions) with respect to space which are given in figure 4.2a.

The solution method involves dividing the length of the beam into equal parts. For any  $i$ -th point along the beam a corresponding vector is defined as follows:

$$Z = [U_2 \ U_3 \ \theta_2 \ \theta_3 \ M_2 \ M_3 \ V_2 \ V_3]^T \tag{3}$$

The spatial equations governing the behavior of the system are rewritten as a function of X in the form of a matrix. In the numerical solution, the Runge Kutta method is responsible for predicting the vector of unknowns and the Adams-Moulton method is responsible for correcting these predictions and of course solving the vector of the unknowns of all the points on the beam. The matrix form of the 8 dynamic equations is as follows (9):

$$\frac{dZ}{dX} = \begin{bmatrix} 0 & \frac{T}{GJ} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-T}{GJ} & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{T}{GJ} & \frac{1}{EI_{22}} & 0 & 0 & 0 \\ 0 & 0 & \frac{-T}{GJ} & 0 & 0 & \frac{1}{EI_{33}} & 0 & 0 \\ 0 & 0 & -J_{22}\omega^2 & 0 & 0 & (\frac{T}{EI_{33}} - \frac{T}{GJ}) & 0 & 1 \\ 0 & 0 & 0 & -J_{33}\omega^2(\frac{T}{EI_{33}} - \frac{T}{GJ}) & 0 & -1 & 0 & 0 \\ -n_o\omega^2 & 0 & 0 & 0 & 0 & \frac{-P}{EI_{33}} & 0 & \frac{T}{GJ} \\ 0 & -n_o\omega^2 & 0 & 0 & \frac{P}{EI_{22}} & 0 & \frac{-T}{GJ} & 0 \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \\ \theta_2 \\ \theta_3 \\ M_2 \\ M_3 \\ V_2 \\ V_3 \end{bmatrix} \tag{4}$$

For any point, the vector  $Z_i = c_1 z_i^{(1)} + c_2 z_i^{(2)} + \dots + c_8 z_i^{(8)}$  corresponds to the *i*-th solution of the governing differential equation. In other words, is of the form  $Z=[K]C$ . At the tip of the beam, considering the physical constraints governing it, Z is as follows:

$$Z_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ M_2 \\ M_3 \\ V_2 \\ V_3 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \dots + C_8 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

The first four coefficients of the solution model are equal to zero:

$$C_1 = C_2 = C_3 = C_4 = 0 \tag{6}$$

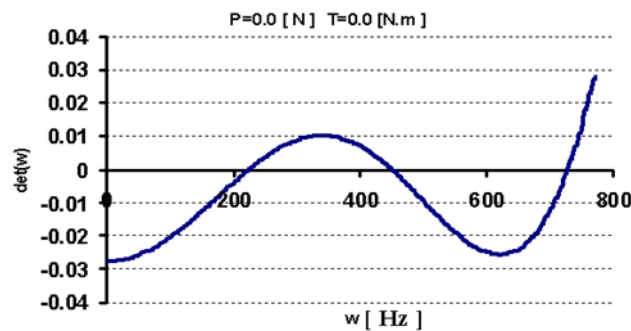
Using the superposition method and by applying eight boundary conditions at the tip of the beam and integrating for each of these conditions along the beam using a predictor- corrector algorithm, the K matrix is obtained at the end of the beam. By applying the boundary conditions at the end of the beam as given below,

$$Z = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{18} \\ k_{21} & k_{22} & \dots & k_{28} \\ \vdots & & & \\ k_{81} & k_{82} & \dots & k_{88} \end{bmatrix}_{100} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \theta_2 \\ \theta_3 \\ 0 \\ 0 \\ V_2 \\ V_3 \end{bmatrix}$$

we arrive at the following equations:

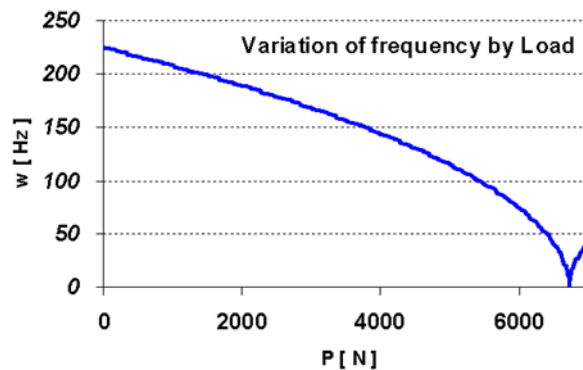
$$\begin{aligned}
 k_{15} C_5 + k_{16} C_6 + k_{17} C_7 + k_{18} C_8 &= 0 \\
 k_{25} C_5 + k_{26} C_6 + k_{27} C_7 + k_{28} C_8 &= 0 \\
 k_{35} C_5 + k_{36} C_6 + k_{37} C_7 + k_{38} C_8 &= 0 \\
 k_{65} C_5 + k_{66} C_6 + k_{67} C_7 + k_{68} C_8 &= 0
 \end{aligned} \tag{8}$$

The condition for the above homogeneous system of equations (of the form  $[K]C=0$ ) to have a solution is that the determinant of matrix  $[K]$  equals zero. The result of this condition implies that  $\det([K])=0$  is a function of  $w$ . The natural frequencies of the model are obtained by numerically solving these equations. The figure below displays the diagram of  $\det([K])(w)$  changes that has been estimated at the points where this function meets the horizontal axis, and of course these frequencies are the natural frequencies of the system.



**Fig. 1:** Det (w)vs.w.

The following diagram shows the variation of the first frequency of the beam with the load applied at the end of the beam. According to the figure, the point where frequency reaches zero corresponds to the critical force for the buckling phenomenon. In the proposed model, the load applied at the end of the beam is the same as the load generated by the mandrel.



**Fig. 2:** The natural frequency of the system in p (power)for finding $P_{cr}$ .

After numerical solution of the system, four natural frequencies are obtained and ultimately the mode shapes related to each of these frequencies are drawn which correspond to beam displacements  $U_2$  and  $U_3$ , beam rotations  $\theta_2$  and  $\theta_3$ , bending moments  $M_2$  and  $M_3$ , and shear forces  $V_2$  and  $V_3$ .

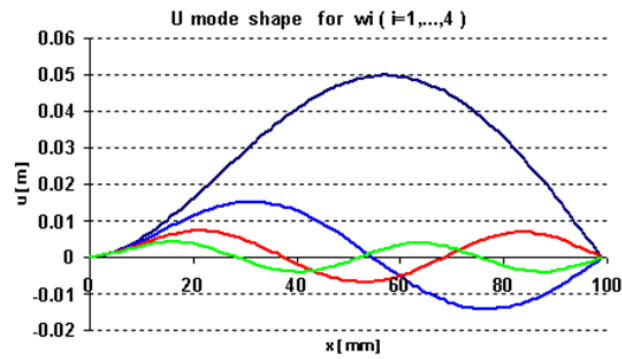


Fig. 3: Mode shapes  $U_2$  and  $U_3$  (displacement in the 2 and 3 directions).

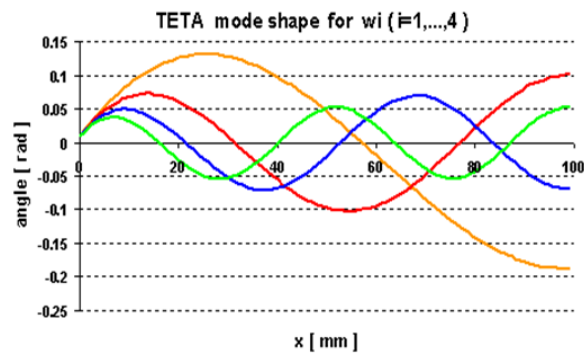


Fig. 4: Mode shapes related to rotations  $\theta_2$  and  $\theta_3$ .

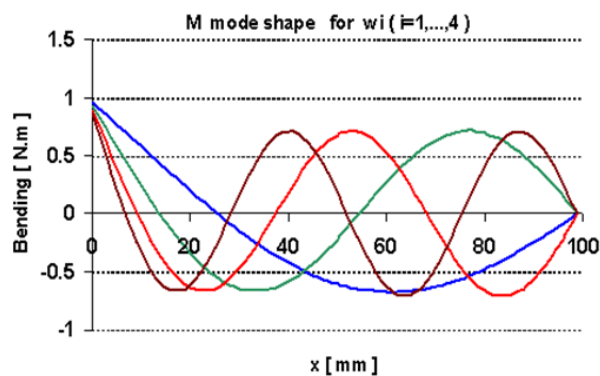


Fig. 5: Mode shapes related to bending moments  $M_2$  and  $M_3$ .

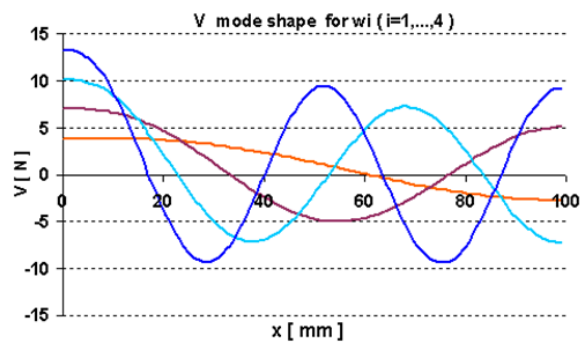


Fig. 6: Mode shapes related to shear forces  $V_2$  and  $V_3$ .

*Conclusion:*

In obtaining the natural frequencies of a system, if the load  $P$  (the force exerted on the cylindrical workpiece due to closing the mandrel) is exerted on the workpiece, the natural frequency of the system ( $\omega$ ) will decrease due to the reduction of beam stiffness.

If we draw the diagram of  $P$  (mandrel force) in  $\omega$  (natural frequency), the critical point for the force  $P$  will be:  $P_{cr}=6730$  N.

**REFERENCES**

- Baker, J.R., K.E. Rouch, 2002. Use of Finite Element Structural Models in Analyzing Machine Tool Chatter. *Finite Elements in Analysis and Design*, 38(11): 1029-1046.
- Chen, M., 1972. Self-induced chatter vibration of lathe tools.
- Chiou, Y.S., S.Y. Liang, 1997. Chatter Stability of a Slender Cutting Tool in Turning with Tool Wear Effect. *International Journal of Machine Tools and Manufacture*, 38(4): 315-327.
- Galewski, M., K. Kalinski, 2009. Vibration surveillance during high speed milling with variable spindle speed (in Polish). The Publication of Gdansk University of Technology, Gdansk.
- Rao, B., Y.C. Shin, 1999. A Comprehensive Dynamic Cutting Force Model for Chatter Prediction in Turning. *International Journal of Machine Tools & Manufacture*, 39: 1631-1654.
- Schmitz, T.L., T.J. Burns, J.C. Ziegerta, B. Duttererc, W.R. Winfough, 2004. Tool Length- Dependent Stability Surfaces. *Machining Science and Technology*, 8(3): 377-397.
- Tang, Y.S., J.Y. Kao, E.C. Lee 2000. Chatter suppression in turning operations with a tuned vibration absorber. *Journal of Materials Processing Technology*, 105.
- Thusty, G., M. Polacek, 1963. The stability of machine tools against self-excited vibrations in machining. In: *Proceedings of the ASME International Research in Production Engineering*, Pittsburgh, USA, pp: 465-474.
- Yosuke, M., M. Takashi, U. Eiji, 2008. Simulation Analysis of Self-Excited Chatter Vibration with Taking the Non-Linearity of Machine-Tool Structure into Account. 3rd Report. *Journal of the Japan Society for Precision Engineering*.