Direct Numerical Simulation of Turbulent Heat Transfer in Pipe Flows with various Prandtl Numbers

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ABSTRACT

The direct numerical simulation (DNS) of heat transfer in a fully developed turbulent pipe flow with for various Prandtl numbers are performed to obtain statistical quantities such as turbulent heat flux, and temperature variance. Main emphasis is placed on Prandtl number effects on turbulent heat transfer in pipe flow. Probability density functions and joint probability density functions of velocity and temperature fluctuations are used to describe the characteristics of the turbulent flow and heat transfer. The scaling temperature and velocity profiles are investigated in order to derive correct logarithmic law for various Pr. Evidence shows that temperature fluctuations and turbulent heat fluxes are increasing when increasing Prandtl number increasing. With decreasing Pr, the conductive sublayer at both walls spreads from the walls to the core region, while the root mean square of temperature fluctuations and the turbulent heat fluxes are reduced near both walls.

Key words: Direct numerical simulation; Prandtl number effect; turbulent heat transfer

Introduction

The problem of heat transfer in turbulent pipe flow is of importance in mechanical and engineering fields and is encountered in a variety of engineering applications such as flow in turbo machines, heat exchange, combustion chambers, nuclear reactors, etc (Redjem-Saad et al., 2007; Ould-Rouiss, 2010a). Direct numerical simulations provide means for obtaining detailed information about turbulent structures and heat transfer (Ould-Rouiss et al., 2010a). It has become a powerful supplement to experimental investigation of turbulence flow and moderate Reynolds number turbulent flows that could be performed with a sufficiently high accuracy between the numerical and analytical differentiation (Kawamura, 1995; Yamamoto et al., 2008). Turbulent heat transfer characterized not only by the Reynolds number (Re) but also the Prandtl number (Pr) of the fluids (Kawamura et al., 1998, 1999; Redjem Saad, 2007). The experimental evidence that \( \nu_f \) values very near the wall which have substantial influence on heat transfer rates at a high Prandtl numbers or at low thermal diffusivity (Thakre and Joshi, 2000; Nakaharaj et al., 2007). In addition, near-wall flow structures are especially important, because the thermal boundary layer is much thinner than the momentum boundary layer. It is often assumed to be constant irrespectively of the wall-normal distance and molecular Prandtl number at least for Pr \( \geq 1 \).

Many numerical works in literature have been devoted to explore the effect of Prandtl number on the thermal statistics in plane channel flows, with the use of DNS (Kawamura et al., 1998, 99; Redjem Saad et al., 2007; Ould-Ruiss et al., 2010). Yakhot et al. (1987) they showed that the proposed relation between turbulent viscosity and turbulent diffusivity gives accurate predictions for Nusselt number and temperature distributions in wide ranges of Prandl and Reynolds numbers (1 < Pr < 106 and 2.5 \times 104 < Re < 2 \times 105). Kim and Moin (1989) made simulations for Prandtl numbers with Pr.0.1, 0.71 and 2 with Re of 180 and assumed a constant volumetric heating with a uniform-wall temperature. Profiles of the mean temperature, temperature variance and turbulent heat flux were obtained.
Recently, Redjem-Saad et al. (2007) compared their DNS results for a pipe flow at Re = 5600 to those of Kawamura et al. (1999) and found small differences even though discrepancies are increasing at low Prandtl number (Pr = 0.026). This is because the increase of the Prandtl number requires a larger mesh number.

There are two approaches such as the eddy diffusivity model and the surface renewal model which is widely used to model heat transfer at wall boundary (Tricoli, 1999). Tricoli, (1999) pointed out that there have no well-based relationships with the correlated turbulent fluctuations in these models which are the fundamental quantities underlying turbulent transport. The most frequently adopted models in the turbulent flows are the k-ε models (Mony et al., 1989; Takagi and Hirai, 1998, Thakre and Joshi, 2000). Thakre and Joshi (2000) analyzed two types models for heat transfer such as twelve versions of low Reynolds k-ε model and two low Reynolds stress turbulence models. They showed that showed that the k–ε models performed relatively better than the Reynolds stress models for predicting the mean axial temperature and the Nusselt number in pipe flows, for different Prandtl numbers. In addition, they concluded that the predictive ability of the k-ε models is expected to improve when turbulent Prandtl number variations near the wall are included. On the other hand, both the k-ε and the RSM models for heat transfer can be attributed to the incorrect near-wall modeling of the dissipation term (Redjem-Saad et al., 2007). They pointed out also the lack of detailed near-wall temperature and scalar flux measurements at higher Prandtl numbers. Bricteux, L., et al. (2011), applied computational fluid dynamics (CFD) codes related to low Prandtl number flows in industrial application. They suggested that the turbulent Prandtl number concept should be used with care and that even recent proposed correlations may not be sufficient.

Despite many investigations, the existing computational and experimental results for heat transfer in pipe flow are quite incomplete. However, a comprehensive literature reveals that investigations on the effect of Prandtl number on turbulent heat transfer flow calculations are few, and when available, they are incomplete and uncertain. In this paper, we present a critical review on turbulent heat transfer flow and their statistics (mean profiles, root mean squares, Reynolds shear stresses, turbulent heat fluxes) at different Prandtl numbers. In the paper, we first discuss about the first DNS which investigates the effects of different Prandtl numbers at different Reynolds number. The present paper is organized as: governing equations and numerical procedure are described in section two. The effects of Prandtl number are Reynolds number on the thermal statistics is investigated and discussed in section three and finally conclusion in section four.

### Governing equations and numerical procedure:

#### Governing equations:

The turbulent flow configuration investigated is a forced, fully developed and the Newtonian fluid is incompressible with a uniform heat flux q_w imposed at the wall (Fig. 1). The fluid properties are assumed constant and the viscous and the wall units viscous term is neglected. The temperature is considered as a passive scalar. The dimensionless temperature \( \Theta \) is defined as:

\[
\Theta = \frac{\left( T_r - T \right)}{T_r}
\]

Where $T_r = \frac{q_w}{\rho C_p U_b}$ is the reference temperature and $\langle T_w \rangle$ denotes the wall temperature averaged in time and circumferential direction.

Fig. 1: Schematic of the computational domain

Using the dimensionless variables $q_r = r \cdot v_r$, $q_\theta = r \cdot v_\theta$, $q_z = v_z$, the energy equation writes as

$$\frac{\partial \theta}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r q_r \theta \right) + \frac{1}{r} \frac{\partial}{\partial \theta} (q_\theta \theta) - \frac{q_z}{r} \frac{\partial \langle T_w \rangle}{\partial x} = \frac{1}{Re Pr} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \theta^2} + \frac{\partial^2 \theta}{\partial x^2} \right]$$  \hspace{1cm} (2)

where $r$ and the velocity components are scaled by the pipe radius $R$ and the centerline streamwise velocity of the laminar Poiseuille profile $U_p$ respectively. The Reynolds number $Re$ is defined as $Re = U_p R / \nu$.

The heating condition imposed on the wall implies a linear increase of the bulk temperature in the streamwise direction. For full developed flows, the following equalities are satisfied

$$\frac{\partial \langle T_b \rangle}{\partial x} = \frac{\partial \langle T_w \rangle}{\partial x} = \frac{2 q_w}{\rho C_p U_b} = 2 T_r$$ \hspace{1cm} (3)

The wall temperature fluctuations are assumed to be zero and the dimensionless temperature boundary condition is $\Theta = 0$. This condition corresponds to the mixed type boundary condition described by Piller (2005). In this case, the time-averaged wall heat flux is uniform in space, and the wall temperature is not time-dependent and varies linearly along the streamwise direction.

**Numerical procedures:**

The governing equations were discretized on a staggered mesh in cylindrical coordinates with different computational length in the axial direction (Redjem-Saad et al., 2007; Ould-Rouiss, 2010b; Kawamura et al., 1998). The numerical integration was performed by a finite difference scheme, second-order accurate in space and in time (Redjem-Saad et al., 2007). The time advancement employed a fractional step method. A third order Runge–Kutta explicit scheme and a Crank–Nicolson implicit scheme were used to evaluate the convective and diffusive terms, respectively (Redjem-Saad et al., 2007; Yamamoto et al., 2008). Uniform computational grid and periodic boundary conditions were applied to the circumferential and axial directions. The influence of different grids on the accuracy of the solution was investigated. Redjem-Saad et al., (2007) carried out a study that considered $129 \times 95 \times 129$ grid for $Pr=0.026$ and $129 \times 129 \times 257$ grid for higher Prandtl numbers. For the high Prandtl numbers, the grid resolution which captures the viscous sublayer can be inadequate and insufficient for the thermal conduction region. For small Prandtl numbers, the opposite behaviour is observed (Montreuil, 2000). Owing to the grid requirement, the DNS of turbulent heat transfer becomes thus much difficult to handle for $Pr \geq 1$. DNS of turbulent heat transfer are conducted for $Pr \leq 1$, less grid resolution is required.

**Results and discussion:**

**Mean velocity profile and root mean square:**

The streamwise mean velocity profile is plotted in Fig. 2 as a function of the distance from the wall $y^+$ along the DNS data by Eggels et al. (1994) obtained in a pipe flow. The comparison between the two DNS profiles exhibits a good agreement between them. It is well known that the viscous sublayer ($0 \leq y^+ \ll 5$) and
the buffer region ($5 \ll y^+ \ll 30$) are well predicted, while in the logarithmic region, the velocity profile slightly deviates from the log law, due to the small value of the Reynolds number.

![Fig. 2: Mean velocity profile at (Re =5500) (Redjem-Saad et al., 2007)](image1)

![Fig. 3: RMS of velocity fluctuations (Re= 5500) (Redjem-Saad et al., 2007)](image2)

Indeed, the log-laws are not observed in pipe flow for $Re \ll 9600$ (Feiz et al., 2005; Redjem Saad et al., 2007). On the other hand, the velocity profile obtained at $Pr=1.0$, Bricteux et al. (2011) found very close to that of the DNS of Kawamura et al. (1999), which validates the numerical approach.

**Mean temperature characteristics:**

Kawamura et al., (1998) show that the mean temperature distributions normalized by the friction temperature ($T_f = q_w / \rho C_p u_*)$ in Fig. 4 for various Prandtl numbers. The profile obtained at $Pr=1.0$, Bricteux et al. (2011) found very close to that of the DNS of Kawamura et al. (1999), which validates the numerical approach. The temperature fluctuations curve is reported in Fig. 4; it is also very close to the DNS result of Kawamura et al. (1999) for $Pr = 1.0$. With increasing Pr, the thermal resistance is mainly concentrated in the conductive sublayer which is immersed in the various sublayer. It indicates a rapid transport of heat in the pipe. Indeed, they showed that the logarithmic region of the temperature can be better distinguished from the wake region with increases in the Reynolds and Prandtl numbers.
It is well known that the near-wall temperature variation can be expanded in terms of $y^+$ as

$$\Theta^+ = Pr y^+$$

Fig. 4: Mean temperature profile with an emphasis on the logarithmic region (Kawamura et al., 1998)

With decreasing the Pr, the conductive sublayer spreads from the wall to the core region. The temperature profile for the smallest Prandtl number (Pr = 0.025) indicates that the molecular heat transfer dominates. For small Prandtl numbers, the logarithmic part of the temperature profile appears only at high Reynolds numbers (Re $>$ 10^5). However, Fig. 5 displays clearly such an asymptotic behaviour, within the near wall region, and confirms that the conductive sublayer is deeply immersed in the viscous sublayer for small Prandtl numbers.

**Turbulent heat fluxes:**

Fig. 6 depicts the distributions of the streamwise turbulent heat flux for different Prandtl numbers. In the case of higher Pr number, the effect of molecular heat conductivity is relatively small, and the heat flux by turbulent transport becomes increasingly important in the near wall region. Like the rms of temperature fluctuations, the same behaviour is detected near the two walls: the maximum of the streamwise turbulent heat flux increases with increasing Pr. The position of this maximum appears to be nearly insensitive to the value of Prandtl number for Pr $\leq$ 1: the peak in the streamwise turbulent heat flux is located at $y^+$ $\approx$ 15, near the inner and outer cylinders, which is close to the peak of the position of the peak of axial velocity fluctuations. Indeed, Ould-Rouiss et al. (2010a) showed that the radius has no effect on the peak in the streamwise velocity fluctuations at both walls. However, when the Prandtl number is larger than 1, the peak shifts towards the wall with increasing Pr, because the similarity of the velocity and thermal fields is lost. When the Prandtl number increases, the conductive sublayer becomes thinner and the peak value in the streamwise turbulent heat flux increases and shifts towards the wall.
The wall-normal turbulent heat flux, Fig. 7, behaves like the Reynolds shear stress: its radial distribution is asymmetric. With increasing Pr, the width of the conductive sublayer and the molecular heat flux diminish while the turbulent wall-normal heat flux increases close to the inner and outer surfaces. With increasing Re, the streamwise turbulent heat flux for Pr = 0.71 is amplified, while the wall-normal heat flux is slightly augmented (not shown here). The effect of Re on the wall-normal heat flux is thus negligible, in comparison to its effect on the streamwise turbulent heat flux. A key finding from this figure is that the Reynolds and Prandtl numbers seem to have practically no effect on the location of zero total heat flux \( q_{\text{total}} = \frac{1}{Pr} \frac{d\delta^+}{dy^+} < v'_{\text{r}} \Theta^+ \). This location is always at \( y/\delta = 0.46 \), except for Pr = 0.026 for which \( y/\delta = 0.37 \).

**Fig. 6:** Streamwise turbulent heat flux (Kawamura et al., 1998)

**Fig. 7:** Wall-normal turbulent heat flux (Kawamura et al., 1998)

**Turbulent Prandtl number:**

The turbulent Prandtl number \( (Pr_t) \) is an important and useful quantity for practiced heat transfer calculation (Srinivasan and Papavassiliou, 2011; Yakhot et al., 1987). The turbulent Prandtl number concept is a structural coupling of velocity and temperature fields; as a result the eddy-diffusivity concept is applied to heat turbulent transport in a similar way as the eddy-viscosity concept is applied to the momentum transport (Briteux et al., 2011). The turbulent Prandtl number, which is defined in equation in Eq. (4) as a function of \( y^+ \)

\[
Pr_t = \frac{\nu}{\alpha_t} = \frac{\bar{u}' \bar{\theta}'}{\bar{\nu} ' (d\bar{\theta}'/dy)}
\]  

Fig. 8 shows the turbulent Pr number \( (Pr_t) \) profiles, where the \( Pr_t \) was defined by turbulent viscosity \( (\nu_t) \) and turbulent diffusivity \( (\alpha_t) \). The knowledge of \( Pr_t \) is used to predict heat transfer from known velocity field, and particularly in the near-wall region. There is an influence of the wall distance on the value of the turbulent Prandtl number which tends to increase Pr, close to the wall (Weigand et al., 1997). This increase in Pr near the wall is especially important for high Prandtl number fluids because of the very thin thermal boundary layer.
Redjem-Saad et al., (2007) conducted a study and show that Pr = 0.71 coincide well with the results of Piller, although there is a slight discrepancy far away from the wall. The best agreement is obtained near the wall. In the near-wall region, $Pr_t$ is consistent with the well known limiting behavior for Pr >> 0.2. For Pr < 0.2, the turbulent Prandtl number increases ($Pr_t \approx 1.4$ for Pr = 0.1, $Pr_t \approx 2.7$ for Pr = 0.026). Far from the wall $y^+ > 50$, $Pr_t$ decreases. These trends indicate that near the wall, the momentum diffusion is much larger than heat diffusion.

Conclusions:

In this paper, DNS of turbulent heat transfer in pipe flow under isoflux conditions have been performed at various Prandtl numbers. Different statistical turbulence quantities including the mean and fluctuating temperatures, the heat transfer coefficients and the turbulent heat fluxes are obtained and analyzed. When Pr increases, the temperature profile reveals that the conductive sublayer is gradually reduced, inducing a rapid transport of heat in the annular space. The rms of temperature fluctuations and the turbulent heat fluxes increase near the inner and outer walls with increasing Pr. The Prandtl number seems to have practically no effect on the position of zero total heat flux. A detailed study of the asymptotic behaviours of the turbulent variables in the close vicinity of the walls shows that the rms of the temperature fluctuations is constant in the near-wall region, while the turbulent streamwise and wall-normal heat fluxes behave as $y^+$ and $y^+2$ respectively. With increasing Re, the convective effects induce an extended logarithmic region, and more intensified temperature fluctuations and turbulent heat fluxes. The present review would be useful of prediction accuracy for turbulent heat transfer in a pipe with different Prandtl numbers. A rational approach to transport in turbulent flow must be based on the primary ingredients of turbulent transport, i.e., the turbulence fluctuations and the molecular momentum, heat or mass diffusivity. The conclusion gives a theoretical basis on the widely employed practice to use a constant Pr in the heat transfer calculation of the fluids with the normal to high molecular Prandtl number.

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Reference