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ORIGINAL ARTICLE

Adomian Decomposition Method to Study Mass Transfer from a Horizontal Flat Plate Subject to Laminar Fluid Flow

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ABSTRACT

Adomian Decomposition Method (ADM) was utilized to tackle a famous and important problem encountered in chemical engineering, viz, mass transfer during the contact of a solid slab with an overhead flowing fluid. After a brief presentation of the governing mathematical model, ADM was exploited to investigate the solution. Following ADM procedure, a recurrence relation was found to establish the Adomian series form of the solution. Once the final obtained series solution was compared with the Taylor series expansion to an analytical (exact) solution of the problem (obtained via the Combination of Variables method ahead), it was interesting to prove that ADM had led to the identical exact solution.

Key words: Adomian Decomposition Method, Laminar Mass Transfer, Concentration Boundary Layer, Combination of Variables Method

Introduction

Adomian Decomposition Method (ADM) was developed and initially introduced by the acknowledged mathematician George Adomian in mid-80s (Adomian, 1984) and basics of it has been fully covered (Adomian, 1994, 1998; Wazwaz, 2002). Being powerful, effective, and able to handle an extensive class of linear, nonlinear differential/integral equations with less mathematical complexity, ADM has gained intense popularity among numerous researchers since then. In this regard and just for the sake of exemplification, we make reference to a limited number of extensions/modifications made to ADM. Wazwaz and El-Sayed performed modifications to improve the original ADM technique (Wazwaz, 1999; Wazwaz and El-Sayed, 2001). The convergence (and its order) of ADM was explored (Babolian and Biazar, 2002). Ibijola *et al*. and Casasús and Al-Hayani provided insights in application of ADM to solution of ODEs (Ibijola *et al*., 2008; Casasús and Al-Hayani, 2001). Pamuk successfully extended ADM to linear and nonlinear heat equations (Pamuk, 2005). Wazwaz and Gorguis achieved analytical solution of Fisher's equation via ADM (Wazwaz and Gorguis, 2004). Bougoffa and Bougoffa proposed ADM to solve coupled linear and nonlinear ODEs of first and second order (Bougoffa and Bougoffa, 2006). Adjedj exploited ADM to model HIV immune response (Adjedj, 1999). Layeni and Akinola managed to use ADM in solving the mathematical model (a PDE) pertaining to a water filtration process (Layeni and Akinola, 2008).

In this paper, we present a simplified model for mass transfer phenomenon from a horizontal flat plate fixed along a laminar fluid flow and then extend ADM to solution of the model. Subsequently, it is showed that ADM leads to an analytical solution identical to the one resulted from Combination of Variables Method which is an exact method.

Problem Description:

A schematic illustration of the problem is depicted in Figure 1.

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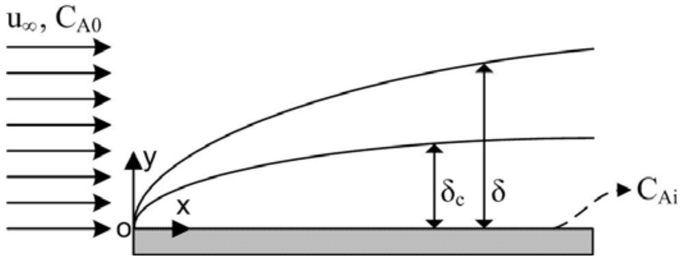

Fig. 1: A schematic image of the problem. Molecules of species A from the solid slab diffuses along the y-axis to be drifted away by the fluid stream flow of the hydrodynamic boundary layer.

Mass transfer under the mentioned settings/conditions is often encountered in the realm of chemical engineering (Mori *et al*., 1991; Nassif *et al*., 1995; Sharma and Rahman, 2002). The following realistic assumptions were made to set a mathematical model for the phenomenon.

Model Assumptions:

1- The fluid flow is laminar and steady state situation is reached.
2- The solid surface is so smooth that no ripples or surface waves exist.
3- The fluid is Newtonian and, as a result, has velocity distribution.
4- Mass transfer rates are so small that no sharp density gradients occur along the x-axis, causing the natural convection be negligible.
5- The Flow is one-dimensional, so there is no mass transfer due to bulk convection along the y and z axes.
6- Diffusional mass transfer in all directions is negligible except for that of in y-axis direction.
All over the surface has a constant interface concentration of species *A*, namely, C_A.
Physical properties such as mass diffusivity are uniformly constant.
No reactions take place between the surface and fluid.
The end effects are neglected.
Schmidt number is assumed to be larger than 1 (Sc>1).

Formulation of Model:

Mass balance for species *A* over an infinitesimal element based on the Cartesian coordinates while applying the mentioned assumptions leads to:

$$D\frac{\partial^2 C}{\partial y^2} = u\frac{\partial C}{\partial x} \tag{1}$$

Assuming a third degree polynomial form for velocity distribution inside velocity boundary layer yields (Holman, 2002):

$$\frac{u}{u_\infty} = \frac{3}{2}(\frac{y}{\delta}) - \frac{1}{2}(\frac{y}{\delta})^3 \tag{2}$$

For any point inside concentration boundary layer thickness with any arbitrary height of y, $y<\delta$ making y/δ less than unity. Accordingly, the cubic term of Eq. (2) can be neglected with respect to the linear term for the sake of simplification, approximating that:

$$\frac{u}{u_\infty} = \frac{3}{2}(\frac{y}{\delta}) \tag{3}$$

It is worthwhile to mention that such a simplification becomes very accurate for the cases with the fluid having Schmidt numbers larger than unity (i.e. when hydrodynamic boundary layer thickness δ is larger than concentration boundary layer thickness δ_c, which is the domain for y, at any given x. Such a deduction is inferred from the analogy between heat and mass transfer, knowing that Prandtl number is heat transfer analog of Schmidt number (Holman, 2002).

From Fluid Mechanics it can be proved that velocity boundary layer thickness for laminar flow over flat plates is governed by (Bird *et al*., 2001):

$$\delta = \frac{4.64x}{\sqrt{\text{Re}_x}} = 4.64\frac{\upsilon^{0.5}x^{0.5}}{u_\infty^{0.5}} \tag{4}$$

Combining Eq. (4), (3) and (1) yields:

$$D\frac{\partial^2 C}{\partial y^2} = \frac{3}{9.28}\frac{u_\infty^{1.5}}{\upsilon^{0.5}}x^{-0.5}y\frac{\partial C}{\partial x} \tag{5}$$

By taking $\alpha = \frac{3u_\infty^{1.5}}{9.28D\nu^{0.5}} = cte$, we have:

$$C_{yy} = \alpha x^{-0.5} y C_x \tag{6}$$

with below boundary conditions

BC1: $C(x,\delta) = C_{A0}$

BC2: $C(x,0) = C_{Ai}$

BC3: $C(x,\delta) = C_{A0}$ (7)

Analysis by ADM:

The eq. (6) can be written in an operator form by

$$L_y C = \alpha x^{-0.5} y L_x C \tag{8}$$

where the differential operators L_x and L_y are given by

$$L_y = \frac{\partial^2}{\partial y^2}, L_y = \frac{\partial^2}{\partial y^2} \tag{9}$$

and therefore the inverse operators L_x^{-1} and L_y^{-1} are defined by

$$L_x^{-1}(.) = \int_0^x (.)dx, L_y^{-1}(.) = \int_0^y\int_0^y (.)dydy \tag{10}$$

applying the inverse operator L_y^{-1} to both side of eq. (8) gives

$$C(x,y) = C(x,0) + yC_y(x,0) + L_y^{-1}\left(\alpha x^{-0.5} y L_x C\right) \tag{11}$$

using boundary condition no. (2) while assuming $f(x) = u_y(x,0)$ we obtain

$$C(x,y) = C_{Ai} + y f(x) + \alpha L_y^{-1}\left(x^{-0.5} y L_x C\right) \tag{12}$$

using Adomian decomposition series, it follows that

$$\sum_{n=0}^{\infty} C_n(x,y) = C_{Ai} + y f(x) + \alpha L_y^{-1}\left(x^{-0.5} y L_x\left(\sum_{n=0}^{\infty} C_n(x,y)\right)\right) \tag{13}$$

according to decomposition method the recursive relation is found as

$$\begin{aligned} &C_0(x,y) = C_{Ai} + y f(x) \\ &C_{k+1}(x,y) = \alpha x^{-0.5} L_y^{-1}\left(y L_x\left(\sum_{n=0}^{\infty} C_n(x,y)\right)\right) \end{aligned} \tag{14}$$

consequently, we obtain the five first iterations as:

$$C_4(x,y)=\frac{\alpha^4}{7076160}y^{13}\left(x^{-2}f^{(4)}(x)-3x^{-3}f'''(x)+4.75x^{-4}f''(x)-3.5x^{-5}f'(x)\right) \tag{15}$$

$$C_1(x,y)=\frac{\alpha}{12}y^4x^{-0.5}f'(x) \tag{16}$$

$$C_2(x,y)=\frac{\alpha^2}{504}y^7\left(x^{-1}f''(x)-0.5x^{-2}f'(x)\right) \tag{17}$$

$$C_3(x,y)=\frac{\alpha^3}{45360}y^{10}\left(x^{-1.5}f'''(x)-1.5x^{-2.5}f''(x)+x^{-3.5}f'(x)\right) \tag{18}$$

$$C_4(x,y)=\frac{\alpha^4}{7076160}y^{13}\left(x^{-2}f^{(4)}(x)-3x^{-3}f'''(x)+4.75x^{-4}f''(x)-3.5x^{-5}f'(x)\right) \tag{19}$$

and a general relation can be written as:

$$C_m(x,y)=\frac{\alpha^m}{\prod_{j=1}^{m}\left((3j)(3j+1)\right)}y^{3m+1}\sum_{i=1}^{m}\left\{(-1)^{i+m}A_{m,i}x^{-\frac{3}{2}m+i}f^{(i)}(x)\right\},\quad m\geq 1 \tag{20}$$

where $A_{m,n}$ is calculated through the recursive formula as follows

$$A_{m,n}=A_{m-1,n-1}+A_{m-1,n}(m-n-1+\frac{m-1}{2})\ ,\quad \begin{cases}A_{1,1}=1\\ A_{m,n}=0 \quad if \quad m<n\end{cases} \tag{21}$$

In the next step, we have to find a representative for $f(x)$ somehow. Since $f(x)$ is not directly given by the problem, we have to set an equation for it with the help of known parameters. As a very straightforward and rough estimation from the definition of derivative of the function C with respect to y one can write

$$f(x)=C_y(x,0)\propto\frac{C_{A0}-C_{Ai}}{\delta} \tag{22}$$

As noted before, the laminar momentum (or hydrodynamic) boundary layer thickness can be obtained from

$$\delta=\frac{4.64x}{\sqrt{\mathrm{Re}_x}}=4.64\frac{\upsilon^{0.5}x^{0.5}}{u_\infty^{0.5}}=\frac{x^{0.5}}{\beta} \tag{23}$$

so

$$f(x)\propto\beta(C_{A0}-C_{Ai})x^{-0.5} \tag{24}$$

It is known that when D, the diffusion coefficient or diffusivity, goes to infinity, the concentration of component A must become uniformly equal all over the plate (i.e. $C_A(x,y)=C_{Ai}$). As a consequence, $f(x)$ and its higher derivatives shall become zero once D inclines towards infinity. Thus, a proportion coefficient like λ should be multiplied to the right side of relation no. (24) having the characteristic of $\lim_{D\to\infty}\lambda=0$. Moreover, it is wise to add a correction or tuning factor to the relation and not to lose generality, we propose that every order of derivative of $f(x)$ have its own correction factor, namely, w_i. Applying these two aforesaid factors to the relation no. (24), the equation below is achieved.

$$f^{(i)}(x)=\lambda\beta(C_{A0}-C_{Ai})(-1)^i w_i\left(\prod_{k=1}^{i}\frac{(2k-1)}{2}\right)x^{-0.5-i}, \tag{25}$$

Substituting eq. (25) into eq. (20) gives

$$C_m(x,y)=\frac{\lambda\alpha^m\beta(C_{A0}-C_{Ai})}{\prod_{j=1}^{m}\left((3j)(3j+1)\right)}y^{3m+1}\sum_{i=1}^{m}\left\{(-1)^{2i+m}A_{m,i}x^{-\frac{3}{2}m+i}w_i\left(\prod_{k=1}^{i}\frac{(2k-1)}{2}\right)x^{-0.5-i}\right\},\quad m\geq 1 \tag{26}$$

and therefore

$$C_5(x,y)=-\frac{\lambda\alpha^5\,\beta(C_{A0}-C_{Ai})}{14929920}w_5\,y^{16}x^{-8} \tag{27}$$

$$C_0(x,y)=C_{Ai}+\lambda\beta(C_{A0}-C_{Ai})w_0\,y\,x^{-0.5} \tag{28}$$

$$C_1(x,y)=-\frac{\lambda\,\alpha\,\beta(C_{A0}-C_{Ai})}{24}w_1\,y^4x^{-2} \tag{29}$$

$$C_2(x,y)=\frac{\lambda\alpha^2\,\beta(C_{A0}-C_{Ai})}{504}w_2\,y^7x^{-3.5} \tag{30}$$

$$C_3(x,y)=-\frac{\lambda\alpha^3\,\beta(C_{A0}-C_{Ai})}{12960}w_3\,y^{10}x^{-5} \tag{31}$$

$$C_4(x,y)=\frac{\lambda\alpha^4\,\beta(C_{A0}-C_{Ai})}{404352}w_4\,y^{13}x^{-6.5} \tag{32}$$

$$C_5(x,y)=-\frac{\lambda\alpha^5\,\beta(C_{A0}-C_{Ai})}{14929920}w_5\,y^{16}x^{-8} \tag{33}$$

...

$$\frac{C(x,y)-C_{Ai}}{C_{A0}-C_{Ai}}=\lambda\beta\left(\begin{array}{l}w_0yx^{-0.5}-\frac{\alpha}{24}w_1y^4x^{-2}+\frac{\alpha^2}{504}w_2y^7x^{-3.5}-\frac{\alpha^3}{12960}w_3y^{10}x^{-5}+\frac{\alpha^4}{404352}w_4y^{13}x^{-6.5}\\-\frac{\alpha^5}{14929920}w_5y^{16}x^{-8}+...\end{array}\right) \tag{34}$$

(34)

where

$$\beta=\frac{u_\infty^{0.5}}{4.64v^{0.5}},\alpha=\frac{3u_\infty^{1.5}}{9.28D\,v^{0.5}} \tag{35}$$

As proved in Appendix A, an analytical (exact) solution to the problem can be obtained, through combination of variables method, as below:

$$\frac{C-C_{Ai}}{C_{A0}-C_{Ai}}=\frac{\sqrt[3]{\frac{\alpha}{3}}}{\Gamma(4/3)}\int_0^{yx^{-0.5}}\exp\{\frac{-\alpha\theta^3}{3}\}d\theta \tag{36}$$

substituting the integrand with its Taylor's expansion series we get

$$\frac{C-C_{Ai}}{C_{A0}-C_{Ai}}=\frac{\sqrt[3]{\frac{\alpha}{3}}}{\Gamma(4/3)}\int_0^{yx^{-0.5}}\left(1-\frac{\alpha\theta^3}{3}+\frac{\alpha^2\theta^6}{9\times 2!}-\frac{\alpha^3\theta^9}{27\times 3!}+\frac{\alpha^4\theta^{12}}{81\times 4!}-\frac{\alpha^5\theta^{15}}{243\times 5!}+...\right)d\theta \tag{37}$$

and performing the integration within the limits it is obtained that

$$\frac{C-C_{Ai}}{C_{A0}-C_{Ai}}=\frac{\sqrt[3]{\frac{\alpha}{3}}}{\Gamma(4/3)}\left[\theta-\frac{\alpha\theta^4}{3\times 4}+\frac{\alpha^2\theta^7}{9\times 2!\times 7}-\frac{\alpha^3\theta^{10}}{27\times 3!\times 10}+\frac{\alpha^4\theta^{13}}{81\times 4!\times 13}-\frac{\alpha^5\theta^{16}}{243\times 5!\times 16}...\right]_0^{yx^{-0.5}} \tag{38}$$

and finally

$$\frac{C-C_{Ai}}{C_{A0}-C_{Ai}}=\frac{\sqrt[3]{\sqrt[3]{\frac{\alpha}{3}}}}{\Gamma(4/3)}\left(yx^{-0.5}-\frac{\alpha y^4x^{-2}}{3\times 4}+\frac{\alpha^2y^7x^{-3.5}}{9\times 2!\times 7}-\frac{\alpha^3y^{10}x^{-5}}{27\times 3!\times 10}+\frac{\alpha^4y^{13}x^{-6.5}}{81\times 4!\times 13}-\frac{\alpha^5x^{16}y^{-8}}{243\times 5!\times 16}\cdots\right) \tag{39}$$

Comparing the eq. (39), which is the exact solution, with the eq. (34), which is yielded from Adomian decomposition method, easily it is deduced that

$$\lambda=\frac{4.64}{\sqrt[3]{9.28}}\frac{1}{\Gamma(\frac{4}{3})}\sqrt[3]{\frac{\nu}{D}}=\frac{4.64}{\sqrt[3]{9.28}}\frac{1}{\Gamma(\frac{4}{3})}Sc^{\frac{1}{3}}\cong 2.4726\times \mathrm{Sc}^{\frac{1}{3}} \tag{40}$$

and

$$w_i=2^i \tag{41}$$

where $\frac{\nu}{D}$ is a famous dimensionless group called the Schmidt number which appears in many chemical engineering problems and Γ is the Gamma function ($\Gamma(\frac{4}{3})\cong 0.8929795$).

As anticipated and discussed before, it is obvious from eq. (39) that $\lim_{D\to\infty}\lambda=0$.

As demonstrated above, the Adomian decomposition method has led to the exact solution to the PDE introduced in this paper.

Conclusion:

The efficient and powerful method of Adomian Decomposition was successfully applied to solve a problem of practical significance in chemical engineering, namely, mass transfer from a flat slab toward a fluid flowing over it. It was proved that ADM resulted in the exact solution cross-checked by the analytical method of Combination of Variables. Due to its simplicity as well as precision, ADM is recommended for solution of PDEs arising in engineering/scientific modeling problems.

Appendix A (solution of problem by Combination of Variables Method):

A proposed combination of the variables x and y is as follows:

$$\eta=y^nx^m \tag{A.1}$$

Using the chain rule of differentiation in Calculus, it yields:

$$\frac{\partial C}{\partial y}=\frac{\partial C}{\partial \eta}\frac{\partial \eta}{\partial y}=ny^{n-1}x^m\frac{\partial C}{\partial \eta} \tag{A.2}$$

$$\frac{\partial^2 C}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial C}{\partial y}\right)=n(n-1)y^{n-2}x^m\frac{\partial C}{\partial \eta}+n^2y^{2(n-1)}x^{2m}\frac{\partial^2 C}{\partial \eta^2} \tag{A.3}$$

$$\frac{\partial C}{\partial x}=\frac{\partial C}{\partial \eta}\frac{\partial \eta}{\partial x}=mx^{m-1}y^n\frac{\partial C}{\partial \eta} \tag{A.4}$$

Substituting Eq. (A.2) and (A.4) into Eq. (6) results in:

$$n(n-1)y^{n-2}x^m\frac{\partial C_A}{\partial \eta}+n^2y^{2(n-1)}x^{2m}\frac{\partial^2 C}{\partial \eta^2}=\alpha mx^{m-1.5}y^{n+1}\frac{\partial C}{\partial \eta} \tag{A.5}$$

By letting n=1 the Eq. (A.5) reduces to:

$$x^{2m}\frac{\partial^2 C}{\partial \eta^2}=\alpha mx^{m-1.5}y^2\frac{\partial C}{\partial \eta} \tag{A.6}$$

According to Eq. (A.1) $\eta^2=y^{2n}x^{2m}=y^2x^{2m}$, this transforms Eq. (A.6) to:

$$x^{3m+1.5}\frac{\partial^2 C}{\partial \eta^2} = \alpha m \eta^2 \frac{\partial C}{\partial \eta} \tag{A.7}$$

To obtain an ODE, the exponent term $3m+1.5$ must be let zero, that is $m=-0.5$.

$$\frac{d^2 C}{d\eta^2} = -\alpha\eta^2 \frac{dC}{d\eta} \tag{A.8}$$

By defining $\zeta = \frac{dC}{d\eta}$ we have:

$$\frac{d\zeta}{d\eta} = -\alpha\eta^2\zeta \tag{A.9}$$

Indefinite integration yields:

$$\zeta = \frac{dC}{d\eta} = A\exp\{\frac{-\alpha\eta^3}{3}\} \tag{A.10}$$

in which A is an arbitrary integration constant.

Integrating Eq. (A.10) from the plate surface, at any x and y=0 which forces η=0, to an arbitrary position above the plate, that is any x and y or any η, yields:

$$C - C_{Ai} = A\int_0^\eta \exp\{\frac{-\alpha\theta^3}{3}\}d\theta \tag{A.11}$$

Since it was found that $m=,-0.5$ both B.C.1 and B.C.2 make $\eta = \infty$. Applying this to Eq. (A.11):

$$C_{A0} - C_{Ai} = A\int_0^\infty \exp\{\frac{-\alpha\theta^3}{3}\}d\theta \tag{A.12}$$

Dividing Eq. (A.11) by Eq. (A.12) yields:

$$\frac{C - C_{Ai}}{C_{A0} - C_{Ai}} = \frac{\int_0^\eta \exp\{\frac{-\alpha\theta^3}{3}\}d\theta}{\int_0^\infty \exp\{\frac{-\alpha\theta^3}{3}\}d\theta} \tag{A.13}$$

After some easy mathematical effort, the denominator of the fraction appeared in Eq. (A.13) will be written in form of the Gamma function:

$$\frac{C - C_{Ai}}{C_{A0} - C_{Ai}} = \frac{\sqrt[3]{\frac{\alpha}{3}}}{\Gamma(4/3)}\int_0^{yx^{-0.5}} \exp\{\frac{-\alpha\theta^3}{3}\}d\theta \tag{A.14}$$

Nomenclature:

A integration constant
C_A concentration of component A, mol/m^3

C_{Ai} concentration of component A over the plate interface, mol/m^3

C_{A0} initial concentration of component A, mol/m^3

D diffusion coefficient, m^2/s
m,n auxiliary variables

R_{ex} local Reynolds number, $(u_\infty x/\upsilon)$

Sc Schmidt number, (υ/D)

u x-component of fluid velocity, *m/s*
u_∞ free stream velocity, *m/s*
x streamwise (horizontal) coordinate, *m*

y traverse (vertical) coordinate, m

Greek letters:

$\alpha, \beta, \eta, \zeta$ auxiliary variables

δ_c concentration boundary layer thickness, m

δ Momentum (or hydrodynamic) boundary layer thickness, m
Γ Gamma function
υ kinematic viscosity, m^2 / s

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