Optimal Placement of Phasor Measurement Units With Considering Weight Matrix and the Average Probability of Observability

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ABSTRACT
Phasor measurement units are the linchpin of a comprehensive monitoring system what enable to the initial review on measured data and the results of this analysis to form the data ready to send to the control center. In this paper an attempt has been made to determine the best placement of PMU in minimum numbers for total observability of network. Since the phasor measurement units with more measuring directions costs more, the investigation of weight matrix is dealt with here. In conditions in which matrices with the same weight are available, in order to select the optimum result, calculation of average probability of observability index is also suggested. The solution of the problem of this article is simulated with linear integer program on MATLAB software in IEEE standard 57-bus system. The results indicate the optimum placement of PMUs with less cost and efficient average probability of observability index in this issue.

INTRODUCTION
In addition to SCADA system, recently is raised another system with called the Wide Area Monitoring System (WAMS). The main idea WAMS is centralized processing of data that are collected from the different parts of the system simultaneously and synchronized with each other. The concept that offers to WAMS for power system is very close to the actual conditions of the system. The reason for this can be simply expressed as that due to constant changes in the frequency of power system and its impact on the calculation of angles, buses angles are comparable to each other only when the exactly related to the identical of a moment. Which this is possible in WAMS to using the phasor measurement units (PMU)[10].

But due to the high cost of this equipment and the fact that installing PMU on each bus enables voltage phasor of that bus and current phasor of all attached branches to it to measured, is attention the optimal specification of these measuring instruments in power networks according to their place and number for observability of the whole network and also the least amount of cost are remarkable [4,5,6,7].The cost of installation and operating of PMU in different places is, however different, so that the installed PMU on the bus which is connected to more branches and has more measuring directions, is higher weight (i.e:cost) than other PMUs [12].

In reference [7] placement to gain a complete set of answers by ILP method is introduced. Also in the reference [13] for a sample network with using particle swarm optimization (PSO) the investigation of weight matrix in placement of PMU in normal stage has been done without the comparison and ranking of answers. In reference [5] with algorithm of mixed integer programming(MIP) operating PMU in system has been done by using the index of average probability of observability(APO).

In this paper, by integer linear programming (ILP) method, according to the need of workers of power industry who risk to have a technical and economic optimal choice, a complete answer set of placement with minimum number of PMU considering weight matrix are specified and with indices of APO the answer set forthe IEEE standard 57-bussystems are compared.
The Formulation of Optimal Placement PMU:

The formulation of the optimal placement of PMU in a system with n buses is presented as equation (1)[1,2,5,6,9]:

\[
\text{min} \sum_{i=0}^{n} w_i x_i \\
\text{s.t.} \quad y = Ax \geq b
\]

Which w is the cost function for the installed PMUs, and in normal stage, placement equaling to matrix of unit is considered. A is connection matrix of n×n which reveals the way of connection of buses which is defined as (2) [3,4,6,8]:

\[
A_{n \times n}(i,j) = \begin{cases} 
1 & \text{if buses } i \text{ and } j \text{ are connected} \\
0 & \text{otherwise} 
\end{cases}
\]

The discrete nature of the optimal placement of PMU make it necessary that X vector to be defined as equation (3) such that the elements of that position show the installation of this equipment in each bus [13]:

\[
[x]_i = \begin{cases} 
1 & \text{if PMU is install at bus } i \\
0 & \text{otherwise} 
\end{cases}
\]

Also b matrix for at least one observability is shown as (4)[6,7,8,11]:

\[
b_{(n+w)1} = [1 \ 1 \ldots \ n]^{T}
\]

So if the \(x_{1} = [x_{1,1} \ x_{1,2} \ldots \ x_{1,n}]^{T}\) is optimal answer of placement of PMU with m number, to gain \(w\) matrix, optimal answer without the repetition of similar answers, \(A_{n \times n}\) matrix and \(b_{(n+w)}\) vector will be modified as equation (5)[7]:

\[
A_{(n+w) \times n} = \left[ \begin{array}{c} A_{n \times n} \\
-x_{1,1} \\
-x_{1,2} \\
\vdots \\
-x_{sw,1} \\
-x_{sw,2} \\
\vdots \\
-x_{sw,n} \end{array} \right]_{(n+w) \times n}
\]

\[
b_{(n+w)1} = [1 \ 1 \ldots - (m - 1) \ldots - (m - 1)]^{T}
\]

Therefore the optimal placement of PMUs is calculated to achieve a complete set of answers as equation (6)[7]:

\[
\begin{align*}
\text{min} \sum_{i=1}^{n} w_i x_i \\
\text{subjecto:} \quad A_{(n+w) \times n} \times nX_{n \times 1} \geq b_{(n+w)1} \\
X_{n \times 1} = [x_{1,1} x_{1,2} \ldots x_{1,n}]^{T} \\
x_i \in \{0, 1\}
\end{align*}
\]

In this equation the A matrix and b vector will be modified at each repetition. Also the repetition will be stops if the number of PMU is larger than m which is the number of optimal PMUs calculated in the first replication.

Optimal placement of PMU considering the effect of the weight matrix (w):

Weight matrix in the equation (3) of PMU placement is related to the number of connected branches to each bus. Equation (7) shows the value of bus with the number of connected branches[11]:

\[
w = \begin{cases} 
1 & \text{if } \text{NUM}_{j} < 3 \\
1.5 & \text{if } \text{NUM}_{j} \geq 3
\end{cases}
\]

In this equation \(\text{NUM}_{j}\) reveals the number of connected lines to bus \(j\) that if it is less than 3 the weight of bus \(j\) is one, and if it equals 3 or is more than it, the weight of bus \(j\) is 1.5.

Average Probability of Observability Index:

The value of average probability of observability of all buses is considered as the system index that makes a value checking on observability of power system for operator and is calculated as the equation (8)[4]:

\[
\text{APO} = \frac{1}{N_B} \sum_{i=1}^{n} \text{PO}_{i}
\]

In which \(N_B\) is the number of system buses and \(i\) is a set of buses. \(\text{PO}_{i}\) is the probability of observability related to bus \(i\) which is achieved form equation (8)[4]:

\[
\text{PO}_{i} = 1 - \prod_{j=1}^{n} (1 - \mu_j A_{ij}); \forall i
\]

Where \(\mu_i\) is a binary variable and investigates PMU installed in bus \(j\) and \(A_{ij}\) examines probability of observability of bus \(i\) by installing PMU in bus \(j\). This amount is defined as the equation (10)[4]:

\[
A_{ij} = a_{ij} A_{ij}^{PMU} + a_{ij}^{Lin} A_{ij}^{Lin}; \forall i, \forall j
\]
In which $A_{ij}^{vm}$ and $A_{ij}^{cm}$ show availability of voltage measurement respectively by installing PMU in bus $j$ and current measurement with PMU in line $ij$ which can be calculated in availability conditions of voltage transformer (PT) and current transformer(CT)[1,17]. $A_{ij}^{PMU}$ and $A_{ij}^{Link}$ are the probability of successful PMU in bus $j$, and transmission link respectively. Also $A_{ij}^{Link}$ specifies the availability of line $ij$. In equation (11) $A_{ii}^{line} = A_{ii}^{cm} = 1$; $\forall i$ and $a_{ij}$ are the parameters of binary connections and is shown as equation (11) [4]

$$a_{ij} = \begin{cases} 
1 & i = j \\
1 & \text{if buses } i \text{ and } j \text{ are connected} \\
0 & \text{otherwise}
\end{cases}$$

Therefore every answer which has higher APO is selected as an optimal answer.

**The Simulation Results:**

In this section In order to evaluate the potential of the proposed formulation, investigated the optimal placement of phasor measurement units for the IEEE 57-bus system are given in figures (1), with the purpose of full observability with the lowest cost and select the most appropriate response in two stages aided by using integer linear programming algorithm in MATLAB software environment.

![Diagram of IEEE 57-bus system](image)

**Stage1) the complete observability in normal stage:**

In this stage in table (1) according to the formulization of optimal placement of PMU in equations (1)-(4) optimal placement of PMU in minimum number has been done for system observability by specifying the optimal place and the number. The results of this table show the complete observability of system buses while almost 30% of system buses are equipped with PMU in normal stage of placement.

**Table 1:** The optimal number and location of PMU in normally state system.

<table>
<thead>
<tr>
<th>Location of PMUs</th>
<th>No. of PMUs</th>
<th>Test system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IEEE 57-Bus</td>
<td>17</td>
<td>1,4,6,13,20,22,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25,29,32,36,39,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>41,45,47,51,54</td>
</tr>
</tbody>
</table>

**Stage2) the complete observability of network by considering the effect of weight matrix (w):**

In this stage in the tables (4) according to the equations (5) and (6) the set of answers with the minimum number of PMU is calculated by taking the effect of weight matrix into consideration.

Also in order to choose the more optimal answer and the ranking of answers by prevailing the minimum amount of weight matrix the index APO by using the equations (8)-(11) and the data of tables (2) and (3) based on reliability and availability of transmission lines for IEEE 57-bus is used. As it can be seen weight matrix has no effect on the minimum number of PMU.

**Table 2:** Reliability data of 57-Bus system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{i}^{PMU}$</td>
<td>.9954978</td>
<td>$A_{i}^{CM}$</td>
<td>$(.99958447)^{1}$</td>
</tr>
<tr>
<td>$A_{i}^{VM}$</td>
<td>$(.99854238)^{1}$</td>
<td>$A_{ij}^{Link}$</td>
<td>.999</td>
</tr>
</tbody>
</table>

According to the results of table (4) the answer rows 1,2,3,4 in this stage with the minimum weight matrix and the equality of APO index is the best place to PMU installation in IEEE 57-bus network for complete system observability. In this way the answers can be ranked with the minimum weight matrix and then high index APO, since if there is limitation in installing on one row answer there will be a better choice among other
answers. Next to them, answer row 5 and 8 with the higher number of indices compared with row 6 and 7 has priority to be selected and so row 7 is better than row 8.

Table 3: Availability of transmission lines of the IEEE 57-Bus.

<table>
<thead>
<tr>
<th>Place</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9966</td>
</tr>
<tr>
<td>5</td>
<td>0.9966</td>
</tr>
<tr>
<td>9</td>
<td>0.9966</td>
</tr>
</tbody>
</table>

Table 4: Review SORI and APO index compared to answers placement PMU with weight matrix, in IEEE 57-bus network.

<table>
<thead>
<tr>
<th>Number of PMU</th>
<th>Rows</th>
<th>Location of PMU</th>
<th>W</th>
<th>APO</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1</td>
<td>2,6,12,19,22,25</td>
<td>20</td>
<td>0.9869</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2,6,12,19,22,25</td>
<td>20</td>
<td>0.9869</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2,6,12,19,22,25</td>
<td>20</td>
<td>0.9869</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2,6,12,19,22,25</td>
<td>20</td>
<td>0.9869</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2,6,12,19,22,25</td>
<td>20.5</td>
<td>0.9877</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>2,6,12,19,22,25</td>
<td>20.5</td>
<td>0.9873</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>2,6,12,19,22,25</td>
<td>20.5</td>
<td>0.9871</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>2,6,12,19,22,25</td>
<td>20.5</td>
<td>0.9877</td>
</tr>
</tbody>
</table>

Results:

In this paper the answer set of placement phasor measurement unit with minimum number is calculated for complete observability of the IEEE 57-bus system when the weight matrix is taken into consideration. And by prevalence of the minimum amount of this matrix the optimal answer is selected. So there is the probability of answer row in equal weight matrix, to select the optimal answer row and ranking of the answers for conditions in which because of limitations there is not any opportunity to select one answer, average probability of observability index were proposed to be studied. And by them the answers are ranked with the prevalence of the minimum of weight matrix and the maximum number for these index.
REFERENCES


