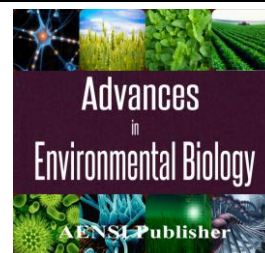




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A Model of Portfolio Optimization using Multivariate GARCH Models: Evidence of Tehran stock Exchange

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ABSTRACT

Background: In this paper, appropriate allocation of a portfolio consisted of stocks of selected industries 'automobile and part manufacturing', 'pharmaceutical materials and products', 'chemical products', 'tile and ceramics' and 'sugar' as members of Tehran Stock Exchange (TSE) are analyzed using MGARCH models. These kinds of models are well-suited tools for analyzing and forecasting time series volatility, which are fluctuating over time. They are also used in econometric literature in order to estimate conditional covariance, by which and by using expected return of existing portfolio investments, optimal weight of portfolio investments are determined. **Objective:** In this paper, conditional time-varying matrix covariance was separately estimated using four multivariate GARCH models- Diagonal-BEKK, Diagonal-VECH, CCC and DCC. Then the portfolio optimization with an approach to risk minimization and the optimal time-varying weights for aforementioned industries were found for each model. **Results:** The results also show that during periods of high volatility, optimum share of the industry from portfolio decreases and inversely an increase in optimum share from portfolio relates to periods of low volatility. The results recommend investing in industries, where there is higher stability in stock prices and low volatility in returns, and give them a higher priority. **Conclusion:** The results of this research indicate compatibility to portfolio risk minimization and thereby high accuracy of multivariate GARCH models in estimating dynamic covariance matrices. Since these models have been fitted to estimate conditional covariance matrix, we can hopefully rely on results of conditional covariance matrix predicted by these models to form an optimum portfolio investment.

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INTRODUCTION

Optimal selection of portfolio is among the most important issues for stock investors, so that by investing in multiple stocks rather than a particular one, they can reap maximum profits at a given risk level or face minimum risk for a given profit. In early 1950s, Markowitz proposed a model to predict portfolio based on mean as profit and variance as risk index [1]

Thus, one major challenge in portfolio selection is determination of an optimal ratio or weight for existing stocks in a portfolio in order to minimize the risks. It is worth noting that, the studies on financial behavior suggest that as opposed to traditional theories, the investor may make decisions, which are not economically justified. [2]

According to financial behavior theory, the investor has priorities that make him/her not to be risk-averse but loss-averse and he/she has willing to take high risks. One, under the influence of society or individuals or in opposition with traditional theories, may make some decisions. [3]

By accepting the traditional theory, investment and the fundamental hypothesis of the investor's risk aversion, selection problem of an optimal portfolio could be solved.

Three main factors involved with portfolio management- individuals or decision-makers; tools, techniques and selection models; and the process or framework used in project selection. Even though there are few frameworks to organize these tools and techniques in a logical manner. Therefore, one important point in projection selection for a portfolio is selection and creation of an appropriate framework to assess proposed projects and select a portfolio synchronized with corporation strategies. [4]

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To obtain an optimal weight for investment in a portfolio (A portfolio that gives maximum return for a given risk, or minimum risk for given return is an efficient portfolio), Markowitz designed and solved a constrained optimization problem by means of which we can obtain the vector of weight of existing investments in a portfolio. In fact, Markowitz determined optimal allocation of an investor's wealth to a variety of investments he/she is willing to hold to maximize the return for a given level of risk or minimize the portfolio risk for a given level of return. The most important idea of Markowitz was application of standard deviation of the portfolio as a risk criterion. Thus, to use Markowitz theory, first it is necessary to calculate standard deviation of the portfolio, which requires estimation of conditional covariance for investments in a portfolio. In econometric literature, Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) was used to estimate conditional covariance. [5]

With the increase in the complexity of the instruments in the risk management field, huge demands for the various models which can simulate and reflect the characteristics of the financial time series have expanded. One of the significant features of financial data that has won much attention is the volatility; because it is a numerical measure of the risk faced by individual investors and financial institutions. It is well known that the volatility of financial data often varies over time and tends to cluster in periods, i.e., high volatility is usually followed by high volatility, and low volatility by low volatility. This phenomenon corresponds to the fluctuating volatility. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and its extensions have been proved to be able to capture the volatility clustering and predict volatilities in the future.

Specifically, when analyzing the co-movements of financial returns, it is always essential to estimate, construct, evaluate, and forecast the co-volatility dynamics of asset returns in a portfolio. This task can be fulfilled by multivariate GARCH (MGARCH) models. The development of MGARCH models could be thought as a great breakthrough against the curse of dimensionality in the financial modeling. Many different formulations have been constructed parsimoniously and still remain necessary flexibility. MGARCH models can be applied to asset pricing, portfolio theory, VaR estimation and risk management or diversification, which require the volatilities and co-volatilities of several markets. [6]

Problem definition and the research objective:

Modeling of Uncertainty in financial time series in the form of Autoregressive Conditional Heteroskedasticity was compared to that of Engle (Engle, 1982). Then, numerous ARCH models were taken into consideration, most of which were univariate, and their generalization to GARCH and MGARCH was considered. [7]

One of the most important applications of MGARCH models is estimation of conditional covariance, which is critically important in portfolio selection and evaluation of stock pricing models. When stipulating a MGARCH model, it should be enough flexible to show dynamics of a conditional covariance matrix. Moreover, since the quantity of MGARCH parameters increase as model dimensions increase quickly, model stipulation should meet cost effectiveness conditions. Of course, it should be noted that meeting cost effectiveness condition will often accompany model error stipulation. [8]

It is also noteworthy that one other condition of stipulating a MGARCH model is that the conditional covariance matrix must be positive definite. Although combination of these parameters in the context of a MGARCH model is difficult, it can be fulfilled through meeting some conditions. Multivariate MGARCH models are also able to analyze obvious characteristics of stock markets including Kurtosis, Leverage Effect, Volatility clustering.

A conditional covariance matrix is a $n \times n$ matrix, which is individually calculated for each model and the weight of each group is then calculated. [7]

In this research, the matrix is 5×5 :

$$H_t = \begin{bmatrix} h_{11t} & h_{12t} & h_{13t} & h_{14t} & h_{15t} \\ h_{21t} & h_{22t} & h_{23t} & h_{24t} & h_{25t} \\ h_{31t} & h_{32t} & h_{33t} & h_{34t} & h_{35t} \\ h_{41t} & h_{42t} & h_{43t} & h_{44t} & h_{45t} \\ h_{51t} & h_{52t} & h_{53t} & h_{54t} & h_{55t} \end{bmatrix} \quad (1)$$

Introduction of GARCH multivariate models:

- *Diagonal-Vech(p,q) model:*

This model was first introduced by Engle and Woolridge (1988) [9].

Diagonal-Vech(p,q) model is expressed as follows:

$$\begin{aligned} r_{i,t} &= \mu_i + x_{i,t} \quad i = 1, 2, \dots, N \\ E(x_{i,t}) &= 0 \quad \forall i = 1, 2, \dots, N \end{aligned} \quad (2)$$

where,

$r_{i,t}$ = ith investment in time t.

Using matrix algebra

$$r_i = \mu + x_i \quad (3)$$

$$\text{Vech}(H_t) = c + \sum_{j=1}^p A_j \text{vech}(\varepsilon_{t-j} \varepsilon'_{t-j}) + \sum_{j=1}^q B_j \text{vech}(H_{t-j}) \quad (3)$$

$$\varepsilon_t = D_t' r_t \quad (4)$$

$$D_t = \begin{bmatrix} \sqrt{h'_{11t}} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{h'_{22t}} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{h'_{33t}} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{h'_{44t}} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{h'_{55t}} \end{bmatrix} \quad (5)$$

$$h'_{ii,t} = \omega_i + \sum_{i=1}^p \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^q \beta_i h'_{ii,t-1} \quad (6)$$

where,

x_t = remaining models vector

A and B = parametric matrix $\frac{N(N+1)}{2} \times \frac{N(N+1)}{2}$ and $C = 1 \times \frac{N(N+1)}{2}$

There are two rules in a Vech model:

1. The quantity of parameters to be estimated is limited.
2. There are constraints to ensure that the conditional covariance matrix is positive definite.

Maximum likelihood method is used for estimating the parameters, and unconditional residual variance as a conditional variance of prototype to ensure definite positivity of H_t . In addition, A_j and B_j are hypothesized as diagonal matrices.

The model obtained from these hypotheses is known as Diagonal-Vech(p,q). This model is easier to estimate because the number of estimated parameters is $\frac{(p+q+1)N(N+1)}{2}$. [6]

- *Diagonal-BEKK(p,q) model:*

In 1991, another class of Diagonal-Vech model was introduced referred to as Diagonal-BEKK. [10]. This model has interesting characteristics, which by some constraints creates a positive and distinct conditional covariance. The matrix is calculated as follows:

$$H_t = CC' + \sum_{i=1}^p A_i' \varepsilon_{t-1} \varepsilon'_{t-1} A_i + \sum_{j=1}^q B_j' H_{t-1} B_j \quad (7)$$

BEKK is not linear in any parameter and the model is difficult to converge.

Advantage of Vech to BEKK is that if the conditional covariance matrix contains more than two variables, it is more flexible, but due to difficulty of ensuring semi-definite positivity of variance matrix, conditional covariance becomes limited.

- *CCC(p,q) model:*

Another type of Vech model, called Conditional Constant Correlation (CCC) was developed in 1990, which hypothesized correlation matrix is time-independent and it is constant over time. And, conditional covariance matrix is indirectly calculated by estimating conditional correlation matrix.

Conditional correlation is assumed to be constant, while conditional variance is variable. Obviously, this assumption is impractical for real time-financial series [11].

$$H_t = D_t P D_t' \quad (8)$$

$$D_t = \begin{bmatrix} \sqrt{h'_{11t}} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{h'_{22t}} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{h'_{33t}} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{h'_{44t}} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{h'_{55t}} \end{bmatrix} \quad (9)$$

$$P = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} & \rho_{15} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} & \rho_{25} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} & \rho_{35} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 & \rho_{45} \\ \rho_{51} & \rho_{52} & \rho_{53} & \rho_{54} & 1 \end{bmatrix} \quad (10)$$

$$h'_{ii,t} = \omega_i + \alpha_i \varepsilon_{t-1}^2 + \beta_i h'_{ii,t-1} \quad (11)$$

$$\varepsilon_t = D_t' r_t \quad (12)$$

r_t = performance of indices

- *DCC(p,q) model:*

DCC-GARCH model was suggested by Engel (2002) [12]. It is a GARCH model, where relationships between the variables are analyzed by considering events during the period. In this case, correlation between variables may be direct, inverse or null. When two series are in the same direction, the correlation increases and the relationship will be direct. If directions are different, the correlation decreases and the relationship will be inverse. DCC is useful, when there is diversion in data. This model is a highly authentic research conducted in relation to modeling time-varying correlational parameters for multivariate portfolios.

DCC is a generalized form of CCC, in which the volatilities vary by time, while they are assumed constant in CCC. An advantage of DCC to CCC is that the number of estimated parameters in a correlation process does not depend on the number of correlated series. Hence, a very big correlation matrix can potentially be estimated.

DCC model can create a definitely positive covariance matrix at any moment [13].

A major problem of GARCH was high number of parameters. To solve this problem, Bollerslev (1990) proposed to consider all correlations constant, and called it Constant Conditional correlation model [14].

But DCC models reserves the ease of estimating CCC and considers the correlations variable over time [12].

Shortage in DCC model is that all conditional correlations follow the same dynamic structure. A number of parameters are estimated, which is less than that of full BEKK model in the same dimensions (if N values are low). When N is high, estimation of DCC model is done in two steps to reduce complexity of estimation processes. Briefly, first the univariate volatilities are modeled for each series of returns. Then, the parameters of the correlation process are estimated [13].

Comparing DCC model to simple multivariate GARCH model and some other estimators suggests that DCC is often more accurate. Therefore, DCC is a generalized form of CCC model, where volatilities vary by time, but conditional correlations are constant. CCC model does not incorporate time variations of asset correlations over stability, growth or recession periods. For this reason, assuming that the correlations vary by time, Engel generalized CCC model. Then, Engel proposed generalization of Bollerslev's model by creating a time-varying conditional correlation matrix. This model is a dynamic conditional correlation (DCC), which is one the most well-known CCC-GARCH models consisted of time-varying conditional correlation matrices [12].

$$H_t = D_t P D_t \quad (13)$$

$$h'_{ii,t} = \omega_i + \sum_{i=1}^p \alpha_i \varepsilon_{t-1} + \sum_{i=1}^q \beta_i h'_{ii,t-1} \quad (14)$$

$$D_t = \begin{bmatrix} \sqrt{h'_{11t}} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{h'_{22t}} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{h'_{33t}} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{h'_{44t}} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{h'_{55t}} \end{bmatrix} \quad (15)$$

$$P = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} & \rho_{15} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} & \rho_{25} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} & \rho_{35} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 & \rho_{45} \\ \rho_{51} & \rho_{52} & \rho_{53} & \rho_{54} & 1 \end{bmatrix} \quad (16)$$

In addition, we find optimum lag for AR models of the data used in this paper to stipulate mean equations. Optimum lag for AR is introduced based on autocorrelation and partial autocorrelation functions as well as Akaike Information Criterion (AIC) for all five time series, i.e. Log Returns of Stock Prices of those four selected industries.

Thus, $p, q=1$

First, we evaluate durability of 72 groups registered in Tehran Stock Exchange. Among groups having durable data, 5 active groups during the time series in question were selected. These 5 groups were 'automobile and part manufacturing', 'pharmaceutical materials and products', 'chemical products', 'tile and ceramics' and 'sugar'.

Multivariate GARCH models must be durable over time, and for this purpose we should change interval variable into logarithm of first order differential equations. To do this, data should be in the following form:

$$r_{i,t} = \text{Log} \left(\frac{p_{i,t}}{p_{i,t-1}} \right) \quad (17)$$

Table1 indicated different statistical attributes in the data used in this research.

Jarque-Bera test shows that for all of these four industries, normality of log return of a time series at %1 confidence level is rejected.

Table 1: indicated different statistical attributes in the data used in this research

Jarque-Bera	Kurtosis	Skewness	Standard Deviation	Mean	Groups
3308.007	10.37348	-.108773	0.006076	0.000354	Automobile and part manufacturing
4966.047	11.98374	.495491	0.004539	0.000717	chemical products
26874.11	23.19214	2.930289	0.002492	0.000563	pharmaceutical materials and products
23366.98	22.50545	.989350	0.006170	0.000689	sugar
73585.19	37.79135	.050984	0.005016	0.000746	tile and ceramics

Then, durability of groups is analyzed by ADF test.

First, four models are estimated by using 4 time-varying conditional covariance, then portfolio optimization is performed with an approach to risk minimization based on Markowitz theory, and finally the optimal weights for the four industries will be identified.

Weights are obtained from the following formulae: [13]

$$W_{ij,t} = \frac{h_{j,t} - h_{ij,t}}{h_{i,t} - 2h_{ij,t} + h_{j,t}} \quad (18)$$

Price index variation diagrams and logarithmic variation rate are drawn in five groups.

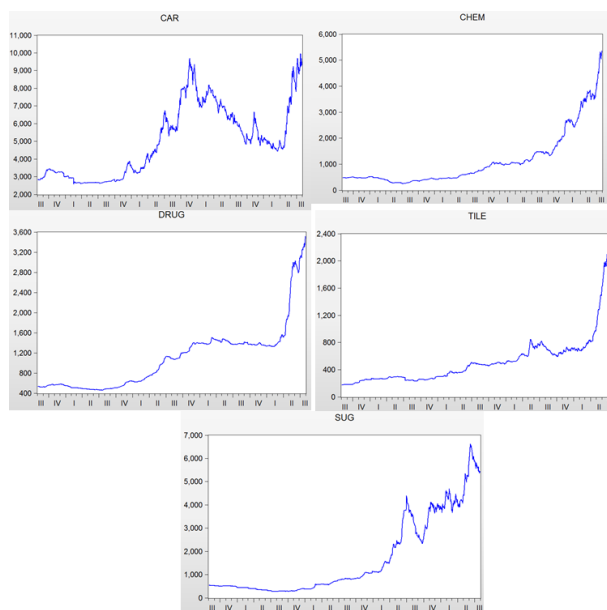


Fig. 1: Price index variation diagrams.

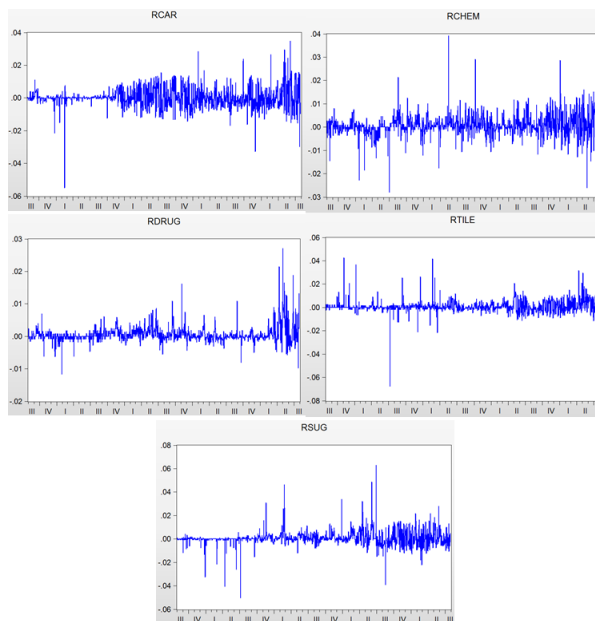


Fig. 2: logarithmic variation rate.

We use a very common test-ADF test as a test for a unit root by Eviews software to evaluate durability.

In unit root tests, the following hypothesis is always is always valid:

$H_0: \Theta = 0$, unit root exists and the variable is nondurable.

$H_1: \Theta < 0$, unit root does not exist and the variable is durable.

Table 2: shows the results of ADF test.

Trend & Intercept	None	Intercept	Groups
-23.71043	-23.63578	-23.70998	Automobile and part manufacturing
-27.28162	-16.61029	-26.78720	chemical products
-16.91325	-15.93240	-16.57119	pharmaceutical materials and products
-27.52613	-27.13687	-27.38248	sugar
-30.30610	-29.69061	-30.20673	tile and ceramics

Table2, results of ADF test

Now, after making sure that the series are durable, we go to 4 GARCH models.

Results:

1. CCC Model Output:

$$\omega_i = (1.98E-07 \quad 2.10E-06 \quad 8.17E-07 \quad 2.07E-05 \quad 3.46E-07)$$

$$\alpha_i = (0.053966 \quad 0.096995 \quad 0.731184 \quad 0.895477 \quad 0.000118)$$

$$\beta_i = (0.950093 \quad 0.810233 \quad 0.346538 \quad 0.007764 \quad 0.986693)$$

$$P = \begin{pmatrix} 1 & 0.106424 & 0.156084 & 0.030439 & 0.110415 \\ 0 & 1 & 0.140868 & 0.035736 & 0.74602 \\ 0 & 0 & 1 & 0.46621 & 0.143105 \\ 0 & 0 & 0 & 1 & 0.073206 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Allocation results are shown below:

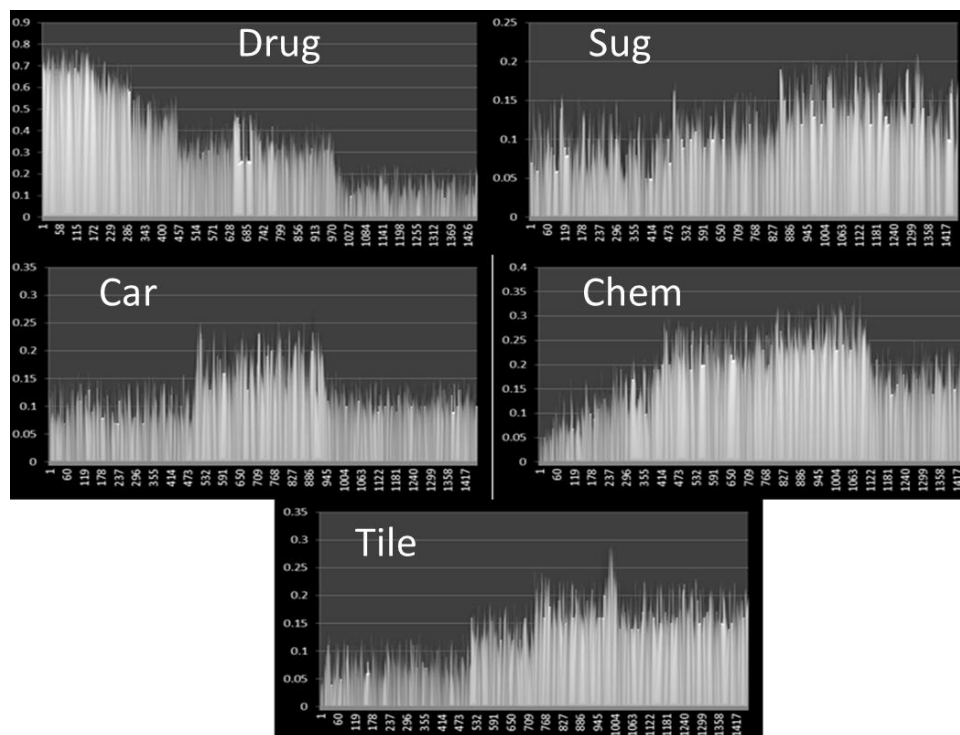


Fig. 3: Allocatio result in CCC.

2. DCC Model Output:

$$P = \begin{pmatrix} 1 & 0.410778 & 0.3010108 & 0.13703 & -0.0118569 \\ 0 & 1 & 0.3685028 & 0.168147 & 0.006344 \\ 0 & 0 & 1 & -0.0006265 & 0.0052756 \\ 0 & 0 & 0 & 1 & 0.0794756 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\omega_i = (9.53E-08 \quad 8.87E-07 \quad 6.554E-07 \quad 0.000019 \quad 0.0000243)$$

$$\alpha_i = (0.0682619 \quad 0.1267871 \quad 0.5699709 \quad 0.9775625 \quad 0.102455)$$

$$\beta_i = (0.9452359 \quad 0.7660006 \quad 0.4532938 \quad 0.0121901 \quad -0.1196506)$$

Allocation results are shown below:

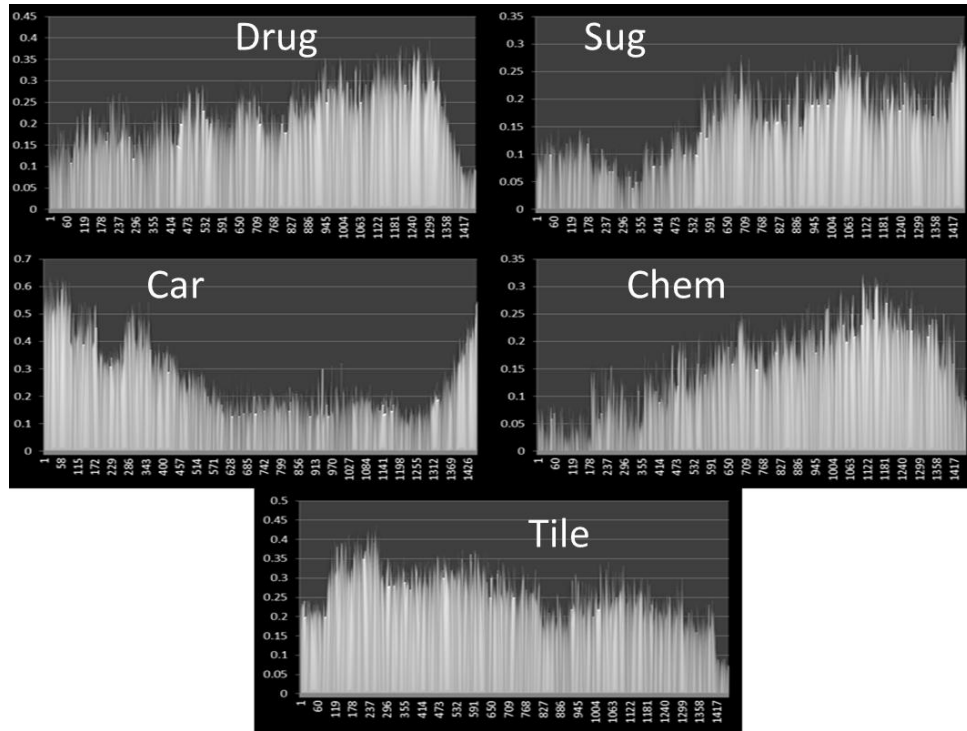


Fig. 4: Allocatio result in DCC.

3. BEKK Model Output:

$$CC' = M = \begin{pmatrix} 1.62E-07 & 3.57E-08 & 1.24E-07 & 2.49E-07 & 6.16E-08 \\ 0 & 1.68E-06 & 1.38E-07 & 3.49E-07 & 1.13E-07 \\ 0 & 0 & 6.40E-07 & -1.08E-07 & 2.02E-07 \\ 0 & 0 & 0 & 2.36E-05 & 1.59E-06 \\ 0 & 0 & 0 & 0 & 1.79E-06 \end{pmatrix}$$

$$A = \begin{pmatrix} 0.235094 & 0 & 0 & 0 & 0 \\ 0 & 0.247550 & 0 & 0 & 0 \\ 0 & 0 & 0.584899 & 0 & 0 \\ 0 & 0 & 0 & 0.807998 & 0 \\ 0 & 0 & 0 & 0 & 0.110335 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.974976 & 0 & 0 & 0 & 0 \\ 0 & 0.928902 & 0 & 0 & 0 \\ 0 & 0 & 0.773116 & 0 & 0 \\ 0 & 0 & 0 & 0.044354 & 0 \\ 0 & 0 & 0 & 0 & 0.957483 \end{pmatrix}$$

Allocation results are shown below:

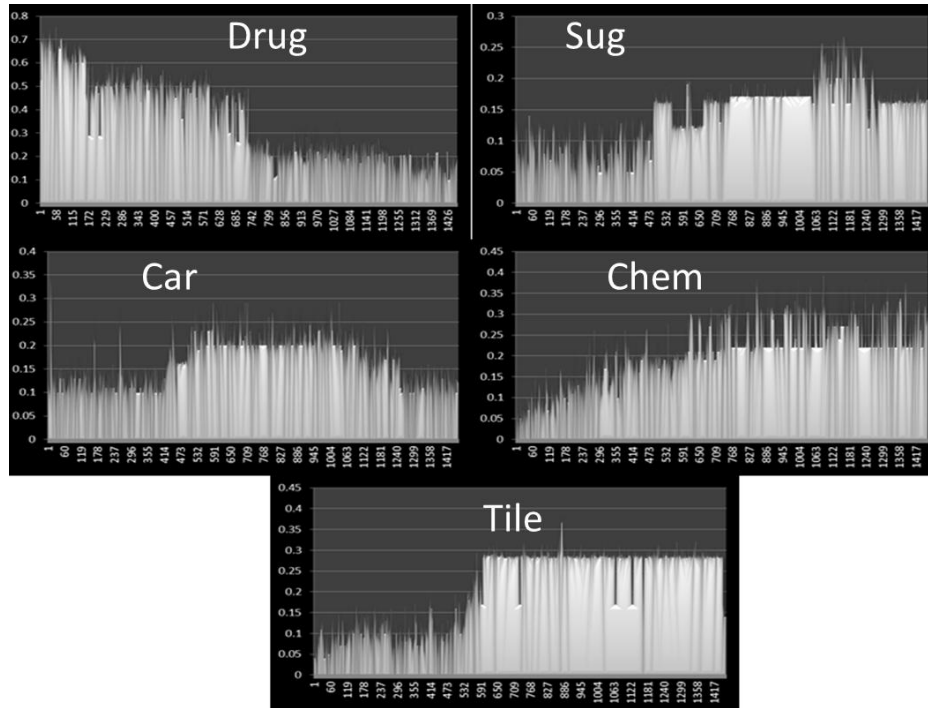


Fig. 5: Allocatio result in BEKK.

4. Vech Model Output:

$$C = \begin{pmatrix} 1.93E-07 & 1.73E-07 & 4.26E-07 & 1.95E-07 & 2.18E-07 \\ 0 & 2.02E-06 & 3.56E-07 & 6.69E-07 & 6.56E-07 \\ 0 & 0 & 7.04E-07 & 8.06E-08 & 1.93E-07 \\ 0 & 0 & 0 & 2.11E-05 & 1.29E-06 \\ 0 & 0 & 0 & 0 & 7.96E-06 \end{pmatrix}$$

$$A = \begin{pmatrix} 0.058777 & 0.037000 & 0.115456 & 0.126525 & 0.030766 \\ 0 & 0.103929 & 0.144664 & 0.157124 & 0.077822 \\ 0 & 0 & 0.475018 & 0.247175 & 0.074524 \\ 0 & 0 & 0 & 0.825788 & 0.123550 \\ 0 & 0 & 0 & 0 & 0.047708 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.946119 & 0.815702 & 0.389159 & 0.132790 & 0.816024 \\ 0 & 0.809668 & 0.495167 & -0.191691 & 0.488617 \\ 0 & 0 & 0.481539 & 0.145021 & 0.728275 \\ 0 & 0 & 0 & 0.001563 & 0.082144 \\ 0 & 0 & 0 & 0 & 0.640750 \end{pmatrix}$$

Allocation results are shown below:

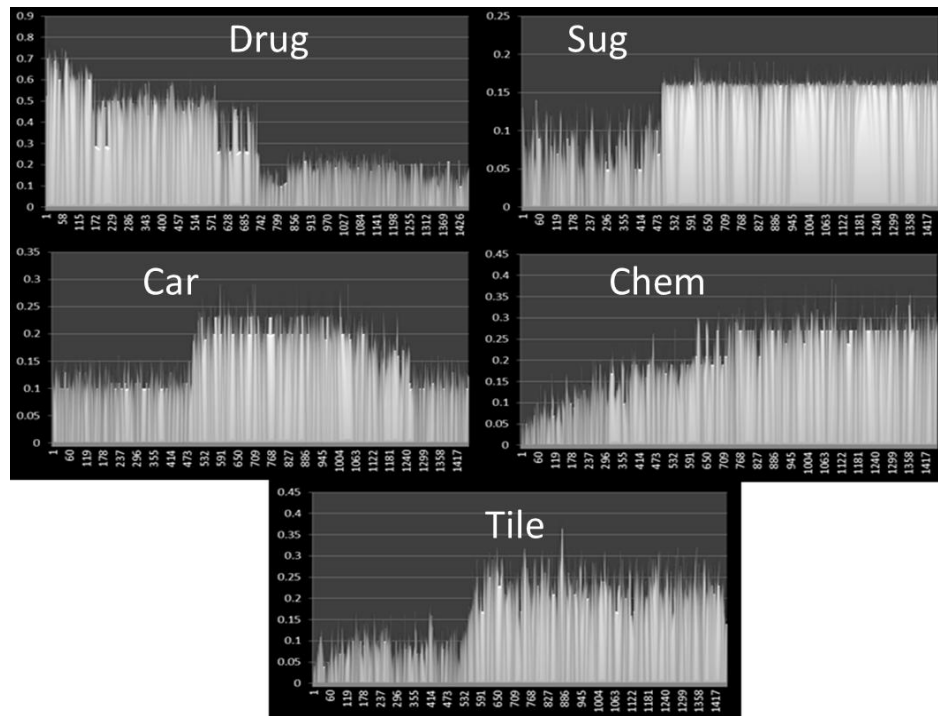


Fig. 6: Allocatio result in Vech.

- Mean optimum share of the industries in CCC:

Table 3: Mean optimum share of the industries in CCC.

Tile & Ceramics	chemical products	Automobile and part manufacturing	Suger	pharmaceutical materials and products
0.148718	0.208081	0.140758	0.132676	0.3847

- Mean optimum share of the industries in DCC:

Table 4: Mean optimum share of the industries in DCC.

Tile & Ceramics	chemical products	Automobile and part manufacturing	Suger	pharmaceutical materials and products
0.302818	0.13625	0.326732	0.142675	0.212544

- Mean optimum share of the industries in BEKK:

Table 5: Mean optimum share of the industries in BEKK.

Tile & Ceramics	chemical products	Automobile and part manufacturing	Suger	pharmaceutical materials and products
0.215759	0.220127	0.174077	0.146341	0.360295

- Mean optimum share of the industries in Vech:

Table 6: Mean optimum share of the industries in Vech.

Tile & Ceramics	chemical products	Automobile and part manufacturing	Suger	pharmaceutical materials and products
0.194078	0.227522	0.172182	0.141742	0.356944

Conclusion:

As the results imply, based on all four models, more weight is allocated to those industries experiencing lower volatilities in their rate of return. The results also show that during periods of high volatility, optimum share of the industry from portfolio decreases and inversely an increase in optimum share from portfolio relates to periods of low volatility. The results recommend investing in industries, where there is higher stability in stock prices and low volatility in returns, and give them a higher priority. It is also observed that there are many differences between the results allocated in DCC and those of CCC, BEKK and Vech models, which can be attributed to their different natures and software.

The results of this research indicate compatibility to portfolio risk minimization and thereby high accuracy of multivariate GARCH models in estimating dynamic covariance matrices. Since these models have been fitted to estimate conditional covariance matrix, we can hopefully rely on results of conditional covariance matrix predicted by these models to form an optimum portfolio investment.

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