On the Comparison of Supply Chain with Sub–Dmus in DEA

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INTRODUCTION

Among the various methods of assessment, Data Envelope Analysis (DEA) is widely used to measure the relative performance of a group of manufacturing processes called the decision making unit (DMU). Conventional DEA models make no assumption about the internal operations of DMUs; they only consider details of the main used inputs and the final outputs produced by DMU. So far, the idea of performance measurement of some sub – DMUs or elements has been discussed in different ways. For example, Charenz, et al., [1] have used two-stage model in military recruitment. Chen and Yan [2] have introduced a network DEA model. Seiford and Zhu, [3] have A DEA approach for evaluating the commercial banks in America, in a multi stage process described by the profitability and ability of supply. Zhu applied the same process for 500 companies of Fortune Global. And, Sexton and Lewis [4] have studied performance level of Major Baseball League in a two stage process and Seiford –Zhu [3] have used the standard DEA method at each stage. CAO – Wong, [5] have developed a different way in which a two stage process can be decomposed to create efficiency of two sub – processes. And thus they obtained the efficiency of both the overall and each stage. Also, the idea was studied by Amirteimoori et al., [6, 7]. In Section 2 of this paper we review the background of the concept. In section 3, the advantages and disadvantages of these two approaches are presented and finally conclusions and recommendations are discussed.

Efficiency Measurements of Multi-Component Dmus:

Assume a DMU$_p$ consists of b DMSU’s. Each DMSU$_i$ transforms resources, or inputs into products, or outputs. In particular, DMSU$_t$, (1 ≤ t ≤ b – 1) producestwo types of outputs: $Y_p^{(1)}$, $Y_p^{(2)}$. $Y_p^{(1)}$ goes on to outside of system and $Y_p^{(2)}$ is used as inputs to later DMSU$_{t+1}$. Also, DMSU$_1$ consumes $X_p^{t}$ to produce $Y_p^{(1)}$, $Y_p^{(2)}$, and DMSU$_b$, (2 ≤ t ≤ b) consumes two types of inputs: $X_p^{t}$, $Y_p^{(1-2)}$. The inputs $X_p^{t}$ coming from outside the whole of DMU$_p$ and $Y_p^{(1-2)}$ is the output produced by the DMSU$_{t-1}$. The production process for DMSU$_t$ is depicted in Fig. 1.

$Y_p^{(1)}$ the R$_{t1}$ dimensional vector of outputs included in the tth component of DMU$_p$, that goes on to outside of system. $Y_p^{(2)}$ the R$_{t2}$ dimensional vector of outputs included in the tth component of DMU$_p$ that is used as inputs of the (t + 1)th component. b number of DMSUs in DMU$_p$, $X_p^{t}$ the I$_t$ dimensional vector of inputs dedicated to the tth component of DMU$_p$. $Y_p^{(b)}$ the final output produced by DMU$_p$. $\mu^{(1)}$ vector of multipliers applied to outputs $Y_p^{(1)}$, $\mu^{(2)}$ vector of multipliers applied to outputs $Y_p^{(2)}$. $\mu^{(b)}$ vector of multipliers applied
to outputs $Y_p^{(b)}, V^{(s)}$ vector of multipliers applied to inputs $Y_p^{(1)}$. $W^{(s)}$ vector of multipliers applied to inputs $Y_p^{(2)}$.

From Fig. 1, we can conclude that since the outputs $Y_p^{(1)}$ and $Y_p^{(2)}$ are produced from the inputs $X_p^{(1)}$ and $Y_p^{(2)}$, the aggregate performance measure of DMU$_p$ is defined as follows:

$$e_p(z) = \frac{\sum_{i=1}^{b} \mu(t)^1 y_p^{(1)} + \sum_{i=t}^{b} \mu(t)^2 y_p^{(2)} y_p^{b} b}{\sum_{i=1}^{b} \sum_{i=t}^{b} w(t)^1 y_p^{(1)} + \sum_{i=t}^{b} w(t)^2 y_p^{(2)}} \quad p=1,\ldots,n$$

(1)

From this representation of $e_p(z)$, the specific-component measures $e_p^{(t)}$ are given by

$$e_p^{(1)} = \frac{\mu(t)^1 y_p^{(1)} + \mu(t)^2 y_p^{(2)}}{y_p^{(1)} x_p^{(1)}}$$

$$e_p^{(t)} = \frac{\mu(t)^1 y_p^{(1)} + \mu(t)^2 y_p^{(2)}}{y_p^{(t)} x_p^{(t)} + w(t)^1 y_p^{(2)}} \quad t=2,\ldots,b-1$$

(2)

$$e_p^{(b)} = \frac{\mu(b)^1 y_p^{(b)}}{y_p^{(b)} x_p^{(b)} + w(b)^2 y_p^{(b-1)}}$$

To derive $e_p^{(b)}, e_p^{(1)}, \ldots, e_p^{(a)}$, we use the following multi-component DEA model:

$$\text{Max} \ e_p^{(a)}$$

St: $e_p^{(t)} \leq 1 \quad t = 1,\ldots,b; \, j = 1,\ldots,n$

(3)

$$(\mu(t)^1, \mu(t)^2, \mu(b)) \in \Omega_1$$

$$e(t)^{(s)} \in \Omega_2$$

Efficiency Measurements of Two-Component Dmus In Supply Chain:

For the simplicity, consider a two stage supplier–manufacturer chain as following Fig. 2. Where, S and M represent the supplier and the manufacturer, respectively.

X is the input vector of supplier (S) and Y$^1$, Y$^2$ are its output vectors which are also input vectors to the manufacturerstage. Z$^1$ and Z$^2$ is the output vector corresponding to manufacturer (M1) and manufacturer (M2), respectively. Consider n same supply chains called Decision Making Units (DMUs) in DEA literatures, denoted by DMU$^1$, DMU$^2$, \ldots, DMU$^n$. Assume that supplier S and manufacturer M are under the decision of a DMU and supplier M2 is under the decision of another DMU. Yan and Chen proposed the following DEA model to measure the efficiency.

$$\text{Min} \theta$$

St: $\sum_{j=1}^{n} x_j \lambda_j^1 \leq \theta x_0$

$$\sum_{j=1}^{n} y_j^1 \lambda_j^1 \geq \sum_{j=1}^{n} y_j^1 \lambda_j^2$$

$$\sum_{j=1}^{n} y_j^1 \lambda_j^1 \geq \sum_{j=1}^{n} y_j^2 \lambda_j^3$$

(4)

$$\sum_{j=1}^{n} y_j^1 \lambda_j^2 \leq y_j^1$$

$$\sum_{j=1}^{n} z_j^1 \lambda_j^2 \geq z_j^1$$

$$\sum_{j=1}^{n} z_j^2 \lambda_j^3 \geq z_j^2$$

$$\lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0 \quad j=1,\ldots,n$$

Comparison of the Advantages and Disadvantages of the Proposed Models in A Complex Dmus:

In traditional DEA models, each DMU is considered as a black box, which, as mentioned earlier, only suffices to the consumed input and the final output for evaluation and efficiency can be calculated by many
models such as SBM [8, 9]. But in the real world, researchers have dealt with a lot of instances suppose which DMU structure is not as simple as assumptions in models. So they tried to fix the problem. As we see, despite all attempts to solve this problem, the proposed solutions have many defects and the models discussed in this article are not accepted.

In Amirteimoori et al. [6], the efficiency is calculated using the help of $\mu^{(1)}$, $\mu^{(2)}$, $\nu^{(b)}$, $w^{(t)}$; since the model has no way to obtain these coefficients, they are applied based on decision maker’s opinions that affect final efficiency. But, $e_p^{(b)}$, $e_p^{(1)}$, .......... $e_p^{(b)}$ give us the efficiency of each sub – component, which having them makes it easier to identify weaknesses in the system's performance in each area.

Also, based on proven theorem, $e_p^{(b)}$ is a complex combination of $e_p^{(i)}$. Given that, using b available components, many convex combinations can be written that $e_p^{(b)}$ is obtained, it cannot be concluded which of these compounds can be Indicative of the effect of each component in unit performance. In the other hand, it is possible that obtained $e_p^{(b)}$ is not relevant to obtained efficiency from DMUs regardless of the sub – components. In the model proposed by Chen and Yan sub – components are not calculated, so the model of Amirteimoori is preponderant. Also, in calculation of overall efficiency of process, given that $Y^1, Y^2$ are considered outputs once they are considered inputs before or vice versa, while in the DEA, the desired goal is to have maximum outputs with minimum inputs. So, in this case we have a conceptual paradox. Additionally, considering three $\lambda_j$ (i.e., $\lambda^{(1)}, \lambda^{(2)}, \lambda^{(3)}$) increases dimensions of the problem dramatically and makes it computationally complex. Of the benefits of above method is obtaining $\theta$ in a logical progression and without personal judgment.

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**Fig. 1:** Production process for a DMU

Conclusions and Recommendations:

Until now, many methods have been proposed to calculate the efficiency of complex DMUs, but with all the advantages they are with numerous disadvantages; this causes for every application of the proposed methods is preponderant. Further, effective researches regarding to returns to scale and ranking DMUs have not been done. And, the way for comprehensive researches in this area is open.

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REFERENCES


