Measurements and Factors Analysis of Fuzzy Numbers: A Case Study Involvinga Clinical Effect

R. Saneifard and K.H. Kamyab

Department of Mathematics, Urmia Branch, Islamic Azad University, Urmia, Iran

**INTRODUCTION**

Defutification plays a central role in the implementation of fuzzy system’s modeling techniques, such as those used in fuzzy logic controllers [6,7,12]. Defuzzification is essentially a process where guided by the output fuzzy subset of the model. One selects a single crisp value as the system output. In fuzzy logic controller applications the typical methods used for this process are the center of area(COA) method, and the mean of maximal (MOM) method. Both of these methods can be seen to be based on a weighted type aggregation which can be seen as a blending or mixing of different solutions. In [10], Filev and Yager show that these are essentially the same approaches, distinguished only by the choice of a parameter. While in many cases these types of blending methods work well, examples have been suggested in the literature which lead to very unsatisfactory solutions when we use these kinds of blending methods [11,12,13]. The problems that usually arise are rooted in situations where two or more distinct modes of solution exist and the combining (blending) of these distinct modes leads to an unsatisfactory conclusion. For example, if two roads exist from point A to point B, and if we try to combine these two roads, we may end up traveling through the woods. A central issue that must be addressed is, when can we and when cannot combine solutions. In this note we attempt to suggest an approach to the defuzzification process which can avoid some of the pitfalls that have been shown to exist in many defuzzification methods. We note that in [12], we have looked at a closely related issue that of defuzzification under constraints. Today, most applications use either mean of maxima(MOM) defuzzification or centroid (COG) defuzzification method. These two algorithms are well known and used in many implementations. They are working well in many fields, but cannot be regarded as general-purpose algorithms [11]. Having reviewed the previous concepts, this article proposes here a method to use the concept of (RMS) root mean square, so as to find the order of fuzzy numbers. This method can distinguish the alternatives clearly. The main purpose of this article is that, (RMS) can be used as a crisp approximation of a fuzzy number. Therefore, by the means of this defuzzification, this article aims to present a new method for ranking of fuzzy numbers. In addition to its ranking features, this method removes the ambiguities resulted and overcome the shortcomings from the comparison of previous ranking.

2 Basic Definitions and Notations:

The basic definitions of a fuzzy number are given in [1,2,3,4,5,6] as follows:

**Definition 1:**

A fuzzy number \( \mathbf{A} \) is a mapping \( \mu(x): \mathbb{R} \rightarrow [0,1] \) with the following properties:
1. $\mu$ is an upper semi-continuous function on $\mathbb{R}$.
2. $\mu(x) = 0$ outside of some interval $[a_2, b_2] \subset \mathbb{R}$.
3. There are real numbers $a_2, b_1$ such as $a_1 \leq a_2 \leq b_1 \leq b_2$ and
   - $\mu(x)$ is a monotonic increasing function on $[a_1, a_2]$,
   - $\mu(x)$ is a monotonic decreasing function on $[b_1, b_2]$,
   - $\mu(x) = 1$ for all $x$ in $[a_2, b_1]$.

Let $\mathbb{R}$ be the set of all real numbers. The researchers assume a fuzzy number $A$ that can be expressed for all $x \in \mathbb{R}$ in the form

$$
\mu(x) = \begin{cases} 
  g(x), & \text{when } a \leq x \leq b, \\
  1, & \text{when } b \leq x \leq c, \\
  h(x), & \text{when } c \leq x \leq d, \\
  0, & \text{otherwise,}
\end{cases}
$$

(1)

where $a, b, c, d$ are real numbers such as $a < b \leq c < d$ and $g$ is a real valued function that is increasing and right continuous and $h$ is a real valued function that is decreasing and left continuous.

Support function is defined as follows:

$$
\text{supp}(A) = \{ x | \mu(x) > 0 \},
$$

where $\{ x | \mu(x) > 0 \}$ is closure of set $\{ x | \mu(x) > 0 \}$.

3 Proposed RMS defuzzification algorithms:

Many defuzzifiers have been proposed so far. But no one method gives a right effective defuzzified output because each method gives different results. This paper introduces new defuzzification algorithms based on RMS value, since RMS always leads to an effective value. This word effective is taken from electrical engineering where RMS is always referred to as an effective value.

3.1 RMS Algorithm 1. [9]:

The root mean square (RMS) of a variate $X$ of the mean squared value of $X$ is the squared value of $X$, given by the following algebraic expression for an input ($X$) varying function:

$$
\text{RMS}_1 = \sqrt{\frac{\int_A^B (f(\mu(x)))^2 \, dx}{B-A}}.
$$

(2)

$A$ and $B$ are the lower and upper limits of the function and $f$ represents the aggregated membership function and $\mu(x)$ is the degree of membership. The $\text{RMS}_1$ value, which is per unit, depends on the interval chosen. If the values of $A$ and $B$ are changed the $\text{RMS}$ value of the function across the interval from $A$ to $B$ will also change. In electrical engineering, this definition is invariably used for a periodic function.

3.2 RMS Algorithm 2. [9]:

The $\text{RMS}$ of a variate $X$ of the mean squared value of $X$ is the squared value of $X$ given by the following algebraic expression:

$$
\text{RMS}_2 = \sqrt{\frac{\int_A^B f(\mu(x))x^2 \, dx}{\int_A^B f(\mu(x)) \, dx}}.
$$

(3)

$f$ represents the aggregated membership function and $\mu(x)$ is the degree of membership. This function calculates the RMS value based on the area under the membership function $f(x)$.

4 A New Method For Ranking Fuzzy Numbers:

In this section, we present a new approach for ranking fuzzy numbers based on the distance method. The method not only considers the RMS of a fuzzy number, but also considers the minimum crisp value of fuzzy numbers.

For ranking fuzzy numbers, this study firstly defines a minimum crisp value $\mu_{\text{min}}(X)$ to be the benchmark and its characteristic function $\mu_{\text{min}}(x)$ is as follows:
When ranking \( n \) fuzzy numbers \( A_1, A_2, \ldots, A_n \) the minimum crisp value \( \mu_{\tau_{\min}} \) is defined as:

\[
\mu_{\tau_{\min}}(x) = \begin{cases} 
1 & \text{when } x = \tau_{\min}, \\
0 & \text{when } x \neq \tau_{\min}.
\end{cases}
\]

When ranking \( n \) fuzzy numbers \( A_1, A_2, \ldots, A_n \) the minimum crisp value \( \mu_{\tau_{\min}} \) is defined as:

\[
\tau_{\min} = \min\{x \mid x \in \text{Domain}(A_1, A_2, \ldots, A_n)\}.
\]

The advantages of the definition of minimum crisp value are two-fold: firstly, the minimum crisp values will be obtained by themselves, and another is it is easy to compute.

Assume that there are \( n \) fuzzy numbers \( A_1, A_2, \ldots, A_n \). The proposed method for ranking fuzzy numbers \( A_1, A_2, \ldots, A_n \) is now presented as follows:

**Step 1.** Use formulas (2) and (3) to calculate the \( \text{RMS} \) of each fuzzy numbers \( A_j \), where \( 1 \leq j \leq n \).

**Step 2.** Calculate the maximum crisp value \( \tau_{\min} \) of all fuzzy numbers \( A_j \), where \( 1 \leq j \leq n \).

**Step 3.** Use the point \( \text{RMS} \) to calculate the ranking value \( d_{\text{RMS}}(A_j, \tau_{\min}) \) of the fuzzy numbers \( A_j \), where \( A_j \), where \( 1 \leq j \leq n \), as follows:

\[
d_{\text{RMS}}(A_j, \tau_{\min}) = \left| \text{RMS}(A_j) - \tau_{\min} \right|.
\]

From equation (6), we can see that \( d_{\text{RMS}}(A_j, \tau_{\min}) \) can be considered as the Euclidean distance between the point \( (\text{RMS}(A_j), 0) \) and the point \( (\tau_{\min}, 0) \). We can see that the bigger the value of \( d_{\text{RMS}}(A_j, \tau_{\min}) \), the better the ranking of \( A_j \), where \( 1 \leq j \leq n \).

Let \( A_j \) is a fuzzy number characterized by (1) and \( d_{\text{RMS}}(A_j, \tau_{\min}) \) is the Euclidean distance between the point \( (\text{RMM}(A_j), 0) \) and the point \( (\tau_{\min}, 0) \) of its.

Since this article wants to approximate a fuzzy number by a scalar value, thus the researchers have to use an operator \( d_{\text{RMS}} : F \to R \) (A space of all fuzzy numbers will be denoted by \( F \) which transforms fuzzy numbers into a family of real line. Operator \( d_{\text{RMS}} \) is a crisp approximation operator. Since ever above defuzzification can be used as a crisp approximation of a fuzzy number, therefore the resultant value is used to rank the fuzzy numbers. Thus, \( d_{\text{RMS}} \) is used to rank fuzzy numbers.

Let \( A_1, A_2 \in F \) be two arbitrary fuzzy numbers. Define the ranking of \( A_1 \) and \( A_2 \) by \( d_{\text{RMS}} \) on \( F \) as follows:

1. \( d_{\text{RMS}}(A_1, \tau_{\min}) < d_{\text{RMS}}(A_2, \tau_{\min}) \) if only if \( A_1 < A_2 \).
2. \( d_{\text{RMS}}(A_1, \tau_{\min}) > d_{\text{RMS}}(A_2, \tau_{\min}) \) if only if \( A_1 > A_2 \).
3. \( d_{\text{RMS}}(A_1, \tau_{\min}) = d_{\text{RMS}}(A_2, \tau_{\min}) \) if only if \( A_1 \sim A_2 \).

Then, this article formulates the order \( \succeq \) and \( \preceq \) as \( A_1 \succeq A_2 \) if and only if \( A_1 > A_2 \) or \( A_1 \sim A_2 \) if and only if \( A_1 < A_2 \) or \( A_1 \sim A_2 \). The new ranking index can sort many different fuzzy numbers simultaneously. In addition, the calculation is simple, and the index also satisfies the common properties of ranking fuzzy numbers:

a) Transitivity of the order relation, i.e. if \( A_1 \leq A_2 \) and \( A_2 \leq A_3 \), then we should have \( A_1 \leq A_3 \).

b) Compatibility of addition, that is if there is \( A_1 \leq A_2 \) on \( [A_1, A_2] \), then there is \( A_1 + A_3 \leq A_2 + A_3 \) on \( [A_1 + A_3, A_2 + A_3] \).

**Remark 3.1.** If \( A \preceq B \), then \( -A \succeq -B \).

Hence, this article can infer ranking order of the images of the fuzzy numbers.

### 4 Case clinical effect:

Based on the appraisal system of the Department of Healthcare of Iran, healthcare institutes are categorized into medical centers and regional hospitals, district hospitals, and primary clinics. District hospitals are responsible for providing primary healthcare services, similar to those offered by primary practitioners in clinics offering outpatient services. Regional hospitals have a minimum of 250 acute beds, are staffed with physicians of various specialties, and provide healthcare services and specialist training. Containing over 500 acute staffed beds, medical centers are delegated responsibility for providing healthcare services, training medical professionals, and conducting research. Since higher accredited hospitals such as medical centers and regional...
hospitals are large scale hospitals with advanced medical equipment, patients with severe or complex conditions are normally drawn to such facilities. However, all patients in Iran can freely access any health care provider of their choice, unlike other countries, which often limit patient choice regarding providers, and which may confound the relationship between provider volumes and patient outcomes. Thus, Iranians hospitals not merely compete with other hospitals in the same category for inpatients, but also with clinics for outpatient services and referral system never prevailed till now. The healthcare institution surveyed in this study is a teaching medical center that accredited as a regional teaching hospital in August 2006, and as a teaching hospital in January 2008 by the Department of Healthcare of Iran. Meanwhile, the case hospital also achieved Joint Commission International Accreditation in November 2008, and is the first JCI accredited hospital in south Iran. Notably, the Joint Commission on Accreditation of Health care Organizations founded in 1951, is an independent, not-for-profit organization that surveys over 15,000 health care programs via a voluntary accreditation process in the United States. JCI has accredited over 220 public and private health care organizations in 33 countries. Owned and operated by the medical school of a university in Tehran, the case study hospital provides a full spectrum of inpatient, outpatient and emergency services and wellness programs. Most attending doctors come from major medical centers in Iran, and can provide quality care in their medical specialties. The case study hospital features a 24-h Emergency Department, 29 operating rooms and six intensive care units equipped with state-of-the-art equipment including PET/CT, Gamma Knife, 64-Slice CT and MRI, and provide customers with high quality medical services. Currently, over 950 nurses and 230 doctors are employed to serve their patients. Additionally, the clinic is fitted with 1,199 beds, including 925 acute beds and 274 special beds. To identify the elements in the RMS process, this study used a questionnaire to identify the necessary activities. Answers to items for each element in the RMS process were presented on a five-point scale for levels of satisfaction and importance, and ranked from 5, indicating “very satisfied” and “very important” to 1, indicating “dissatisfied” and “not very important”. The initial questionnaire draft contained 20 items related to the RMS process for the case hospital. However, three items were eliminated due to their insignificant loading coefficients being statistically tested by factor analysis. Each of the remaining 17 items was modified to fit the current operating status of the case hospital. A total of 155 nurses at the case hospital were surveyed, and 137 valid responses were obtained following careful examination by the research team, which comprised one academic scholar, one information personnel and three administrators, who were mainly responsible for RMS activities in the case hospital. Table 1 in [11] lists the results of factor analysis. The internal consistency of the scales was examined using Cronbach’s α coefficient to assess reliability. In this study, Cronbach’s α for each subscale was 0.807–0.855; thus, the questionnaire had internal consistency. Additionally, the questionnaire contents were established using the studies of [6]; and on interviews with academics, nursing and RMS personnel at the case study hospital. Thus, questionnaire content validity was adequate and satisfactory. Furthermore, factor analysis shows that the questionnaire items can be grouped into four clusters, each comprising a set of items that are highly correlated, but only minimally associated with the remaining items. The four clusters, namely, knowledge identification, knowledge collection, knowledge organization, and knowledge sharing and creation, can help RMS practitioners choose the right direction when developing operational activities for the RMS program. Table 2 in [11] demonstrates the reliability analysis of the RMS process constructs. To ensure the face validity of the questionnaire, initial scales were taken from previously validated measures in the literature whenever possible, and reworded to relate to the study context. Based on the traditional QFD technique [11], the RMS program is deployed via the following steps: determining and measuring elements in the RMS process; evaluating RMS enablers; assessing the interrelationship matrix between elements in the RMS process and RMS enablers; targeting each RMS enabler; and, prioritizing and selecting RMS enablers.

**Stage 1: Determine the elements of the RMS process.**

The necessary elements of RMS process, statistically supported by factor analysis, are listed in Table 1 in [11], as shown in the previous section. This process consists of creating, identifying, collecting, adapting, organizing, applying, and sharing. Notably, APQC has studied and implemented RMS approaches for over a decade, and has learned how to implement best practices from hundreds of organizations that participated in RMS studies. Furthermore, the RMS process developed by [11] is geared towards constant improvements in performance, including streamlining management practices by institutionalizing RMS or systematically blending the RMS process with normal work processes. In light of the managerial relevance of research to the interests and concerns of practitioners or managers, this study uses the RMS process generated by [11]. According to the discussion with RMS personnel employed in the case company, the RMS process consists of different activities that can be further grouped into knowledge identification, knowledge collection, knowledge organization, and knowledge sharing and creation. These four main tasks are covered by the daily RMS activities suggested by [11] and common to most RMS process models in literature.

**Stage 2: Record the importance and satisfaction levels of elements on the RMS process.**

Table 3 in [11] lists the results of average importance and satisfaction for each question, and tested significance differences between levels of importance and satisfaction. With no exceptions, the mean satisfaction levels for each question were significantly smaller than the mean of importance. This analytical
result suggests that the case study hospital should try to improve employee satisfaction levels and reduce the gap between importance and satisfaction levels of elements in the RMS process.

**Stage 3**: Determine the items of RMS enablers.

After examining the elements in the RMS process with QFD team members, and based on discussions with RMS enablers in the literature [11], 19 items of RMS enablers were derived that facilitate the RMS program at the case hospital. Table 4 in [11] lists the details of items of RMS enablers.

**Stage 4**: Calculate the priorities of elements in the RMS process.

In this stage, priorities are assigned to elements in the RMS process by considering their average degrees of importance and satisfaction. Let $X_i$ and $Y_i$, $i = 1, 2, \ldots, n$, denote the average degree of importance and satisfaction for elements in RMS process $E_i$. Since the priorities of elements in the RMS process are directly related to degree of importance and inversely related to degree of satisfaction, the original priority rating $w_i$ of $E_i$, according to [11], can be obtained using $w_i = X_i(5 - Y_i)$.

**Stage 5**: Construct an interrelationship matrix and determine the fuzzy relationship strength.

Typically, the strength of relationships in the interrelationship matrix of the HOQ is assessed subjectively by an evaluation team with expertise in the relevant subject areas [11]. From discussions with QFD team members in this study, the team constructed an interrelationship matrix linking the elements of the RMS process and items of RMS enablers. The correlation strengths in the interrelationship matrix are expressed in linguistic terms, and a pre-defined fuzzy relationship degree set is used to evaluate the fuzzy strength of the relationship between elements in the RMS process and items for RMS enablers.

**Stage 6**: Prioritize RMS enabler items.

In the final stage, the priorities of the items of RMS enablers are assessed using Eq. (2). Notably, a different optimistic index ($\alpha$) in the fuzzy QFD creates different ranking out comes that reflect the degree of optimism. The outputs of FQFD can then be used as a prerequisite for many RMS enablers of the case hospital in planning the RMS program, and are used to assess how effectively items of RMS enablers facilitate RMS process elements.

5 **Analysis and implications:**

Analytical results shown in Table 5 in [11] indicate that items of RMS enablers for the case hospital exhibit the following ranking: first, evaluating leadership performance by feedback and survey data from nursing staff (VOE4); second, top management setting specific action plans for successful implementation of strategic objectives (VOE1); third, creating a pay-for-knowledge system that emphasizes the sharing of knowledge (VOE18); and fourth, having clear rules for formatting or categorizing hospital clinical nursing process knowledge (VOE12). The top four VOEs are further detailed below.

5.1 **Leadership enabler:**

In light of leadership, top executives of case hospitals should carefully define their mission, vision, strategy and the objectives of their RMS initiative. The classical terms of 5W1H in business administration represent useful starting points in assisting top executives in achieving the above. For example, typical RMS mission and vision statements may be useful in explaining why RMS is important for the case hospital in its current and future competitive environment, and what the case hospital is striving for in the long term using their RMS initiative. Similarly, the RMS strategy may be useful for defining the steps and procedures involved in how to become a knowledgeable healthcare institute. This task is not a one-off exercise, but rather involves an iterative process. The defined strategy and aims must be reviewed in the light of future market requirements, the development of knowledge in the respective areas inside and outside the organization and the results of the ongoing RMS initiative. According to the outcomes from the proposed FQFD for RMS, not only should top management regularly review these subjects, but nursing staff should also periodically return feedback to top management. The scenario is reflected by VOE1 and VOE4, respectively.

5.2 **Measurement enabler:**

Meanwhile, the FQFD results for the case hospital also show that a pay-for-knowledge or reward system, an item of the enabler in the measurement category (VOE18), is a key issue when implementing RMS programs. Knowledge sharing is crucial in successful RMS. Currently, only informal monetary reward and incentive are attached to knowledge sharing in the case hospital. Consequently, a formal reward system for information sharing is required, since knowledge sharing is not a common phenomenon. No one wants to share knowledge when there is no benefit associated with doing so. In fact, individuals become motivated to share knowledge only when rewarded [11]. Pfuger et al., stated that knowledge sharing should be encouraged by rewarding workers via a formal incentive system that includes monetary and reputation rewards. Failing to share valuable information can have a number of negative consequences. Notably, information may be lost, or may be possessed by a single individual whose contribution may be lost as a result of disability or retirement.
Additional time and expense may be required to extract information, creating inefficiencies and unnecessary delays. Furthermore, the case study hospital must recognize that it cannot always afford to retain nurses who sometimes work overtime and usually have to work in shifts. Thus, the hospital should seek to retain the knowledge nurses have gained while on clinical practices. Therefore, the case hospital must offer adequate reward programs that motivate nurses to share their knowledge with others in the hospital. The management of the case hospital also must realize that a poorly structured reward system can cause information overload—knowledge gathering activities are frequently not accompanied by clear policies regarding what knowledge to capture and when or how to update it. Without clarification of the benefits and objectives of information and knowledge gathering, a user can face a frustrating and bewildering range of choices, rather than experiencing a streamlined and rapid search for relevant knowledge. Therefore, a reward system that recognizes the importance of information contributions in determining employee bonuses but does not evaluate those contributions is likely to result in a database full of low-quality lengthy reports that offer limited benefit. To resolve the problem of inappropriate reward for knowledge gathering and sharing, [11] stress the link between RM Sand quality management with a particular focus on the role of quality culture. As long as quality information is strongly favored, information overload is unlikely.

5.3 Technology enabler:

According to [11], nurses and patients interact in the process of nursing practices, resulting in nurses identifying new knowledge regarding symptoms, changed conditions, and other issues. However, the response from nurses as a result of this interaction is usually a standard set of clinical activities such as diagnostic tests, medical procedures or clinical interventions. Standardized clinical activities sharpen the need of the case hospital for clear rules for formatting or categorizing hospital clinical nursing process knowledge by utilizing technical systems within the health institute. Generally, technology focuses on information technology support within an organization[11]. According to [11], information technology used by knowledge management enablers mainly refers to fundamental building blocks of IT, which can enable rapid search, access, and retrieval of information for different users. Consequently, the key role of information technology lies in its ability to support communication between organizational members, collaboration and the search for knowledge, and enable collaborative learning. Currently, high care standard is achieved through the more effective use of past experience and experimentation with new technology. For example, a recent and revolutionary method of knowledge sharing is the creation of online forums within the medical community. Nurses are frequently assigned to one of three shifts, working 24-h rotations. Traditionally, nurses share their clinical knowledge with other clinical professionals in multiple ways, such as sharing information during end of shift meetings and rounds, or during patient charting. With online forums, an IT application, case hospital nurses can easily share their experience and knowledge with others when off work. Thus, the itemVOE12 in the measurement enabler category reveals the importance of measurement enabler in implementing the RMS program set by the case hospital.

5.4 Culture enabler:

As for the priority of RMS enablers, the most significant among the four enablers is the factor of measurement. Other than that, the relative importance of these RMS enablers to the case hospital follows the order, from highest to lowest, leadership, information technology, and organizational culture. The culture enabler is less important than the other enablers, because QFD team members believe the case hospital has already implicitly encouraged their health practitioners to work closely with other units to provide satisfactory healthcare for patients. Clearly, even simple surgery may involve people from surgical, medical, anesthesia, and orthopedics departments working together to cure a patient. Thus, it is natural for healthcare practitioners like nurses to discuss their work with people in other workgroups of the hospital. However, according to analysis from the FQFD, the case hospital must still encourage nurses to explore and learn new nursing ideas and techniques externally, indicating the importance of organizational culture enabler in implementing the RMS program. The analytical results of the FQFD indicate that the top four items of the RMS enablers must be established and reinforced first to facilitate elements of the RMS process for the case hospital implementing its own RMS program. Meanwhile, the analytical results of fuzzy QFD for the case hospital also show that the order of priority of the RMS enablers for the case hospital, shown in Table 6 in [7,8,11], is the first measurement, followed by leadership, then technology and finally culture. This study also demonstrates that deployment of RMS activities via QFD with fuzzy logic helps practitioners prioritize the needed RMS enablers given limited resources.

6. Conclusion:

In this paper, the researchers proposed a defuzzification using RMS of the fuzzy number and by using this defuzzification, we proposed a method for ranking of fuzzy numbers. Roughly, there not much difference in our method and theirs. The method can effectively rank various fuzzy numbers and their images.
REFERENCES