### Measurement of Returns To Scale for DEA-based Environmental Assessment

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<table>
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<th>Article Info</th>
<th>Abstract</th>
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<td>Article history:</td>
<td>Data envelopment analysis (DEA) is a branch of management concerned with evaluating the performances of homogeneous decision making units (DMUs). In environmental evaluation, undesirable outputs have to be considered along with desirable outputs. Today, climate change, an increase in the amount of CO₂ emission, and water pollution are major problems all over the world. These worldwide problems indicate the importance of developing firms with less undesirable outputs. This study explores the measurement of returns to scale for environmental issues which have not been given the deserved attention in the last few decades. Corresponding to SE (Scale Economies) and RTS (Returns to Scale) for desirable outputs, the new concepts of ESE (Environmental Scale Economies) and ERTS (Environmental Returns to Scale) are extended for both desirable and undesirable outputs. Applying these concepts in actual applications, this study selects the data set associated with the industry in 31 administrative regions of China. We therefore determine the type of ERTS for efficient industries and suggest increasing, decreasing or maintaining their size.</td>
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<td>INTRODUCTION</td>
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<td>Data envelopment analysis (DEA) is a branch of management concerned with evaluating the performances of homogeneous decision making units (DMUs). It is known that increases in desirable outputs result in an increase in the efficiency. However, in environmental assessment, undesirable outputs have to be considered along with desirable outputs. Therefore, a firm’s provision of undesirable outputs penalizes it and influences its efficiency. Today, climate change, increased emission of CO₂, and water pollution are major problems all over the world. These worldwide problems indicate the importance of developing firms with less undesirable outputs.</td>
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<td>Available online 14 November 2013</td>
<td>Fortunately, there have been some valuable studies for dealing with undesirable outputs within the framework of DEA. For instance, Cooper et al. [2], Dyckhoff and Allen [3], Korhonen and Luptacki [8], Trianits and Otis [16], Zhou and Ang [19], Sueyoshi and Goto [13, 15], Fare et al. [4, 5], Hailu and Veeman [7], Kousmanen [9], Kousmanen and Podinovski [10], and many other studies, to name a few.</td>
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<td>Key words: Data Envelopment Analysis, Environmental performance, Returns to scale, Undesirable output</td>
<td>Acknowledging the valuable contribution of previous DEA researches, this study mentions that all the previous studies, except Sueyoshi and Goto [14], did not discuss how to determine the type of returns to scale in the presence of undesirable outputs. Sueyoshi and Goto [14] studied RTS and SE (Scale Economies) from environmental performance. Corresponding to RTS and SE on desirable outputs, they introduced the new concepts of DTS (Damages to Scale) and SD (Scale Damages) to undesirable outputs. Then, they combined the two concepts of RTS and DTS in a unified treatment and introduced the concepts of RTS unified (RTSU) and DTS unified (DTSU).</td>
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This study explores how to measure the type of returns to scale in the presence of undesirable outputs. Here, we employ the linear unified efficiency measure for evaluating environmental issues proposed by Zare Haghighi et al. [18] for environmental assessment. We introduce the new concept of ESE (Environmental Scale Economies) dealing with both desirable and undesirable outputs. This concept corresponds to SE for desirable outputs. Using this new concept, we determine the type of RTS for environmental issues and call it the Environmental Returns to Scale (ERTS). |

This paper is arranged as follows: In section 2, we review the linear unified efficiency measure presented by Zare Haghighi et al. [18] for evaluating environmental issues. In section 3, the concept of SE is described. |

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and then extended to the new concept of ESE for environmental issues. In section 4, we explain how to determine the type of ERTS, using the ESE measure. Applying these concepts in actual applications, this study selects the data set concerned with the industry in 31 administrative regions of China. The results are shown in section 5. Section 6, provides the summary and conclusions of the study.

Preliminaries:

In this paper, it is assumed that there are n observed DMUs (Decision Making Units) and the jth DMU, \( j \in \{1, \ldots, n\} \), is determined by the vector \( (x_j, g_j, b_j) \), where \( x_j = (x_{1j}, x_{2j}, \ldots, x_{mj}) \in \mathbb{R}^m \), \( x_j \geq 0, x_j \neq 0 \) is the vector of inputs, \( g_j = (g_{1j}, g_{2j}, \ldots, g_{sj}) \in \mathbb{R}^s \), \( g_j \geq 0, g_j \neq 0 \) is the vector of desirable (good) outputs and \( b_j = (b_{1j}, b_{2j}, \ldots, b_{hj}) \in \mathbb{R}^h \), \( b_j \geq 0, b_j \neq 0 \) is the vector of undesirable (bad) outputs. Here, we employ the extended version of Range-Adjusted Measure (RAM) DEA model which was presented by Zare Haghighi et al. [18], and incorporates both desirable and undesirable outputs in a unified manner. The model we use is as follows:

\[
\begin{align*}
\text{Max} & \quad \sum_{i=1}^{m} R_i^e (s_i^{x_i} - s_i^{x_i}^-) + \sum_{r=1}^{s} R_r^e s_r^+ + \sum_{f=1}^{h} R_f^b s_f^-

\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{x_i} - s_i^{x_i}^- = x_{i_k} \quad (i = 1, \ldots, m), \\
& \quad \sum_{j=1}^{n} \lambda_j g_{rj} - s_r^+ = g_{r_k} \quad (r = 1, \ldots, s), \\
& \quad \sum_{j=1}^{n} \lambda_j b_{fj} - s_f^- = b_{f_k} \quad (f = 1, \ldots, h), \\
& \quad \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0 \quad (j = 1, \ldots, n), \\
& \quad s_i^{x_i}^- \geq 0, \quad s_i^{x_i}^+ \geq 0 \quad (i = 1, \ldots, m), \\
& \quad s_r^+ \geq 0, \quad s_f^- \geq 0 \quad (f = 1, \ldots, h).
\end{align*}
\]

In this model, \( \lambda_j \) is the jth structural variable corresponding to the jth DMU. Also, \( s_i^{x_i}^- \) and \( s_i^{x_i}^+ \) are the slack variables for the decrease and increase in the ith input, respectively. All through this paper, the fixed ranges of \( R_i^e \) \((i = 1, \ldots, m)\), \( R_r^e \) \((r = 1, \ldots, s)\), and \( R_f^b \) \((f = 1, \ldots, h)\) are determined as follows:

\[
\begin{align*}
R_i^e &= 1/[(m + s + h)(\bar{g}_i - \bar{x}_{i})] \quad (i = 1, \ldots, m), \\
R_r^e &= 1/[(m + s + h)(\bar{g}_r - \bar{g}_{r})] \quad (r = 1, \ldots, s), \\
R_f^b &= 1/[(m + s + h)(\bar{b}_f - \bar{b}_{f})] \quad (f = 1, \ldots, h).
\end{align*}
\]

Where \( \bar{x}_j = \max_j x_{ij} \), \( \bar{g}_r = \max_j g_{rj} \), \( \bar{b}_f = \max_j b_{fj} \), \( x_{i_k} = \min_j x_{ij} \), \( g_{r_k} = \min_j g_{rj} \) and \( b_{f_k} = \min_j b_{fj} \). After finding an optimal solution of model (1), the unified efficiency score of the kth DMU is measured by:

\[
\theta^* = 1 - \left( \sum_{i=1}^{m} R_i^e (s_i^{x_i}^- + s_i^{x_i}^+) + \sum_{r=1}^{s} R_r^e s_r^+ + \sum_{f=1}^{h} R_f^b s_f^- \right)
\]

Here, the superscript (*) means optimal value. The dual model associated with model (1) is as follows:
\text{Min} \quad \sum_{i=1}^{m} v_i x_d - \sum_{r=1}^{s} u_r g_d + \sum_{f=1}^{h} w_f b_f + \sigma \\
\text{s.t.} \quad \sum_{i=1}^{m} v_i x_d - \sum_{r=1}^{s} u_r g_d + \sum_{f=1}^{h} w_f b_f + \sigma \geq 0 \quad (j=1,...,n), \\
v_i \geq R_i^+, \quad -v_i \geq -R_i^+ \quad (i=1,...,m), \\
u_r \geq R_r^+ \quad (r=1,...,s), \\
w_f \geq R_f^+ \quad (f=1,...,h).

Here, \( v_i \) \((i=1,...,m)\), \( u_r \) \((r=1,...,s)\), and \( w_f \) \((f=1,...,h)\) are the dual variables corresponding to the first, second and third group of constraints in model (1), respectively. Also, \( \sigma \) is the dual variable corresponding to the constraint \( \sum_{j=1}^{n} \lambda_j = 1 \).

In model (2), there are two groups of constraints which indicate \( v_i \geq R_i^+ \) and \(-v_i \geq -R_i^+\) hold for all \( i=1,...,m\). Hence, it is concluded that \( v_i = R_i^+ \) for all \( i=1,...,m\). We therefore replace \( v_i \) with \( R_i^+ \) in model (2). So, we have the following model:

\[
\sum_{i=1}^{m} R_i^+ x_d + \text{Min} \quad -\sum_{i=1}^{m} u_i g_d + \sum_{f=1}^{h} w_f b_f + \sigma \\
\text{s.t.} \quad \sum_{i=1}^{m} R_i^+ x_d - \sum_{r=1}^{s} u_r g_d + \sum_{f=1}^{h} w_f b_f + \sigma \geq 0 \quad (j=1,...,n), \\
u_r \geq R_r^+ \quad (r=1,...,s), \\
w_f \geq R_f^+ \quad (f=1,...,h).
\]

This replacement is very important because it reduces the number of the constraints and the variables of model (2).

In the following two theorems, we prove two properties of model (3). These theorems characterize a supporting hyperplane in the model. This will help us to determine the amount of ESE measure and the type of ERTS in the next sections.

**Theorem 1:**
In the optimality of model (3), at least one of the constraints of the first group for \( j \in \{1,...,n\} \) is binding.

**Proof:**
Let an arbitrary optimal solution of model (1) be \( (\lambda_i^+, s_i^{s+}, s_i^{s-}, s_i^{b+}, s_i^{b-}) \). \( \lambda_i^+ \neq 0 \) since \( \sum_{j=1}^{n} \lambda_j = 1 \).

Suppose that \( \lambda_i^+ > 0 \). Consequently, by the complementary slackness conditions, the constraint corresponding to \( \lambda_i^+ \) in dual problem (2) must have zero slack. So, we have:

\[
\sum_{i=1}^{m} v_i^* x_d - \sum_{r=1}^{s} u_r^* g_d + \sum_{f=1}^{h} w_f^* b_f + \sigma^* = 0
\]

Here, \( (v_i^*, u_r^*, w_f^*, \sigma^*) \) is an optimal solution for dual problem (2). Since \( v_i^* = R_i^+ \) for \( i=1,...,m \), then:

\[
\sum_{i=1}^{m} R_i^+ x_d - \sum_{r=1}^{s} u_r^* g_d + \sum_{f=1}^{h} w_f^* b_f + \sigma^* = 0
\]

Therefore, the \( j \)th constraint of model (3) is binding, and the proof is complete. \( \square \)

**Theorem 2:**
Let an arbitrary optimal solution of model (3) be \( (u_r^*, w_f^*, \sigma^*) \). If the \( k \)th DMU is efficient, then it is on the hyperplane
\[
H = \left\{ (x, g) \in \mathbb{R}^m \times \mathbb{R}^h \left| \sum_{i=1}^{m} R_i^x x_i - \sum_{i=1}^{m} u_i^* g_i + \sum_{j=1}^{h} w_j^* b_j + \sigma^* = 0 \right. \right\}
\]

If the \( k \)th DMU is inefficient, then its projection point is on the hyperplane \( H \).

**Proof:**

Suppose that the \( k \)th DMU is efficient. So, all of the slack variables are zero on the optimality of model (1), and hence, the amount of the objective function of model (1) is zero. According to the strong duality theorem, the amount of the objective function of the dual problem is zero, too. So, we have:

\[
\sum_{i=1}^{m} R_i^x x_i - \sum_{i=1}^{m} u_i^* g_i + \sum_{j=1}^{h} w_j^* b_j + \sigma^* = 0
\]

Hence, \( (x_k, g_k, b_k) \) is on the hyperplane \( H \), and \( DMU_k \in H \).

Now suppose that the \( k \)th DMU is inefficient. Consider \( R = \) Reference set of \( DMU_k = \{ j \mid \lambda_j^* > 0 \} \). Then, the projection point of \( DMU_k \) is \( (\hat{x}_k, \hat{g}_k, \hat{b}_k) = \left( \sum_{j \in R} \lambda_j^* x_{j}, \sum_{j \in R} \lambda_j^* g_{j}, \sum_{j \in R} \lambda_j^* b_{j} \right) \). Since for all \( j \in R \) we have \( \lambda_j^* > 0 \), then the \( j \)th constraint in the dual problem must have zero slack. Therefore, we have for all \( j \in R \):

\[
\sum_{i=1}^{m} R_i^x x_i - \sum_{i=1}^{m} u_i^* g_i + \sum_{j=1}^{h} w_j^* b_j + \sigma^* = 0
\]

Multiplying each of the above equations by \( \lambda_j^* \), and adding them together, we obtain:

\[
\sum_{i=1}^{m} R_i^x (\sum_{j \in R} \lambda_j^* x_{j}) - \sum_{i=1}^{m} u_i^* (\sum_{j \in R} \lambda_j^* g_{j}) + \sum_{j=1}^{h} w_j^* (\sum_{j \in R} \lambda_j^* b_{j}) + \sigma^* (\sum_{j \in R} \lambda_j^*) = 0
\]

Then, \( \sum_{i=1}^{m} R_i^x \hat{x}_k - \sum_{i=1}^{m} u_i^* \hat{g}_k + \sum_{j=1}^{h} w_j^* \hat{b}_j + \sigma^* = 0 \), and \( (\hat{x}_k, \hat{g}_k, \hat{b}_k) \in H \), and the proof is complete. □

**The Environmental Scale Economies:**

The Scale economies (SE) is an economic measure which is defined as "an increase in a sum of weighted outputs due to a proportional increase in all inputs." (Baumol et al. [1] and Forsund [6]). To discuss the concept of SE, consider a production possibility set in the case of one input (\( x \)) and one desirable output (\( g \)). See Fig. 1.

Fig. 1: The Scale Economies.

Suppose that \( a \) is projected onto \( a' \). Here, the supporting hyperplane is \( v^* x - u^* g + \sigma^* = 0 \), which is passing through \( a' \). The SE measure, which is called the scale elasticity (\( e \)) in the case of one input and one output, is measured by \( e = \left( \frac{dg}{dx} \right) \left( \frac{g}{x} \right) \). This formula is the ratio of the marginal productivity to the average...
productivity. Since the supporting hyperplane is $v^* x - u^* g + \sigma^* = 0$, we have $\frac{dg}{dx} = v^*$ and $g = \frac{v^*}{u} + \frac{\sigma^*}{u}$. Hence, the scale elasticity is equal to $e = \left( \frac{dg}{dx} \right) \left( \frac{g}{x} \right) = 1/\left( 1 + \frac{\sigma^*}{v^* x} \right)$. Since $v^* x > 0$, hence, the degree of the scale elasticity depends upon the sign of the intercept of the hyperplane $\sigma^*$.

Consequently, the type of the RTS is determined as follows:

a) $e > 1 \Leftrightarrow \sigma^* < 0 \Leftrightarrow$ Increasing RTS,

b) $e < 1 \Leftrightarrow \sigma^* > 0 \Leftrightarrow$ Decreasing RTS,

c) $e = 1 \Leftrightarrow \sigma^* = 0 \Leftrightarrow$ Constant RTS.

For multiple inputs and desirable outputs, the scale elasticity ($e$) is extended and the concept of SE (Scale Economies) is defined. Here, we denote the scale economies in the case of desirable outputs by $SE_g$ to distinguish it from the scale economies in the case of undesirable outputs $SE_b$. According to Sueyoshi [11] (p. 1603), it can be measured for $DMU_k$ as follows:

$$SE_g = \frac{v^* x_k}{u^* g_k} = \frac{v^* x_k}{v^* x_k + \sigma^*} = \frac{1}{1 + \frac{\sigma}{v^* x_k}}$$

Here, $u^* g_k$ is replaced with $v^* x_k$ because it is supposed that $DMU_k$ is efficient, so it is on the hyperplane $v^* x_k - u^* g_k + \sigma^* = 0$. Based upon the sign of the intercept of the hyperplane $\sigma^*$, the $SE_g$ measure and the type of the RTS are determined as follows:

a) $SE_g > 1 \Leftrightarrow \sigma^* < 0 \Leftrightarrow$ Increasing RTS,

b) $SE_g < 1 \Leftrightarrow \sigma^* > 0 \Leftrightarrow$ Decreasing RTS,

c) $SE_g = 1 \Leftrightarrow \sigma^* = 0 \Leftrightarrow$ Constant RTS.

Extending the concept of $SE_g$ to $SE_b$ for multiple inputs and undesirable outputs, we define it for $DMU_k$ as follows:

$$SE_b = \frac{v^* x_k}{w^* b_k}$$

We now extend the concept of SE to a situation in which there are multiple inputs, multiple desirable outputs and multiple undesirable outputs. In this case, we call SE the Environmental scale economies (ESE). We define the ESE measure as follows:

$$ESE = \frac{SE_g}{SE_b} = \frac{v^* x_k / u^* g_k}{w^* b_k} = \frac{v^* x_k / w^* b_k}{u^* g_k}$$

According to the above ratio, the environmental scale economies is defined as "an increase in a sum of weighted undesirable outputs due to a proportional increase in a sum of weighted desirable outputs". Based upon the value of ESE, the type of the environmental returns to scale (ERTS) is determined as follows:

a) $ESE > 1 \Leftrightarrow$ Increasing ERTS,

b) $ESE < 1 \Leftrightarrow$ Decreasing ERTS,

c) $ESE = 1 \Leftrightarrow$ Constant ERTS.

In the above discussion, the returns to scale in the case of undesirable outputs was called the environmental returns to scale (ERTS). The increasing ERTS implies that a unit increase in desirable outputs produces undesirable outputs "less proportionally" than the unit increase in desirable outputs. This means that if an organization increases its current size, it produces more proportional desirable outputs. Therefore, the suggested strategy is that the organization should increase the current size.
The decreasing ERTS means that a unit increase in desirable outputs produces undesirable outputs "more proportionally" than the unit increase in desirable outputs. Then, the suggested strategy is that such an organization should decrease the current size in order to produce less pollution.

The constant ERTS means that a unit increase in desirable outputs results in a proportional increase in undesirable outputs. Therefore, such an organization has an acceptable size, and it is recommended that it should maintain its current size.

**Measurement of environmental returns to scale:**

This section attempts to explain how to measure the environmental returns to scale using the model (1). Here, the type of the environmental returns to scale only for efficient DMUs is discussed. Since the inefficient DMUs have waste in their inputs or shortfall in their outputs and they should first improve their inputs or outputs.

Now suppose that $DMU_{k}$ is efficient. According to theorem 2, it is on the hyperplane $H$. Hence, we have:

$$\sum_{i=1}^{h} R_i^+ x_{ik} - \sum_{i=1}^{h} u_i^+ g_{iak} + \sum_{j=1}^{k} w_j^+ b_{jk} + \sigma^* = 0$$

In the previous section, we defined ESE as "an increase in a sum of weighted undesirable outputs due to a proportional increase in a sum of weighted desirable outputs". Consequently, we have:

$$ESE = \frac{\sum_{j=1}^{k} w_j^+ b_{jk}}{\sum_{i=1}^{h} u_i^+ g_{iak} - \sum_{i=1}^{h} w_i^+ b_{ik} + \sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik}} = \frac{1}{1 + \left(\frac{\sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik}}{\sum_{j=1}^{k} w_j^+ b_{jk}}\right)}$$

Based upon the sign of the term $\sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik}$, the degree of the scale economies and the type of ERTS is determined as follows:

a) $ESE > 1 \iff \sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik} < 0 \iff$ Increasing ERTS.

b) $ESE < 1 \iff \sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik} > 0 \iff$ Decreasing ERTS,

c) $ESE = 1 \iff \sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik} = 0 \iff$ Constant ERTS.

If model (1) has multiple optimal solutions,

a) Increasing ERTS prevail at $DMU_{k}$ if and only if $\sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik} < 0$, for all optimal solutions of model (1),

b) Decreasing ERTS prevail at $DMU_{k}$ if and only if $\sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik} > 0$, for all optimal solutions of model (1),

c) Constant ERTS prevail at $DMU_{k}$ if and only if $\sigma^* + \sum_{i=1}^{m} R_i^+ x_{ik} = 0$, for at least one optimal solution of model (1).

To solve this problem, we need to identify the sign of the maximum amount of $\sigma^*$ and the minimum amount of $\sigma^*$. Therefore, we introduce the two following problems:
\[
\sigma = \text{Max} \quad \sigma \quad (\sigma = \text{Min} \quad \sigma) \quad (4)
\]

\[
s.t. \quad \sum_{i=1}^{m} R_i^k x_{ij} - \sum_{j=1}^{n} u_j g_{ij} - \sum_{f=1}^{h} w_f b_{if} + \sigma \geq 0 \quad (j = 1, \ldots, n),
\]

\[
\sum_{i=1}^{m} R_i^k x_{ik} - \sum_{r=1}^{s} u_r g_{ir} + \sum_{f=1}^{h} w_f b_{if} + \sigma = 0,
\]

\[
u_r \geq R^k_r \quad (r = 1, \ldots, s),
\]

\[
w_f \geq R^b_f \quad (f = 1, \ldots, h).
\]

After solving the two above models, we let:

\[\bar{\sigma} = \sigma + \sum_{i=1}^{m} R_i^k x_{ik} \quad \text{and} \quad \Delta = \bar{\sigma} + \sum_{i=1}^{m} R_i^k x_{ik}.\]

Based upon these two values, the type of ERTS is determined as follows:

a) If \(\Delta < 0\), then Increasing ERTS prevail at \(DMU_k\).

b) If \(\Delta > 0\), then Decreasing ERTS prevail at \(DMU_k\).

c) Under other circumstances, Constant ERTS prevail at \(DMU_k\).

Based on the above models, we have the following algorithm for determining the type of ERTS for \(DMU_k\).

1) Solve model (3). Suppose that \((u^*, w^*, \sigma^*)\) is an optimal solution of model (3) for evaluating \(DMU_k\).

2) If \(\sigma^* + \sum_{i=1}^{m} R_i^k x_{ik} = 0\), then Constant ERTS prevail at \(DMU_k\), and end.

3) If \(\sigma^* + \sum_{i=1}^{m} R_i^k x_{ik} > 0\), then solve model (4) and determine the amount of \(\Delta\). If \(\Delta > 0\), then Decreasing ERTS prevail at \(DMU_k\), else Constant ERTS prevail at \(DMU_k\), and end.

4) If \(\sigma^* + \sum_{i=1}^{m} R_i^k x_{ik} < 0\), then solve model (4) and determine the amount of \(\bar{\sigma}\). If \(\bar{\sigma} < 0\), then Increasing ERTS prevail at \(DMU_k\), else, Constant ERTS prevail at \(DMU_k\), and end.

Therefore, we need to solve at most two linear programming problems to determine the type of ERTS of each DMU.

**Numerical example:**

Consider the data set in table 1, adopted from Wu et al.’s article [17]. This data set is associated with the industry in 31 administrative regions of China. These data have two inputs: the total investment in fixed assets of industry (TIFA) and the electricity consumption by industry (EC), one desirable output: the gross industrial output value (GIOV), and two undesirable outputs: the total volume of industrial waste gas emission (TWGE) and the total volume of waste water discharge (TWWD).

Here, the GAMS (General Algebraic Modeling System) software is utilized for the computations. Table 2 exhibits the computational results for determining the type of ERTS. The first column from the left shows the amounts of the unified efficiency scores measured by model (1). As it can be seen from this column, six industries (Anhui, Beijing, Guangdong, Jiangsu, Shanghai, and Tibet) attain full efficiency in their performances. This result confirms that these six industries pay attention to decreasing their undesirable outputs along with increasing their desirable output.

The other columns from left to right show the amounts of \(\sigma^* + \sum_{i=1}^{m} R_i^k x_{ik} \cdot \Delta\), and \(\bar{\sigma}\), respectively. We have calculated these amounts only for the six efficient industries, since the type of ERTS is only being discussed for efficient DMUs.

The last column indicates that Tibet belongs to increasing ERTS. Also, Beijing, Jiangsu, and Shanghai and Guangdong Shanghai exhibit decreasing ERTS; therefore, they should decrease their current sizes to avoid producing undesirable outputs more proportionally than desirable output. Since Anhui and Tibet belong to increasing ERTS, they could produce desirable outputs more proportionally than undesirable outputs. Then, it is recommended that they should increase their current sizes.
Table 1: Data set of industry of China in 2010.

<table>
<thead>
<tr>
<th>District</th>
<th>DMU</th>
<th>TIFA</th>
<th>EC</th>
<th>GIOV</th>
<th>TWGE</th>
<th>TWWD</th>
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<td>Anhui</td>
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<td>9121.829</td>
<td>1077.91</td>
<td>18732</td>
<td>17849</td>
<td>70971</td>
</tr>
<tr>
<td>Beijing</td>
<td>D2</td>
<td>4554.356</td>
<td>809.9</td>
<td>13699.84</td>
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<td>8198</td>
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<td>D3</td>
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Summary and conclusion:

This study discussed measuring returns to scale for environmental assessment when we need to incorporate undesirable outputs into performance evaluation. Here, we employed the linear unified efficiency measure presented by Zare Haghighi et al. [18]. Using the dual problem of this model, we characterized a supporting hyperplane at efficient points of model (1) and defined the concept of ESE (Environmental Scale Economies). This concept was further extended to the measurement of the type of ERTS (environmental returns to scale) for environmental issues.

An important feature of this study was that it introduced new concepts of ESE and ERTS in environmental assessment. It is widely known that in the last few decades, many researchers have utilized DEA for environmental assessment. With the exception of Sueyoshi and Goto [14], none of the DEA-based studies take notice of measuring returns to scale for environmental issues.

In section 5, the proposed method for determining the type of ERTS was applied to analyze the industry in 31 administrative regions of China. Applying model (1) to these data, the efficient industries were identified, and the type of their ERTS was determined. The results showed that one industry (Tibet) belonged to increasing ERTS, and it was suggested that it should increase its current size. In this case, it will become more productive. Our recommendation for the other four industries (Beijing, Guangdong, Jiangsu and Shanghai) which belonged to decreasing ERTS was that they should decrease their current sizes to avoid producing undesirable outputs more proportionally than desirable outputs. In this case, they will generate less pollutant and waste.
Table 2: Results of the type of ERTS.

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