Application of Genetic Algorithms in Mathematics Programming

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ABSTRACT

Genetic algorithms are optimizing algorithms, inspired by natural evolution. Investigations on genetic algorithms reveal that these algorithms are different from other search-based optimizing methods. In most optimizing techniques based on a point, the analysis is done according to only some of the decision-making regulations. These techniques could yield an incorrect answer in the searching spaces having several maximum points. In other words, it is possible that the local maximum point be obtained as the answer. Hence, genetic algorithms could also be used in mathematical programming. The common techniques utilized in this field are not effective since they need a series of limitations such as functions continuity and differentiation to be optimized. Moreover, there is no originality in these techniques and this is why the genetic algorithm method could be used in these cases, especially for non-linear programming to reach desirable outcomes.

Key words: Genetic Algorithm, Linear, Non-Linear, Programming, Optimization.

Introduction

Genetic algorithms produce a complete population of answering points. Each point is tested independently and to establish new populations, including modified points, existing points merits could be tested [7]. Genetic algorithms consider many points simultaneously and these characteristics conform them with more parallel processors, since considering each point needs some calculations including the target function differentiation, etc [24].

In this algorithm, different operands and mechanisms are implemented, that is described here to invention of genetic algorithms as optimizing algorithms [3] is mainly according to natural evolution simulation and they have not been based on strong mathematical theories. This technique uses natural genetics, statistical methods [6] and a type of creativity and tries to reach on optimized conclusion by the use of these tools [1].

At the moment, there are 3 general methods for optimizing search that include: analytical, enumeration and random [22]. Analytical techniques have been studied extensively [8]. They are divided into two direct and indirect categories [15,4]. The indirect methods obtain the optimized points by solving a set of non-linear equations that are achieved by putting the target function differentiation equal to zero [13]. The direct methods search the optimized points by the target function and moving in gradient direction [10]. These two techniques are optimized and developed but some proofs to be described later show that they are incapable of optimizing properly. The reasons include [4]:

1. Both techniques have a local field. The optimized point these techniques are searching is the best point in the neighborhood of the present point. For example, if we have a function with two peaks, of which one is false and one is real, then reaching the smaller peak cause us to be hindered from the real point. Therefore, we should use other methods, such as random restarting with other loops to achieve the best response.

2. Analytical techniques depend on existence of differentiations. Even if we use an approximate numerical differentiation in the calculations, we will end up with an intensive shortage. Therefore it is clear therewith that the techniques based on continuation limits and derivatives are not suitable for every case.

Although the random search algorithms have more popularity and perform better than the other two methods, but they do not operate appropriately, when the responsive domain is large. But the genetics algorithm has all the advantages of the mentioned methods, as well as not having their disadvantages, and uses creativity, which is mostly evident in human attitudes. It also gets the aid of probability functions as its tool. Hence the differences of the other techniques with genetic algorithm are dividing into four categories [24]:

a) Genetic algorithms with a set of coding.

b) Genetic algorithms with a set of points, that start search from not only a point.
Non-linear programming:

Most economists [2] have realized that non-linearity of functions in economic programming is not considered as exception, but it is quite general [21]. Therefore, the extent of applying linear programming requires this important subject to be considered, too.

The aim of non-linear programming, as a whole is finding the values of \( x = (x_1, x_2, \ldots, x_n) \), such that [13]:

Maximize \( F(X) \)

\[

g_i(x) \leq b_i \quad \text{for} \quad i = 1, 2, \ldots, m
\]

\( x \geq 0 \)

Where \( f(x) \) and \( g_i(x) \) are definite functions from \( n \) deciding variables.

There is no algorithm capable of solving all the problems within the above framework but by applying specific theories and assumptions about the functions, numerous significant models are obtained that remarkable developments have been achieved for their solutions and new investigations are being done. The extent of non-linear programming [14] is such that not all its types could be considered and therefore some examples of non-linear programming applications are to be analyzed in this part and compared with genetic algorithm technique [17].

Comparison of steepest descent method with genetic algorithm:

As mentioned, this technique uses function gradient to achieve the target and optimum solution, which in turn requires derivability and continuity of the function in all the slope, since if the function does not have such characteristics, we will confront difficulties in obtaining the gradient and in this case, this technique is not applicable, while the genetic algorithm technique has no dependence to derivability and continuing of the function and starts by selecting some points in the slope, and searches the optimal answer by using the primary points. Moreover, this technique does not need additional calculations, such as differentiation and obtaining gradients and also replacing values in complex functions. Meanwhile the solution of different problems by both methods, empirically, indicates that the answer is obtained in a shorter time by genetic algorithm (implementing genetic algorithm technique is done by “C” programming, and implementation of Steepest descent is by BASIC language). Hence, by the above comparisons we conclude that genetic algorithm is a more efficient method than steepest descent [23].

For example, to minimize:

\[
F(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + 10(x_1 - x_4)^3 + (x_2 - x_3)^4
\]

We used the program consisting of Steepest descent method and started the point \((1,0,-1,3)\) as the starting point, and we reached the minimum of 3.663253664E-13 with the following coordinates:

\[
x_1 = 1.550654466E - 04 \quad x_2 = - 1.773445193E - 05 \quad x_3 = 3.891654581E - 04 \quad x_4 = 3.082981066E - 04
\]

On the same computer used previously, and after some repeated procedures after 482ms, although the real min, point is the point \((0, 0, 0, 0)\) and the genetic Algorithm technique obtained the answer after 320ms.

The justification made for this result is that since the genetic Algorithm technique uses the primary population, randomly, it would be possible for the primary selected points to be close to the optimal and we, therefore, could get to the result very soon. In such conditions that the primary points are selected...
farther than the optimal points, it takes more time to get to the answer.

Comparison of Dividen – Fletche – POWELL METHOD with genetic algorithm technique:

As discussed about quadratic functions and assuming continuity, the function starts approximation and obtaining the next points. Thus, continuity of the function is a vital and inseparable condition to get to the answer by such methods. But Dividen-Fletcher-Powell method [20] has a great advantage comparing to the previous method, which is that we needed to obtain Housian matrix and also its inverse case in calculations, that required a long time, but fortunately the aforementioned method does not need those calculations, however, it seems that the derivability and continuity of the function in this method are intensely required. Moreover, this method has complicated and long calculations that may cause inaccuracy in the results, since according to these program logics, the linear search is done by 3rd order internal search and the searches are not continued up to the final convergence. Instead, a smaller amount for the function is searched, which of course it means that the convergence characteristics related to the quadratic functions in this method are lost. But the genetic algorithm technique does not have any of the defects and does not use any approximation in obtaining the value of the functions. Hence, the genetic algorithm technique in this case is a more efficient method, too. For example, to minimize:

\[
F(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + 10(x_1 - x_4)^4 + (x_2 - x_3)^4
\]

We used the program consisting of Dividen-Filcher-Powell method and started the point (1,0,-1,3) as the starting point, and we reached the minimum of 3.046283678E-13 with the following coordinates:

\[
x_1 = 1.8568677E - 04 \\
x_2 = -1.85806523E - 05 \\
x_3 = 3.51518448E - 04 \\
x_4 = 3.51564962E - 04
\]

Comparison of genetic algorithm technique with simplex method:

Simplex method [9] is a useful method that could be used for a wide variety of optimizing problems. But it should not be viewed as the general remedy, in this aspect. With convex target function and constraint area, this technique could be successful, although it is possible that specific characteristics of a problem cause us to slightly change the pausing criterion. If the target function is concave or if the constraint area is not convex, it could simply be observed how this method could fail. Therefore, this technique is not comparable with genetic algorithm, since it could be applied for only one field of problems and is not applicable for concave functions, although genetic algorithm has no limits. Hence the genetic algorithm is suggested here, too.

Comparison of Fletcher-Reeves method with genetic algorithm:

The direction of sequential search in Fletcher-Reeves method [16] is pained and this method finds the minimum of quadratic function with “n” variables, after “n” searches. This is particularly true if the linear searches are properly and accurately done and there is no error due to rounding up. Of course this method is used for non-quadratic functions, too. When the approximation gets close to minimum, such that the 2nd order approximation is allowed, then we will have the 2nd order convergence characteristic. Fletcher and reeves suggest that in this case the position of the nth search should be in the direction of the Steepest descent. Also the construction of paired directions should start all over again, in which case it will possess all the defects of the used methods and hence it will not have the applicability of Fletcher-Powell technique. Therefore, obviously the generic algorithm will be more efficient as compared to this method. For example, to minimize:

\[
F(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + 10(x_1 - x_4)^4 + (x_2 - x_3)^4
\]

We used the program consisting of Steepest descent method and started the point (1,0,-1,3) as the starting point, and we reached the minimum of 3.79638338E-13 with the following coordinates:

\[
x_1 = 1.146311886E - 04 \\
x_2 = 1.786272352E - 05 \\
x_3 = 3.824374431E - 04 \\
x_4 = 3.835257332E - 04
\]

On the same computer used previously, and after some repeated procedures after 480ms, although the real min, point is the point (0, 0, 0, 0) and the Genetic Algorithm technique obtained the answer after 320ms.

Comparison of genetic algorithm technique with Hook and jeeves evolved technique and its comparison with genetic algorithm method:

The previous methods that were used for solving unconstraint optimizing problems were not effective. It could be imagined that they could be changed in such a way to include the constraints, too. Indeed, it
is suggested that only the relativity of very large values to the target function (in minimizing problem) is sufficient, when the constraints are not considered. Certainly, this idea is an evident and practical suggestion and could be programmed simply. The examining point is considered whether it is within the constraint area or not.

If it is within the area, than the target function is calculated normally and otherwise, we consider a very large value for the target function. In this way, the searching would be possible and conducted towards the main point within the area. But this method is not a completely logical method, since sometimes, it reaches to wrong answers [12]. For example, to minimize:

\[
F(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + 10(x_1 - x_4)^4 + (x_2 - x_3)^4
\]

We used the program consisting of Steepest descent method and started the point (1, 0, -1, 3) as the starting point, and we reached the minimum of 3.046283678E-13 with the following coordinates:

\[
x_1 = 1.235485796E - 04 \quad x_2 = -1.164908927E - 05 \\
x_3 = 3.527013664E - 04 \quad x_4 = 3.186153137E - 04 
\]

On the same computer used previously, and after some repeated procedures after 450ms, although the real min, point is the point (0, 0, 0, 0) and the genetic Algorithm technique obtained the answer after 320ms.

Results and Discussion

Linear programming is a valuable technique, dealing with limited resource allocation to competitive activities and also other problems that their mathematical models are similar to allocation problems. This technique is considered as an important and competent tool for industrial and commercial organizations. Simplex method is a competent and effective algorithm that solves problems regarding linear programming, with hundreds or thousands of limitations and variables, by the aid of computer.

Nevertheless, all the allocation problems could not be formulized, even with logical approximation, with linear programming models. However, all the linear programming problems could be solved by genetic algorithm method, but we get to the answer in most cases, later than the ordinary methods. Therefore, we do not propose genetic algorithm technique for solving linear programming problems. Some general conclusion could be observed by the analysis of all the techniques and all these techniques have common problems and some specific matters. But, genetic algorithm has none of these defects and moreover, the speed of operation in most of these methods is low, due to some complicated operations that increases the calculation time, but the calculation time in genetic algorithm is very low. About the defects of the ordinary techniques and the advantages of genetic algorithm could be stated that all these methods depend on the existence of derivatives, which by itself is a imitation, since even if we use approximate numerical derivatives in the calculations, we will end up with an intensive shortage. Meanwhile, some of the scientific spatial parameters have a fewer relations, for expressing a derivative. It is why that the techniques, based on continuing limits and existence of derivatives, are not suitable for every type of problems. Moreover, since these techniques are the search methods, searching the vest neighboring point to the present one, it would be possible that a false peak or a false optimal point prevent us from reaching the real optimal point. Therefore, to solve these problems, non-linear functions are suggested and this is where the genetic algorithm is used, since it has none of the above problems, and secondly, it has a higher accessing speed to reach the optimal point, as compared to ordinary techniques. It also has higher operating speed, since there is no need, in genetic algorithm for differentiation calculations and extra operations, and thirdly, it has all the advantages of genetic algorithm, which include:

1- A set of codes are used in genetic algorithm, instead of parameters and in each time the algorithm

![Fig. 1: Comparison between methods by solving times (ms).](image-url)
is repeated, these codes may change and indeed a parameter is changed with a better parameter.

2- In genetic algorithms a set of points is used instead of one.

3- Genetic algorithm uses the target functions to obtain the final result, not the derivatives.

4- Genetic algorithm uses probability laws, instead of a specific law.

The dynamic programming method has problems, too that includes finding the appropriate revocable relation for each specific problem that causes the lack of having a general method to solve such problems and since no general procedures could be found in this respect, then no algorithm and computer programs could be invented to solve such problems in classical way. Moreover, we observed in this method that we started from one process and by using the target function; we decided which others process to tend to. Again, such problems are referred to the target function and its continuity or discretion, and whether it is derivable in all points. If the target function is non-linear, then the previous section discussion will be considered and the methods to solve the non-linear functions that as defined, using the algorithm technique is the optimal way to solve the problems are also taken into account.

References


