Optimal Design of Water Distribution Networks considering Reliability Based on Variance of Discharge Distribution

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ABSTRACT

This paper introduces a method for optimal design of water distribution networks with desired reliability. The design approach consists of two steps: the first step involves an algorithm in which pipe flows are so distributed that the variance of flows is minimized and the second step uses a non-linear optimization formulation to obtain a global optimal design. A comparison among classical optimal design methods and the proposed method is illustrated through a detailed simple example.

Key words: Water distribution network, optimization, variance, reliability, optimization.

Introduction

In the last three decades, a significant number of researches for the design of water distribution networks have been developed. One approach used in these researches was the optimization of the water supply network without consideration of reliability. Several techniques in this regards have been proposed. The common method of optimization in the context of water distribution network design is linear programming. Gupta [11], Gupta and Hassan [12], Alperovits and Shamir [1], Quindry et al. [22] and Bhave and Sonak [2] used linear programming. Kessler and Shamir [15] have proposed the linear programming gradient method for optimal design of water supply networks. For this purpose, other investigators such as Chiplunkar et al. [4], Walski [34], Ormsbee [20], and Samani and Naeini [27] also used nonlinear programming. Walski et al. [35] developed the WADISO pipe network design program in which the hydraulic analysis was linked to the linear programming for optimization analysis. Also, Morgan and Goulter linked a Hardy-Cross network hydraulic analysis with linear programming while Zick employed GIS and AUTO-CAD and linked them with the WADISO computer program. Taher et al. developed a computer program in which a nonlinear programming algorithm is used for hydraulic analysis of the network coupled with a linear programming algorithm through GIS. Simpson et al. and Savic and Walters [29] used genetic algorithms which represents a search method for nonlinear optimization problems. The latter has the merit of reducing the tendency to become entrapped in local minima. Cunha and Sousa [5] employed the simulated annealing approach for optimization of water distribution networks. Mair and Simpson employed the ant colony optimization method, Eusuff and Lansey used the shuffled frog leaping algorithm and Liong and Atiquzzman used the shuffled complex evolution method. In all these methods, only pipe sizes are considered as decision variables in the optimization analysis. Samani and Mottaghi [26] used the integer linear programming for obtaining the optimum pipe sizes and reservoir elevations in pipe networks. Samani and Zanganeh [28] modified the integer linear method to a simpler one in which the integer-real-linear programming is used.

Application of optimization algorithm based on minimization of the design cost leads to the opening of the loops, giving rise to pseudolooped networks [36,1,24,30,10,17,16]. This type of loop will not behave properly because the minimum diameter pipe is not of sufficient capacity to convey large flows when any of other loop pipes have to be temporarily drawn out of service. This leads researchers to the subject of reliability based optimization of water distribution networks pointed out by [9,10]. In this method, the optimization algorithm is combined with
a method for estimating reliability. Goulter and Bouchart [10] have solved a reliability constrained least cost optimization problem. Multiobjective optimization formulation is another approach which is able to simultaneously consider the cost and reliability of the network. In a multiobjective optimization problem, there exists a set of solutions called Pareto optimal solutions [14]. The multiobjective optimization method has been employed by (Halhal et al., [13], Xu and Walters [37], Toodini [31], Tolson [32], Farmani et al., [6], Moneim [19]). It must be recognized that optimization only assist the designer to decide about the best solution and engineering judgment and experience is still required to provide a practicable solution. The Pareto solutions give the designer more flexibility in the selection of practicable design. Toodini [31] introduced the concept of resilience index which is a measure of the capability of the network to cope with failures of pipes and is related indirectly to the network reliability. A third method of optimization of water distribution networks has been proposed by Martinez [18]. In this method, a single objective function in which a new term represents costs of reliability is added to the total cost. This term accounts for the expected annual cost involved in a pipe breakage. This expected cost includes the cost of failure repair and the cost of supplying affected consumers. Every time a pipe breakage occurs, the broken pipe should be removed from the network and the network has to be analyzed in order to determine the total head required to convey water for the affected consumers and consequently the costs of pumping and repairs. This should be done in a number of the pipes that are expected to break and the costs have to be summed. It is obvious that this method requires huge calculations and therefore it is time consuming.

In this paper a simple method has been proposed for the design of water distribution networks, in which adequate desired reliability is considered and requires one time analysis of the network.

Material and Methods

Proposed Model:

The first step in the proposed model is to calculate pipe flows under maximization of their uniformity which can be achieved by dividing flows in pipes in such a way as to minimize the variance of them. Having obtained the pipe flows, pipe diameters are determined via the minimization of the total cost subject to pressure constraints. As it is clear the proposed model results in an optimum and reliable network. This is due to the fact that the total cost of the water distribution network is minimized and pipe flows are uniformly distributed which in its term keeps the network from having opening of loops.

Determination of Pipe Flows:

The variance of the pipe flows can be calculated as:

\[ \text{Variance} = \frac{1}{NP} \sum_{k=1}^{NP} (Q_k - \overline{Q})^2 \]

where \( NP \) = number of pipes in the network; \( Q_k \) = flow in any pipe \( k \); and \( \overline{Q} \) = average of pipe flows.

The function given in equation (1) is considered as the objective function which should be minimized to obtain the pipe flow distribution subject to the constraints of continuity equations at nodes. This objective function is minimized by the transformed Powells’ nonlinear optimization method (Box, 1966).

Total Cost:

The total cost of a municipal water distribution network includes the annualized capital costs and annual energy cost and can be defined as:

\[ TC = K_1 \beta \sum_{k=1}^{NP} L_k D_k^\beta + C_u t \sum_{s=1}^{NS} Q_s H_s \]

where \( L, D \) = pipe length and diameter; \( NP \) = number of pipes; \( NS \) = number of source nodes; \( k, s \) = subscripts for pipes and source nodes; \( \beta, \beta_s \) = coefficient and exponent, respectively of pipe cost formula obtained by curve fitting; \( K_1 \) = annualizing factor; \( C_u \) = unit price of energy multiplied by the water specific weight; \( t \) = annual pumping time in hours; \( Q_s \) = flow from source node; and \( H_s \) = total head at the source node and is defined as:
\[ H_S = \frac{P_{\min}}{\gamma} + \sum_{i=1}^{M} h_{f_i} \]

in which:

- \(P_{\min}\) = minimum nodal pressure in the network;
- \(h_{f_i}\) = head-loss in pipe; \(i\) = subscript represents the pipes in the path between the source node and the node with minimum nodal pressure in the network.

Substituting for \(H_S\) from equation (3) and for the diameters from Hazen-Williams relation in equation (2), results in:

\[ TC = K_i \beta \sum_{k=1}^{NP} L_k \left( 10.7 L_k C_{\gamma H_k}^{1.82} Q_k^{1.852} / (H_{kup} - H_{kdown}) \right)^{4.871} + C_u \sum_{S=1}^{NS} Q_S \left( \frac{P_{\min}}{\gamma} + \sum_{i=1}^{M} h_{f_i} \right) \]

where \(H_{kup}\) and \(H_{kdown}\) are the heads upstream and downstream of pipe \(k\), respectively and \(H_{kup} - H_{kdown}\) represents the head-loss of that pipe. The decision variables in the above objective function are the nodal heads. Minimization of the objective is performed by Powell's transformed non-linear optimization method [3]. Calculating nodal heads by the referred optimization method, pipe diameters are computed in terms of determined head-losses using Hazen-Williams relation and rounded to the nearest greater commercial ones.

For the purpose of comparison with other common design methods in which optimization and reliability are considered, a simple water distribution network is introduced as an example and designed by three methods. These methods include the proposed model in which the variance of pipe flow and the total cost are minimized, Martinez method in which the cost of failure repair and the cost of supplying affected consumers by other means is added to the total cost objective function [18] and the method in which a multi-objective approach is considered where the objectives are minimization of the network costs and maximization of reliability measure [37,7,8,32]. The example chosen herein is the one introduced by Martinez [18] consists of two loops and is introduced in Figure 1. Length of all pipes is 2000 m, Hazen-Williams coefficient is 100 for all pipes, and elevation of all nodes is 100m. Commercial diameters are available from 200 mm on, in increments of 50 mm. Other data of the network are given in Table 1.

Fig. 1: Two loop example for comparison.

**Design of the Network Using the Proposed Model:**

Distribution of the pipe flows obtained by the minimization of pipe flow variance explained in the proposed model is given in Table 2.

Results of minimization of the objective function of equation (4) are given in Table 3.

<table>
<thead>
<tr>
<th>(K_i)</th>
<th>(\beta)</th>
<th>(n)</th>
<th>(C_u) ($/kw.h))</th>
<th>(t) (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>160.69</td>
<td>1.5</td>
<td>0.05</td>
<td>7000</td>
</tr>
</tbody>
</table>
Design of the Network Using Martinez Method:

The reliability in this method is considered somehow in the total cost, thus the expected annual cost involved in pipe breakage which includes the cost of failure repair and the cost of supplying affected consumers by other means is added to the total cost objective function. The total cost in this method can be expressed as:

\[
TC = K \beta \sum_{k=1}^{NP} L_k D_k^n + C_t \sum_{s=1}^{NS} \frac{Q_{\min}}{\gamma} + \sum_{i=1}^{M} 10.7L C_{H_i}^{\alpha} Q_i^{1.852} D_i^{-4.87} + \sum_{k=1}^{NP} a t_f (C_f + C_f)V_k L_k D_k^n
\]

Variables in the third term of the above equation are explained in (Martinez, 2007) and they are given in Table 4. Results of minimization network total cost are given in Table 5.

Table 4: Data of the third term of equation (5).

<table>
<thead>
<tr>
<th>(a)</th>
<th>(t_f) (day)</th>
<th>(C_f) ($/day)</th>
<th>(C_a) ($/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.27</td>
<td>2</td>
<td>500</td>
<td>2</td>
</tr>
</tbody>
</table>

Design of the Network Using Multiobjective Approach:

In this method a multi-objective approach to design a water distribution network is employed. The objectives considered are minimization of the network costs and maximization of the reliability measure. The network costs considered herein are those introduced in equation (2) and the procedure used to calculate the reliability is that of Gupta and Bhave [12]. In this procedure, the reliability is equal to the multiplication of volume reliability factor, time factor, and node factor which are defined in the following:

Node Reliability Factor = \(\frac{\sum_{j=1}^{V_{\text{available}}} V_{j}}{\sum_{j=1}^{V_{\text{required}}} V_{j}}\) (6)

Volume Reliability Factor = \(\frac{\sum_{s=1}^{S} \sum_{j=1}^{V_{\text{available}}} V_{j}}{\sum_{s=1}^{S} \sum_{j=1}^{V_{\text{required}}} V_{j}}\) (7)

Node Factor = \(\left[ \prod_{j=1}^{I} \right. \text{(Node Reliability Factor)} \left. \right]^{1/J}\) (8)
Time Factor = \frac{\sum_{s} \sum_{j} a_{js} \times t_{js}}{JT}

where the node reliability factor is defined as the ratio of the total available outflow volume at a node to the required outflow volume at that node for all states during the period of analysis, volume reliability factor is the ratio of the total available outflow volume to the required outflow volume for the entire network, and the node factor is the geometric mean of the node reliability factors. \( j \) and \( s \) are subscripts represent node number and state number, respectively. The number of states is equal to the number of different combinations of pipe failures. Thus, a simulator is introduced and run \( NP+1 \) times for all possible states of the network: one state with full layout, and one state with each pipe being considered out of service. The Pareto front for the total costs versus reliability computed by this approach is depicted in Figure 2.

**Fig. 2:** Pareto front of the total cost versus network reliability of the example.

**Conclusions:**

A simple method for the design of water distribution networks with a desired reliability and optimum cost is proposed in this research. The proposed model yields high network reliability by distributing the flows in the pipes in such a way that the variance of them is minimized. The pipe diameters were so calculated that the total network costs is minimized. A simple two-loop network which was solved previously by Martinez has been chosen to be designed by the proposed model and another common method for the purpose of comparison. The other method is the one in which a multi-objective approach is considered. Complexities of Martinez and the multi-objective approach method are respectively the calculation of the expected annual cost in which pipe breakage for all pipes in the network is considered and calculation of network reliabilities for the set of Pareto optimal solutions which clearly includes huge computational operations because of the large number of states of pipe failures.

Network costs and reliabilities of these three methods are given in Table 6 for the purpose of comparison.

Comparison of the proposed and Martinez methods referring to Table 6 indicates that reliabilities have no meaningful difference, but the proposed method shows 4 percent less costs. The multi-objective method introduces reliabilities in the range of 0.69-0.925. This indicates that there is no unique reliability to be considered for the client and additional information are required for an acceptable decision. In addition, comparison of the proposed model costs with minimum costs of the multi-objective approach which has a low reliability shows that the difference is about 4 percent. Therefore it
can be concluded that the proposed model is preferable due to the facts that it needs no complicated and time consuming computations, gives very good reliability with rational costs not much greater than the alternative with the least costs.

References


